Large Scale Flows From Eigenfunction Fitting

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Abstract

It is well known that the eigenfunctions, as well as the eigenfrequencies, of normal modes are perturbed by flows and other asphericities. In the case of meridional flows, it is known that while the perturbations to the frequencies are second order, the eigenfunctions are perturbed at first order in the strength of the meridional flow. This leads to the question of whether one might be able to measure meridional flows using the observed eigenfunction perturbations. Here we discuss how well we expect to be able to do this and show some preliminary analysis of MDI data.

Background

The horizontal dependence of the eigenfunctions of the normal modes of the Sun are often approximated as spherical harmonics. It is however well known that the eigenfunctions, as well as the mode frequencies, are significantly modified by various asphericities. In particular Woodard (1989) and Vorontsov (2007) demonstrated how the eigenfunctions are perturbed by differential rotation while Woodard (2000) calculated the effect of meridional circulation.

Due to symmetry constraints the lowest non-vanishing order for various perturbations are different:

- Differential rotation on frequencies: First
- Differential rotation on eigenfunctions: First
- Meridional flow on frequencies: Second
- Meridional flow on eigenfunctions: First

This difference for the mode frequencies (on which past global mode results have been based) is not present in local helioseismic techniques. The magnitude of the frequency shifts for ring diagrams as well as the time shifts for time-distance are independent of the direction of the flow. Since the global techniques are generally more sensitive than the local techniques this means that the meridional circulation is less well constrained than the rotation.

Given this it seems worthwhile to attempt to constrain the meridional flow by measuring the eigenfunction perturbations. Rather than going directly to fitting for the meridional flow this poster describes our progress in fitting for the differential rotation using the eigenfunction perturbations as well as what needs to be done to fit for the meridional flow.

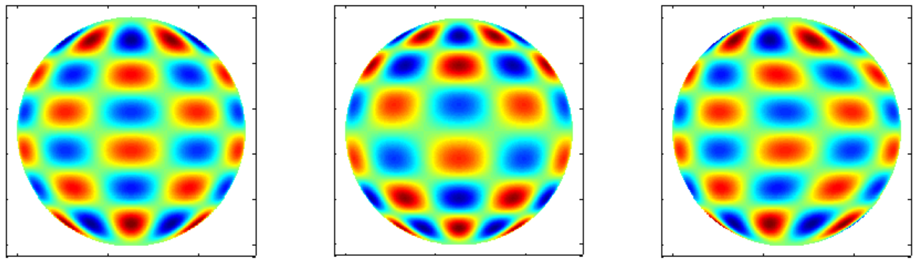

Figure 1: The effect of large scale flows on eigenfunctions. Left panel shows the unperturbed eigenfunction, the middle panel that for differential rotation and the right panel that resulting from a meridional flow. For clarity the magnitudes of the flows were grossly exaggerated.

To illustrate these effects, Figure 1 shows the changes in the eigenfunctions resulting from grossly exaggerated differential rotation and meridional flow.

In the case of the differential rotation a wave crossing the equator at an angle between 0 and 90 degrees to the equator will see a different advection velocity for different parts of the wavefront and will thus turn faster or slower depending on the direction of the differential rotation relative to the direction of propagation. This in turn causes the eigenfunctions to be stretched or compressed in latitude.

In the case of a meridional flow the effect is to introduce a latitude dependent phase factor to the $\exp(im\phi)$ part of the eigenfunction, in other words to advance or retard the temporal dependence.

In both cases the perturbed eigenfunctions can be written as a sum of the unperturbed eigenfunctions. Due to the symmetry of the flows the sums are only over modes with the same m, substantially simplifying the problem.

Fitting Procedure

In order to model the spectra given a known perturbation it is necessary to model several other effects. Most notably we can only observe a fraction of the solar surface which, even in the absence of other perturbations, leads to large leaks (modes with different l and m) in the spectra. Other effects include that we only observe the line-of-sight velocity, optical distortions, the finite PSF and other instrumental effects.

Figures 2 and 3 show model spectra with and without the distortion caused by the differential rotation and compare the results to an observed spectrum.

As can be seen the effect of the differential rotation on the observed spectra is quite noticeable. It is also clear that including the effect improves the fit significantly.

An important point to note is that the sign changes between positive and negative m as well as between positive and negative Δl. This is in contrast to geometric and instrumental problems which in general are symmetric in m.

Instead of proceeding straight to doing a formal fit for the meridional flow a quick proof of concept fit for the differential rotation was performed. This was done partly because the differential rotation is large and partly for technical reasons.

