Development and Nondevelopment of Binary Mesoscale Vortices into Tropical Cyclones in Idealized Numerical Experiments

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Abstract

The evolution of two symmetric midlevel mesoscale vortices situated above a warm ocean is examined with a basic cloud resolving model. Idealized numerical experiments provide insight into how the evolution may vary with the initial vortex separation distance $D$ and other parameters that influence the time-scale for an isolated vortex to begin rapid intensification. The latter parameters include the ambient middle-tropospheric relative humidity (RH) and the initial midlevel wind speed of each vortex. At relatively low RH, there exists an interval of $D$ where binary midlevel vortex interaction prevents tropical cyclone formation. While tropical cyclones generally develop at high RH, similar values of $D$ can delay the process if the vortices are initially weak. Prevention or inhibition of tropical cyclone formation occurs in association with the outward expulsion of lower tropospheric potential vorticity anomalies as the two vortices merge in the middle-troposphere. It is proposed that the primary mechanism for midlevel merger and low-level potential vorticity expulsion involves the excitation of rotating misalignments in each vortex. An analogue model based on this premise provides a good approximation for the range of $D$ in which the merger-expulsion scenario occurs. Relatively strong vortices in high RH environments promptly develop vigorous convection and begin rapid intensification. Differences between the interaction of such diabatic vortices and their adiabatic counterparts are briefly illustrated. In systems that generate tropical cyclones, the mature vortex properties (size and strength) are found to vary significantly with $D$. 
1. Introduction

The nature and consequences of binary mesoscale vortex interactions during tropical cyclone formation are not fully understood. One fundamental issue is how the interaction of neighboring vortices affects the moist convective development of the system. Another fundamental issue is how baroclinic vortex structure and moist convection alter the dynamics governing basic processes such as vortex merger. This paper reports on an effort to gain insight into the preceding issues by way of idealized numerical experiments with a cloud resolving model. Before elaborating on the experiments, it is necessary to provide some general background information on the subject at hand.

Tropical weather systems with the potential to become tropical cyclones are commonly observed to have multiple mesoscale vortices with variable characteristics [Ritchie and Holland 1997; Simpson et al. 1997; Reasor et al. 2005; Sippel et al. 2006; Houze et al. 2009; Tory and Frank 2010]. Some of the more prominent midlevel vortices are believed to develop in association with the stratiform components of the mesoscale convective systems that constitute the broader disturbance [cf. Zhang and Fritsch 1987; Hertenstein and Schubert 1991; Chen and Frank 1993; Bister and Emanuel 1997]. Other mesoscale vortices with stronger signatures near the surface may arise from convective updrafts in larger areas of moderately enhanced vorticity [Montgomery et al. 2006; Kilroy and Smith 2013; Wang 2014]. The diversity of vortices in tropical weather systems implies that there exists a wide variety of vortex-vortex interactions to investigate. The present study focuses on the theoretical interactions of 100-km scale vortices whose maximal cyclonic winds are in the middle troposphere.

Historically, such midlevel vortices have been viewed as typical precursors of tropical cyclones [Bister and Emanuel 1997; Simpson et al. 1997; Gray 1998]. It has been suggested that a midlevel vortex in approximate thermal wind balance may provide conditions that are especially conducive to a mode of convection that enhances lower tropospheric convergence and spinup of low-level winds [Raymond et al. 2011; Raymond et al. 2014]. While the general
importance of pronounced midlevel vorticity to the low-level spinup of a cyclonic disturbance is currently under debate [cf. Dunkerton et al. 2009; Wang 2012; Lussier et al. 2014], the potential for a midlevel vortex in contact with the sea-surface to become a tropical cyclone under favorable ambient conditions seems fairly well established.

Details of the development of relatively weak mesoscale vortices into tropical cyclones have been studied extensively with cloud resolving numerical models. Of note, Montgomery and coauthors [2006] analyzed an idealized simulation of the transformation of an isolated midlevel mesoscale cyclone (with appreciable low-level vorticity) into a surface-concentrated tropical depression vortex. They found that the transformation occurs “in association with the generation of intense vortical hot towers (VHTs), mid-to-upper level moistening by the VHTs, diabatic mergers between VHTs, and the near-surface and mid-to-upper level inflow on the system scale that is induced by the ensemble of VHTs.” The term VHT here refers to a deep convective cloud with intense vertical vorticity in its core. Wang [2014] more recently discussed how “vortical congestus clouds,” which are not as deep as VHTs, may contribute significantly to near-surface spinup during the early stage of tropical cyclogenesis. Nolan [2007] furthermore discussed the possibility of a certain “trigger for tropical cyclogenesis,” in which a robust updraft (appearing after near saturation) rapidly creates a small but prominent surface-concentrated vorticity anomaly that becomes the convective core of an intensifying system.

It is of interest to understand how the interaction of neighboring mesoscale vortices might help or hinder the development process. Earlier work on this subject focused on how the merger of 100-km scale midlevel vortices may directly assist the amplification of cyclonic surface winds. The idea of merger-induced surface spinup was encouraged by observations of apparent vortex mergers preceding the emergence of tropical cyclones [Ritchie and Holland 1997; Simpson et al. 1997; Venkatesh and Mathew 2004]. A notable theory put forth by Ritchie and Holland [1997] begins with the plausible assumption that the merger of midlevel vortices creates a broader vortex in the middle troposphere. The authors reasoned
that the velocity field of the broader vortex should have a stronger surface signature than
either of the original vortices, on the basis that the penetration depth of the balanced
flow is proportional to the width of the potential vorticity (PV) anomaly [cf. Hoskins et
al. 1985; Shapiro and Montgomery 1993]. Several dry-adiabatic simulations were presented
as evidence supporting this theory, in addition to the notion that the final surface signature
is enhanced under conditions of greater ambient rotation [Ritchie and Holland 1997]. Kieu
and Zhang 2010 more recently demonstrated that elimination of a major mesoscale merger
event in the simulated development of tropical storm Eugene (2005) substantially inhibited
storm intensification. Here the merger seemed to involve only one quasi-midlevel vortex,
with the other concentrated in the lower troposphere.

In the present study, the potential consequences of midlevel vortex merger are reexam-
inied in a context where the vortices have sufficiently strong surface winds to develop in
isolation in a reasonable time-frame. While the specific problem investigated is too idealized
for direct applicability to a real tropical weather system, its study will uncover new fluid
dynamics with elements that may be relevant to what occurs in nature.

Even when moist convection has minimal influence, the merger process may differ consid-
erably from the classical two dimensional paradigm. To begin with, each baroclinic vortex
imposes vertical shear on its neighbor, encouraging the midsection of that neighbor to
separate from its vertical extremities. The subsequent interaction of the midsection with
the displaced lower and upper sections of the vortex theoretically factors into whether or not
midlevel merger occurs. Note also that the merger of PV anomalies need not extend beyond
the middle troposphere. Results presented herein give reason to believe that future convec-
tive development of the system may depend on where the lower (and upper) tropospheric
PV anomalies end up.

The numerical experiments conducted for this study make use of a basic cloud resolving
model with warm-rain microphysics and a simple parameterization of air-sea interaction.
The time-scale for a midlevel vortex to become a rapidly intensifying tropical cyclone in
isolation is readily controlled by adjusting the initial vortex properties or the environmental parameters. Some of the experiments are designed to delay the onset of rapid intensification and allow quasi-adiabatic vortex dynamics to operate. Comparison with an analogue model will suggest that the process leading to merger in such experiments is largely governed by the growth and rotation of misalignments expounded elsewhere in the context of vortex motion under the influence of ambient vertical wind-shear [Jones 1995; Walsh and Pratt 1995; Smith et al. 2000; Vandermeirsch et al. 2002; Reasor et al. 2004, henceforth RMG04; Reasor and Montgomery 2015].

Contrary to earlier studies, it will be shown that the rearrangement of PV resulting from quasi-adiabatic midlevel merger can actually inhibit future vortex intensification. The inhibition is found to occur over a distinct range of initial separation distances between the vortices. For such initial conditions, lower tropospheric PV anomalies are ultimately expelled from the region underneath the merged midlevel system. If the expulsion is thorough and the initial relative humidity (RH) of the middle troposphere is sufficiently low, the vertically dissociated system fails to generate a tropical cyclone.