The main effect of the meridional flow is to introduce complex phase terms into the leaks and while the MDI fitting code (Schou 1992) fits the Fourier transforms (as opposed to power spectra) and assumes the correct statistics it currently can't use complex leaks and thus modifications will be required. The MDI code also does not have the capability to fit for the relevant components of the leakage matrix or to fit multiple l's simultaneously The latter is important since the leaks between

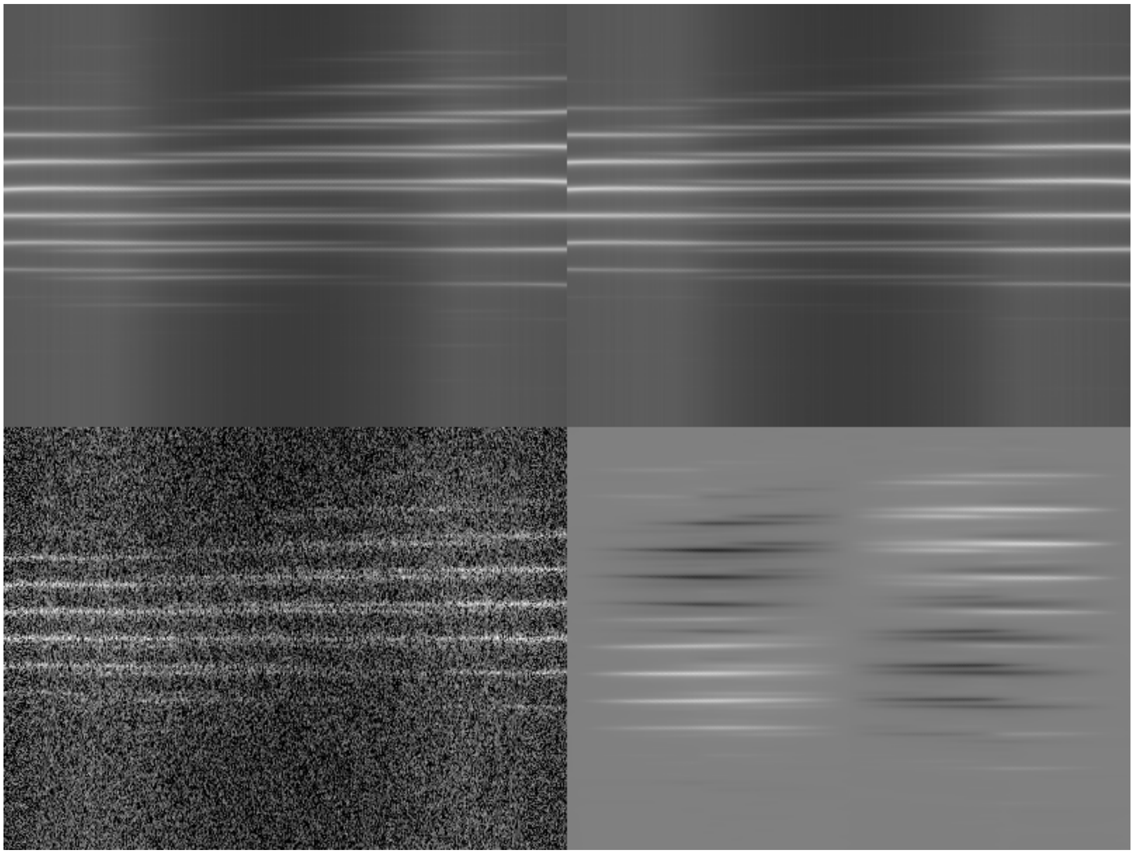

Figure 2: Power spectra for an l=200 f-mode. Each panel has $-200 \leq m \leq 200$ horizontally and frequency $-24\mu HZ < \Delta\nu < 24\mu Hz$ vertically. The spectra were shifted in ν to compensate for the frequency splittings. Panels are from upper left to lower right: Model spectrum including all effects, spectrum without the effect of distortion by differential rotation, an observed spectrum from 72 days of MDI Medium-l data and the ratio of the upper left to the upper right spectrum. The gray scales are logarithmic.

l's are the ones affected by the perturbations and doing a simultaneous fit exploiting the consistency between the l's is thus likely to improve the errors significantly.

On the other hand the signature is likely to be very different from that of other physical, instrumental and data analysis effects and so it is likely to be easy to isolate.