If the time-scale for the onset of convective vortex intensification is less than that of the quasi-adiabatic process leading to midlevel merger, the dynamics of the binary system changes considerably. Mesoscale vortices with substantial moist convection are known to interact quite differently than their dry-adiabatic counterparts. This has been clearly demonstrated in previous numerical studies involving well-developed tropical cyclones [Wang and Holland 1995; Falkovich et al. 1995; Khain et al. 2000], vortical hot towers [Hendricks et al. 2004; Montgomery et al. 2006] and generic convective vorticity anomalies in developing tropical systems [Tory et al. 2006b; Fang and Zhang 2011]. Moist convection generally changes the structure and intensity of each vortex during the interaction period, and may also modify the background vorticity distribution. Furthermore, convection adds a substantial contribution to the irrotational component of the horizontal velocity field not present in classical two-dimensional or quasigeostrophic (QG) models of the adiabatic problem [cf. Melander
et al. 1988; Verron and Valcke 1994; Von Hardenberg et al. 2000]. A related complication is that persistent convection in the form of an overturning secondary circulation involves continual fluid exchange between a vortex and its environment. This paper will briefly discuss the process by which vortices merge in a simulation where they promptly develop compact convective cores whose maximal wind speeds are near the surface. The discussion will be limited, however, because the merger process does not seem remarkably different from that analyzed elsewhere.

The final topic addressed in this paper is how prior mesoscale vortex interactions affect the properties of mature tropical cyclones. The experiments conducted herein will illustrate the potential consequences on size and intensity.

The remainder of this paper is organized as follows. Section 2 explains the computational configurations and initializations of the numerical experiments. Section 3 describes the vortex intensification process that occurs in experiments where tropical cyclones develop. Section 4 briefly describes how the time-scale for the onset of rapid intensification varies with the initial vortex separation distance in relatively dry and humid environments. Section 5 explains the midlevel merger and surface PV expulsion associated with the delay or prevention of tropical cyclone formation in a subset of the experiments. Section 6 briefly discusses the merger of vortices that develop compact convective cores. Section 7 describes how the properties of a mature tropical cyclone emerging from a binary system of midlevel mesoscale vortices depends on the initial separation distance. Section 8 summarizes the main results of this study.
2. Experimental Setup

The numerical experiments are conducted with version 16 of Cloud Model 1 [CM1; Bryan and Fritsch 2002]. The model is configured on the periodic $f$-plane. The Coriolis parameter is given by $f = 5 \times 10^{-5}$ s$^{-1}$, which is representative of 20 °N. Moisture is parameterized with Kessler warm-rain microphysics, and atmospheric radiation is switched off. Subgrid turbulent transport is parameterized by an anisotropic Smagorinsky-type scheme and dissipative heating is activated. The underlying sea-surface temperature is held fixed at 26.1 °C. Surface fluxes are parameterized with a constant drag coefficient $C_D = 0.002$ and a constant enthalphy-flux coefficient $C_k = C_D$. Both $C_D$ and $C_k$ are within observational bounds relevant to tropical cyclones, but the ratio $C_k/C_D = 1$ is relatively high [Bell et al. 2012].

The model discretizes the dynamical system on a stretched rectangular grid whose center coincides with that of the binary system of interacting vortices to be studied. The horizontal grid spans 3600-3700 km in each orthogonal coordinate. The central square of the horizontal domain has increments of 1.67 km. The length of the central square is 574 or 902 km, depending on whether or not $D \leq 300$ km. At the four corners of the horizontal domain, the grid increments are 14.3 km. The vertical grid has 68 points and extends upward to $z = 30$ km, in which $z$ denotes the height above sea-level. The vertical grid spacing increases from 50 to 540 to 750 m as $z$ increases from 0 to 12 to 30 km. Rayleigh damping of upward propagating waves is imposed above $z_d = 25$ km.

Figures 1a and 1b show the ambient soundings used for two classes of experiments where the vortices are placed in relatively humid or dry environments. The temperature profiles resemble the Jordan [1958] mean sounding for hurricane season in the West Indies. The pseudoadiabatic CAPE and CIN of a surface parcel are $2.1 \times 10^3$ and 19-20 J kg$^{-1}$ in either environment.\footnote{The convective available potential energy (CAPE) and convective inhibition (CIN) are here defined for undiluted surface parcels undergoing pseudoadiabatic ascent with liquid-only condensate. CAPE is the} Similarities in CAPE and CIN between the humid and dry experiments
sharply contrast critical differences in RH above the lowest kilometer of the troposphere. The high RH environment seems to better reflect conditions in which substantial midlevel vortices may have developed and rapid intensification might shortly ensue. While the high-RH setup is not precisely modeled after observations, environments of notable CAPE and substantial middle tropospheric relative humidity are commonly observed in connection to vigorous tropical disturbances [e.g., Smith and Montgomery 2012; Davis and Ahijevych 2013]. Moisture in the low RH environment is taken directly from the Jordan mean sounding. This climatological sounding incorporates several distinct air-mass types, and is substantially drier in the middle troposphere than typical environments that produce tropical cyclones [Dunion 2011]. It is used here as a means for slowing down convective changes to the vortices. Regardless of how it is achieved, such slow-down is required to clarify how variation of quasi-adiabatic dynamics (advective rearrangement of PV) early on affects the time-scale for the onset of future intensification.

Each binary system is initialized with two identical vortices having maximum wind speeds in the middle troposphere. Each vortex is characterized by a cyclonic vertical vorticity distribution of the form

\[
\zeta(r, z) = \left[ \zeta_c H(r_c - r) - \zeta_h H(r_h - r) \right] Z(z),
\]

in which \(r\) denotes radius from the center of the vortex, and \(\zeta_h = \zeta_c r_c^2 / r_h^2\). The Heaviside step function is defined such that \(H(r' - r) = 1\) or \(0\) if \(r\) is less than or greater than \(r'\), respectively. The radial profile of vorticity consists of a uniform positive core out to \(r_c = 50\) km, surrounded by a negative halo that brings the circulation to zero at \(r_h = 500\) km. The vertical structure function is given by

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vertical integral of positive buoyancy between the level of free convection (LFC) and the level of neutral buoyancy. CIN is minus the vertical integral of negative buoyancy between the surface and the LFC. The computations of CAPE and CIN are based on the “getcape” subroutine included in the CM1 software package.
\[ Z = \frac{\exp(10/7)}{2 \{ \cosh [(z - z_m)/\delta h] + \cosh(10/7) \}}, \]

in which \( z_m = 5.5 \) km. Note that \( Z \) is symmetric about \( z_m \). The vertical velocity field and the divergence of horizontal velocity are initially set to zero. The horizontal velocity, pressure and potential temperature fields are obtained from equations of nonlinear balance as explained in appendix B1 of Schecter 2011. The balance conditions are applied to the superposition of the two vorticity fields representing the mesoscale vortices, not to each vortex separately. The centers of each vortex are initially separated by a distance \( D \) in the horizontal plane. The system is considered binary only if \( D \geq 2r_c = 100 \) km.

The binary systems are distinguished not only by the ambient RH and \( D \), but also by the values of \( \zeta_c \) and \( \delta h \). Two combinations of the latter two parameters are considered. The first corresponds to a relatively strong vortex with \( \zeta_c = 0.001 \) s\(^{-1}\) and \( \delta h = 1.75 \) km. The second corresponds to a weaker vortex with \( \zeta_c = 0.0005 \) s\(^{-1}\) and \( \delta h = 2.47 \) km. The smaller value of \( \zeta_c \) reduces the midlevel wind speed, but the greater value of \( \delta h \) keeps the surface velocity unchanged.

Figures 1c-1h illustrate the initial states of the strong and weak vortices when isolated from their binary partners. The maximum midlevel wind speeds of the strong and weak vortices are approximately 15 m s\(^{-1}\) and 7.8 m s\(^{-1}\). The weaker vortex may better resemble those typically observed to transform into tropical cyclones [Raymond et al. 2011; Davis and Ahijevych 2012; Komaromi 2013]. Nevertheless, maximum midlevel wind speeds between 10 and 15 m s\(^{-1}\) commonly emerge prior to genesis in computational studies [Bister and Emanuel 1997; Nolan 2007; Schecter 2011; Nicholls and Montgomery 2013].

Imposing nonlinear balance is seen to adjust the thermodynamic state of each vortex, with the greatest changes at the center. The virtual potential temperature \( \theta_v \) is reduced in the lower troposphere and elevated in the upper troposphere.\(^2\) The saturation equivalent potential temperature \( \theta_e^* \) exhibits a similar lower tropospheric reduction and upper

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\(^2\)Similar anomalies seem to commonly precede tropical cyclone formation in nature [cf. Davis and Ahijevych 2013; Raymond et al. 2014].
tropospheric enhancement. The changes in the actual equivalent potential temperature $\theta_e$ are smaller in magnitude. Notable consequences are a central lowering of the level of free convection, and a central decrease (increase) of the lower tropospheric (upper tropospheric) saturation deficit. These modifications are subtle in the weak vortex, but clearly discernible in the strong vortex. In the high RH environment, the saturation deficit at the center of the strong vortex becomes zero between $z \approx 2.5$ and 4 km, causing the appearance of a small cloud. In the low RH and weak vortex experiments, the initial conditions are entirely cloud-free. Note that the equivalent potential temperatures in Fig. 1 and the moist entropies appearing elsewhere in this paper are approximated as in Bryan 2008 for pseudoadiabatic thermodynamics.