For the differential rotation a 72 day MDI Medium-l time

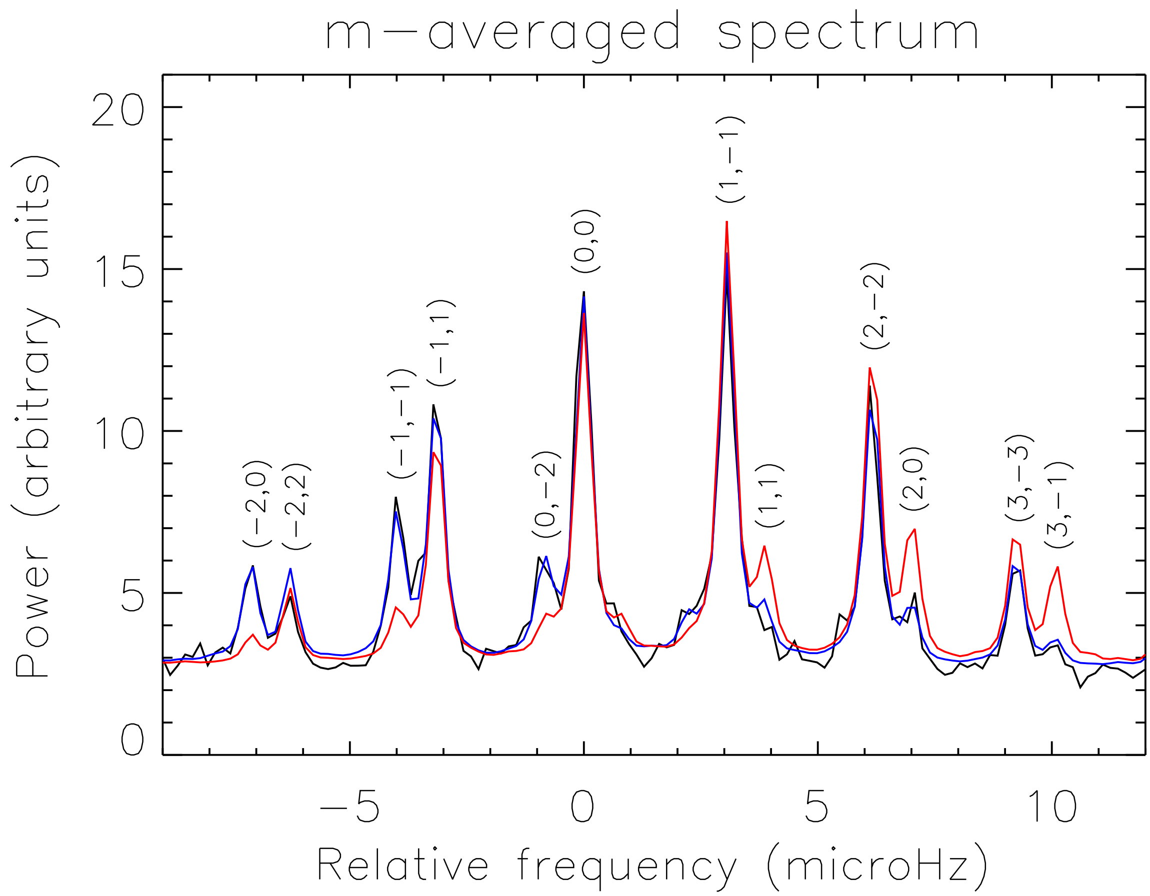

Figure 3: Averages of the spectra from Figure 2 over $-150 \leq m \leq -50$. Black line is the observed spectrum, blue the spectrum including all effects and red the spectrum without distortion by differential rotation. $(\Delta l, \Delta m)$ are identified for selected leaks.

series was fitted with the MDI code including all relevant effects. The amount of differential rotation used for calculating the distortion of the eigenfunctions was multiplied by a factor, a new model spectrum generated and a crude goodness of fit calculated. The scaling parameter was then varied to find the best fit. Results are shown in Figures 4-6.

Several things may be observed from these figures. First of all the fitted values are quite close to 1. Since a crude model of

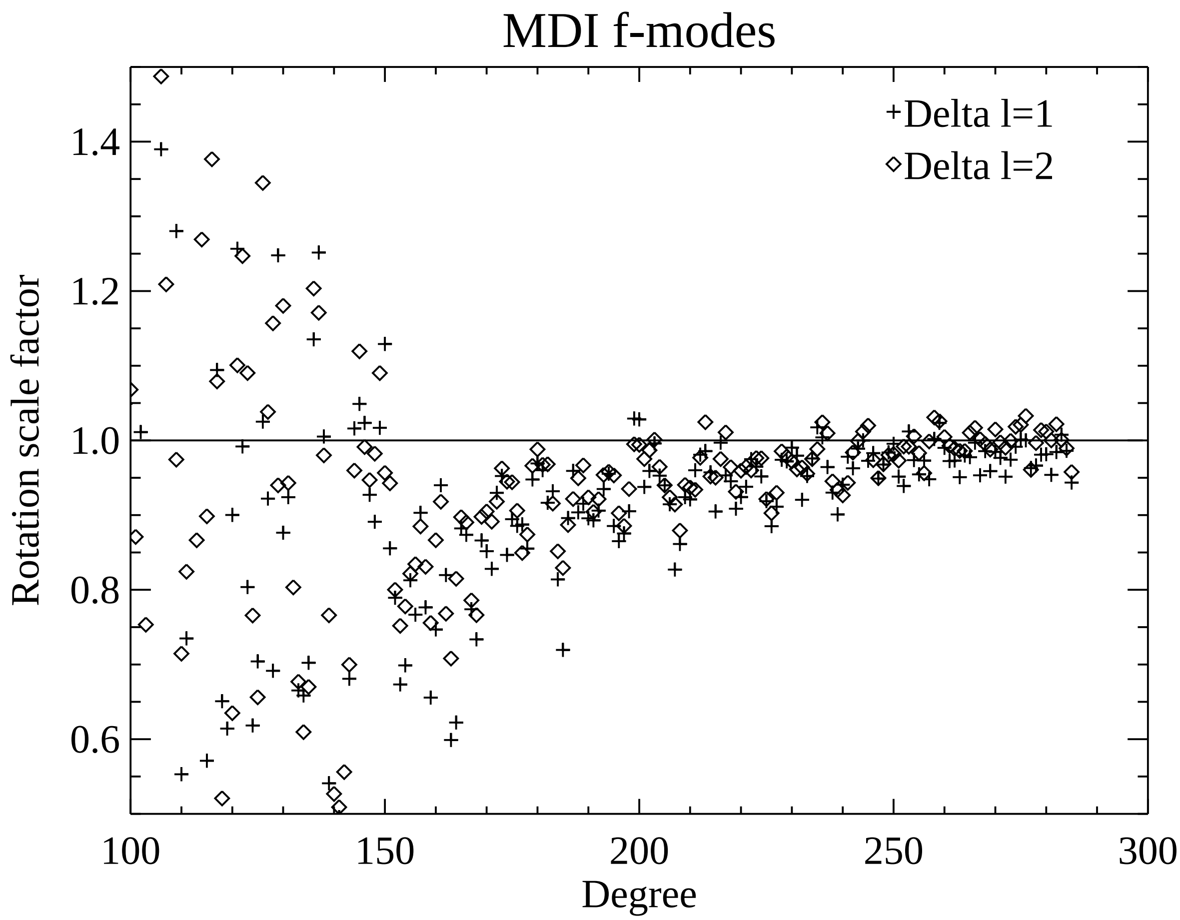

Figure 4: Result of fitting a 72d MDI Medium-l time series. Shown is the scale factor which when applied to the amount of distortion by differential rotation gives the best fit. Results are shown for a fitting interval including the $|\Delta l| \leq 1$ leaks (pluses) and $|\Delta l| \leq 2$ leaks (diamonds).

the rotation was used to calculate the distortion the remaining discrepancy is unlikely to be significant. Second, the scatter is only about 2%. Considering the crude nature of the fitting this is quite competitive with the accuracy of 0.5% on a_3 (the main a-coefficient describing the differential rotation). Third, an offset is seen between the fits including ± 1 in l versus those including ± 2 in l. This likely indicates that other aspects of the leaks have not been properly modeled.

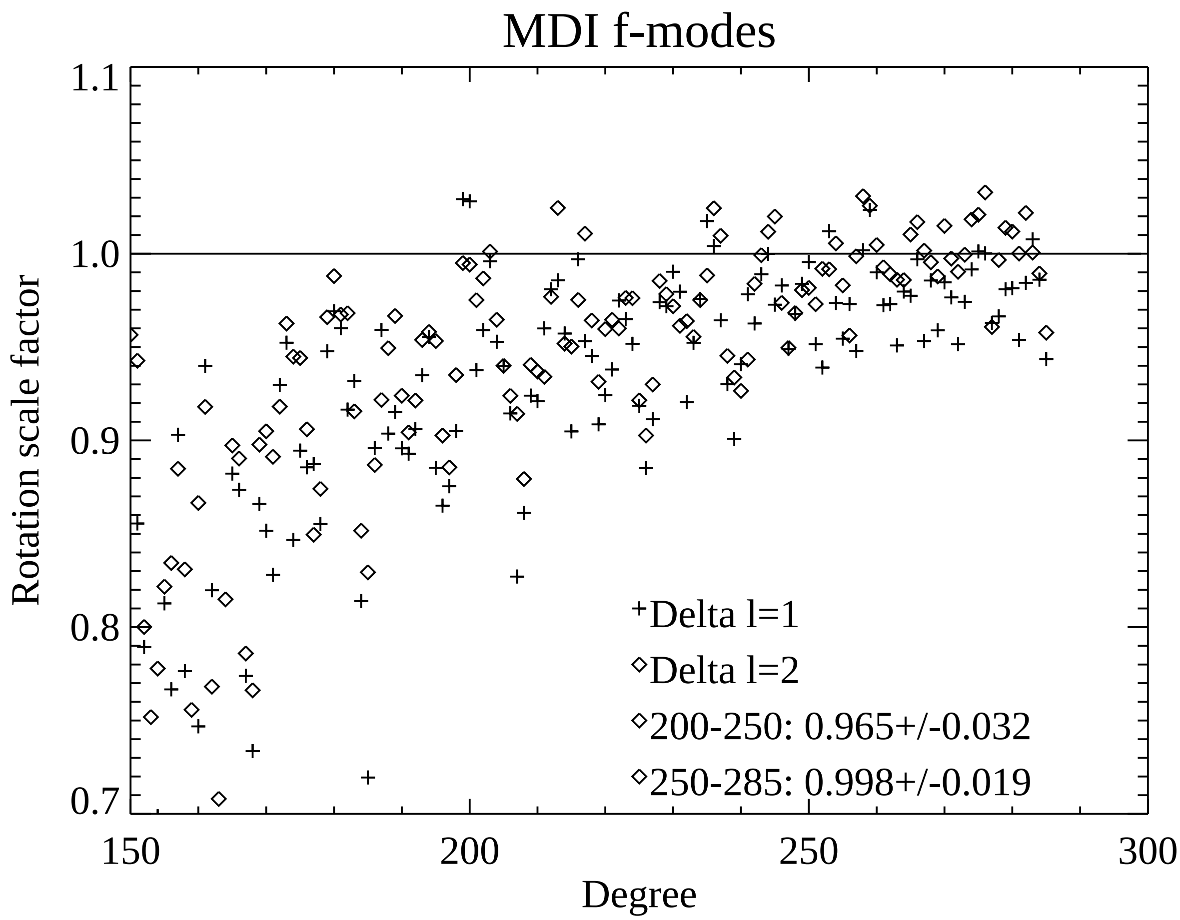

Figure 5: As Figure 4, but on an expanded scale. Also shown is the mean and standard deviation over a couple of l ranges. For comparison the error on a_3 is about 0.5%.

The smaller scatter at high l is likely due to the effect being an increasing function of l and that there are more m's to average over.

It should be noted that the fitting procedure described here leaves a lot to be desired. The fitting is done external to the main fitting code and thus it does not fit for other parameters simultaneously. Also, unlike the MDI code, the likelihood function is not the one appropriate for the data and it thus does not make the best use of the data and the random errors are

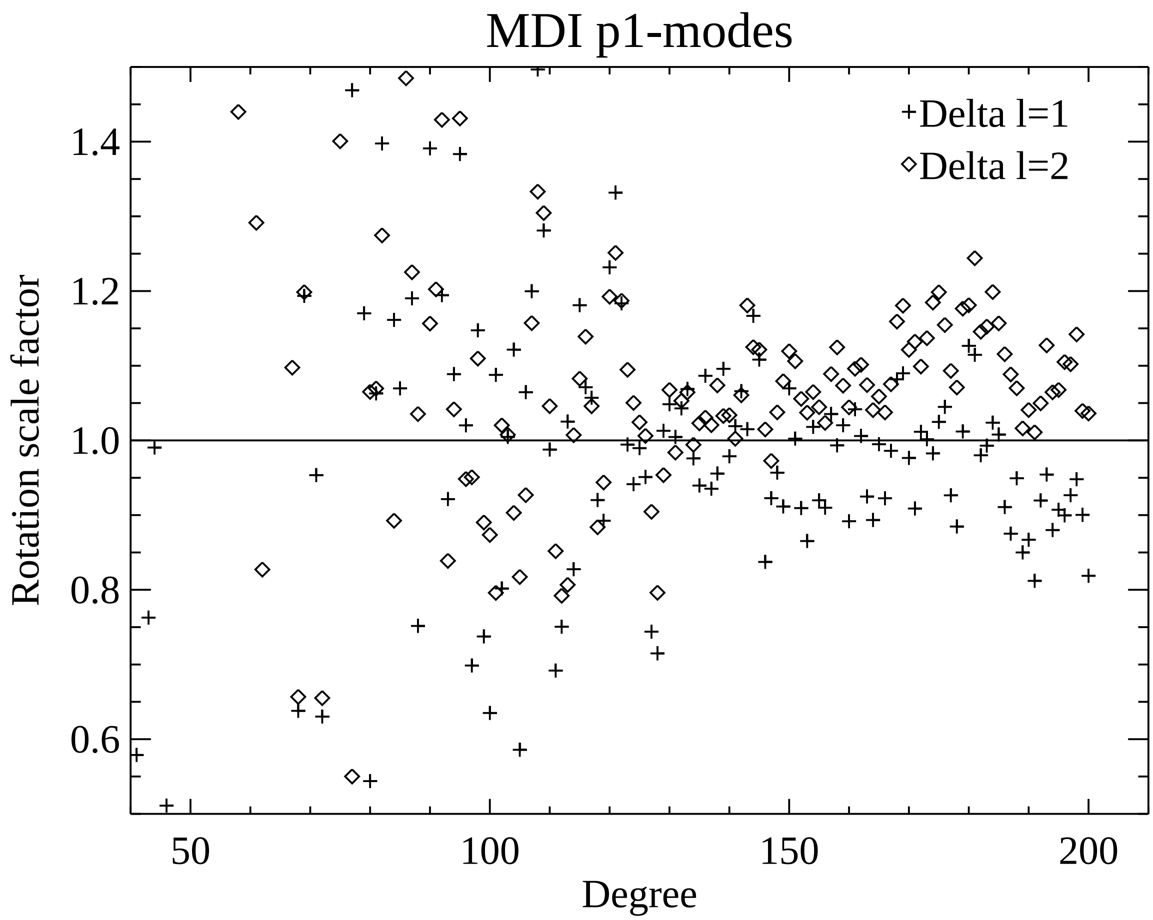

Figure 6: As Figure 4 but for the p_1 mode.

likely to be larger than those from a proper fitting code.

For further entertainment the amount of horizontal displacement was varied relative to the expected value. Results are in Figures 7-9. Given the discrepancies between the $\pm 1l$ and $\pm 2l$ results it is not clear that the differences from 1 are significant.

It should be noticed that Jefferies and Vorontsov have implemented a similar method to fit for various effects. However, their fits are in the power spectrum domain while the MDI fitting code operates in the Fourier domain. Considering that the meridional flows leads to complex leaks it is likely that the MDI method can be more straightforwardly adapted.

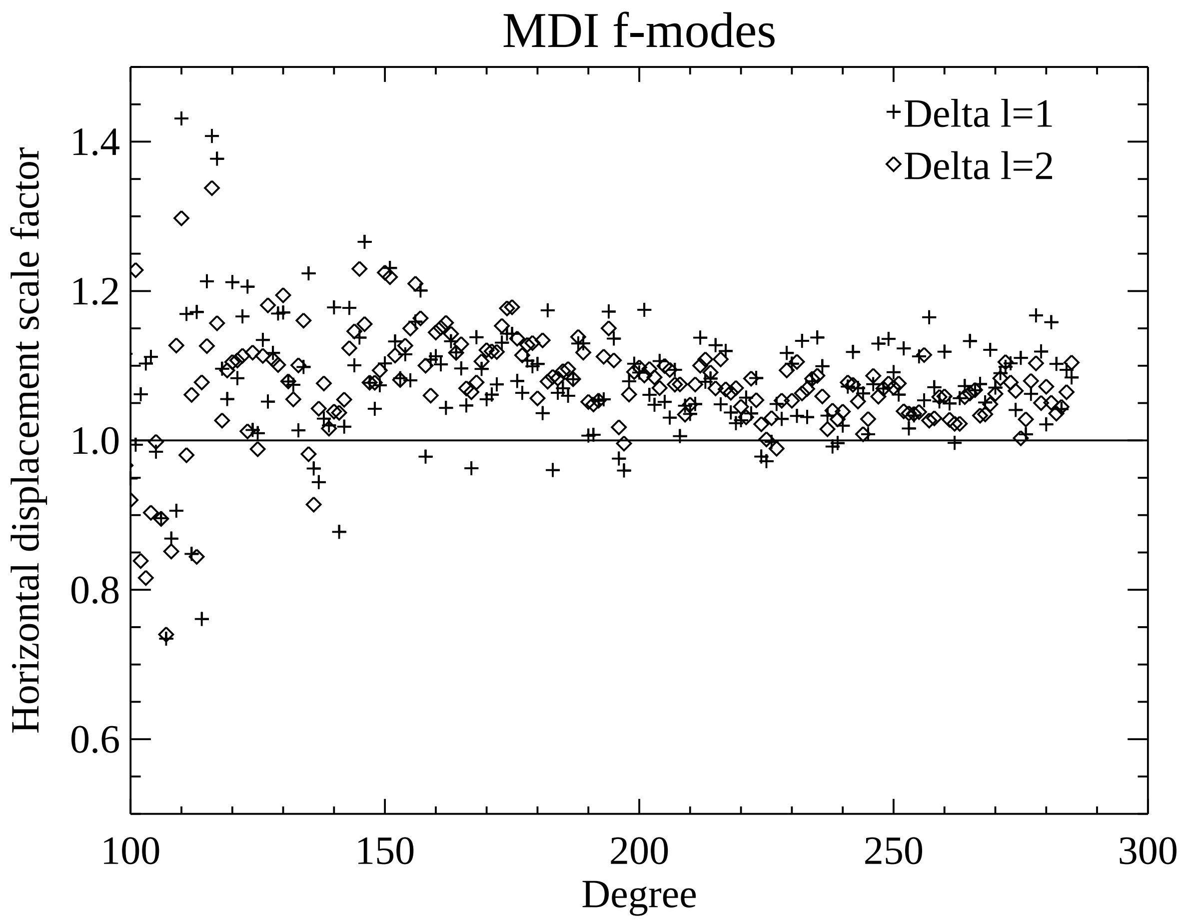

Figure 7: As Figure 4, but fitting for a scaling factor on the ratio of horizontal to vertical mode displacement.

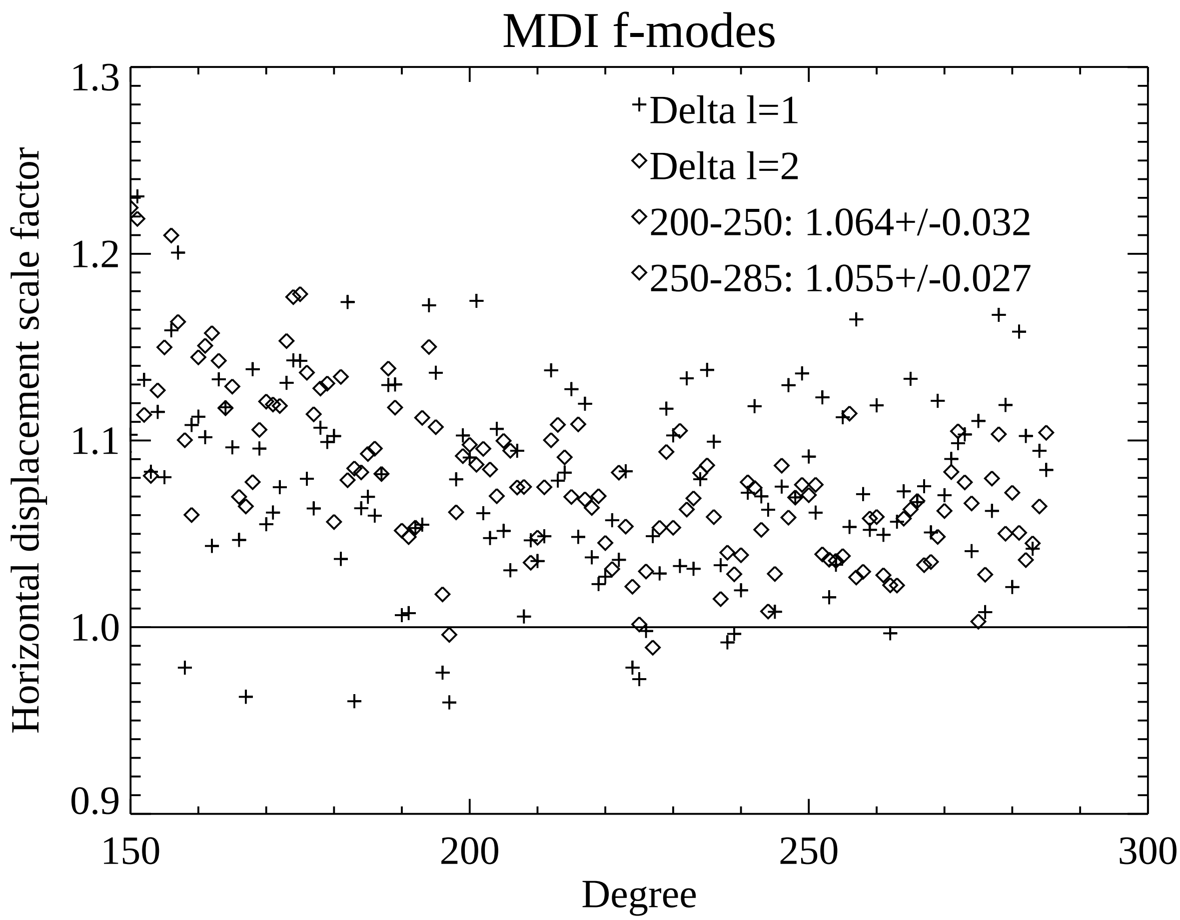

Figure 8: As Figure 7, but at an expanded scale.

Conclusion

It appears that the differential rotation can be measured quite well using the leaks. Considering the crudeness of the fitting method employed here the factor of 4 worse errors seems quite promising.

The code still needs to be modified to incorporate the effects of meridional flows.

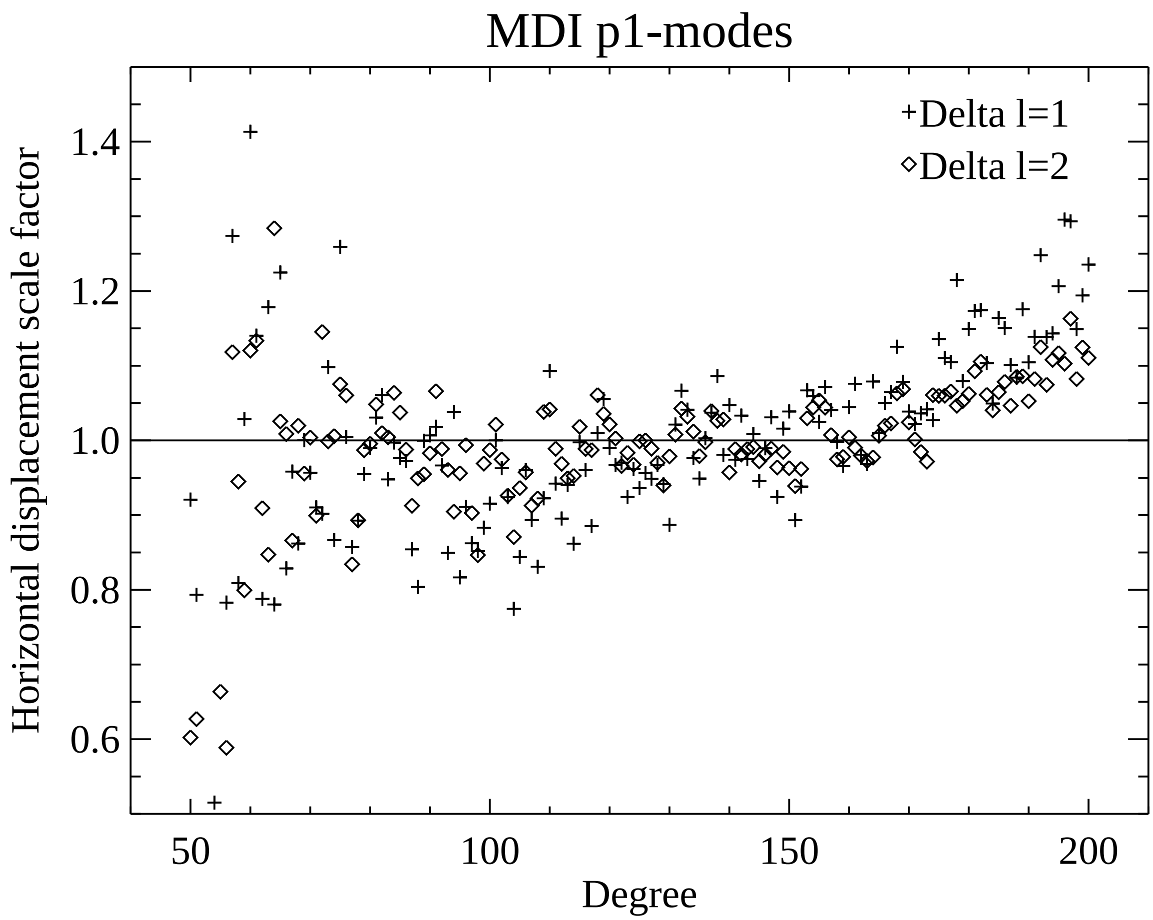

Figure 9: As Figure 7, but for the p_1 mode.

Acknowledgments

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A proposal to work on various aspects of this problem has recently been accepted in the NASA TR&T program.

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