A relatively complete survey of system evolution, from inception to maturation, has been conducted for the strong vortex scenario. The original reason for focusing on the strong binary system was to accelerate the evolution and thereby economize limited computational resources. The author presumed that the acceleration would not fundamentally alter the dynamics. The weak vortex simulations, which are fewer in number and not all carried out to maturation, largely validate this assumption. One notable exception will be discussed at length in section 5d. Basic behavioral similarity seems reasonable given that a factor-of-two midlevel wind speed difference does not change the order of magnitude of the following dimensionless parameters:

\begin{align}
\text{Ro} & \equiv 2V_c/(fr_c), \\
\text{Fr} & \equiv V_c/(Nz_m), \\
L_d & \equiv l_d/r_c, \\
\gamma & \equiv V_s/V_c.
\end{align}

In the preceding equations, $V_c \equiv 0.5(\zeta_c - \zeta_h)r_cZ(z_m)$ is the characteristic (maximal) wind speed of the vortex, $V_s \equiv V_c[1 - Z(0)/Z(z_m)]$ is a measure of internal vertical wind shear, $N = 0.011 \text{ s}^{-1}$ is the approximate Brunt-Väisälä frequency, and $l_d$ is the intrinsic deformation radius [Eq. (B12)]. The Rossby numbers of the strong and weak vortices are respectively $\text{Ro} = 13$ and 6.5, whereas the Froude numbers are $\text{Fr} = 0.27$ and 0.13. The deformation radii
of the strong and weak vortices normalized to the core radius are respectively $L_d = 3.4$ and 5.2. The internal shear parameters given in the same order are $\gamma = 0.77$ and 0.54.

Figure 2 illustrates the basic spatial configuration of a binary vortex experiment and the potential evolution of the simulated system. This particular example involves strong vortices under high RH conditions with $D = 200$ km. The top panel shows the evolution of total rain mass above the surface per unit area ($\sigma_r$). The brightest regions correspond to deep convective updrafts. The bottom panel shows the evolution of near-surface horizontal wind speed. It so happens in the depicted simulation that the two vortices rapidly intensify to hurricane strength and subsequently merge (see section 6). Gradual intensification continues after merger until a peak azimuthal velocity of 83 m s$^{-1}$ (59 m s$^{-1}$) is achieved approximately 400 m above sea level (at the surface). The relatively strong winds produced by the model are analyzed in section 7.

3. Preliminary Description of Convective Vortex Intensification

The vortex intensification process in each numerical experiment where it occurs has an opening act (prelude) and a main event. Both parts of the process are briefly described below. For simplicity, the discussion focuses on the intensification of a strong vortex at high RH [Figs. 1c and 1d] that is isolated by removing its companion from the simulation. The basic phenomenology appears to apply to the constituent vortices of all other developing systems, whether intensification gets underway before or after merger.

Standard conventions are used herein to denote the flow fields associated with a given vortex. The symbols $u$, $v$ and $w$ respectively represent the radial, azimuthal and vertical velocity fields. Overbars denote azimuthal averages, and primes denote asymmetric perturbations. Central to the discussion is the mean azimuthal velocity field $\bar{v}(r, z, t)$, in which $t$ is time. The cylindrical coordinate system $(r, \varphi, z)$ used for analysis is centered so as to maximize the radial maximum of $\int_0^h dz \bar{v}/h$, in which $h = 4$ km. In practice, the central axis
of this coordinate system shifts from the original $z$-independent circulation center to that of a surface-concentrated vortex when the latter begins to dominate the flow.

The opening act begins with minor frictional decay of the mean azimuthal velocity field near the surface. The air-sea coupling responsible for the slight spindown simultaneously acts to increase water vapor in the boundary layer. Sporadic cumulus convection eventually arises around the periphery of the vortex core, and then penetrates inward. The initial state of high RH in the middle-to-upper troposphere is conducive to relatively deep updrafts. The emergence of deep convection coincides with moderate system-scale spinup in the lower troposphere. For systems that do not begin near saturation, the invigoration of convective processes would simultaneously bring about humidification of the vortex core. Another consequence of the early convection is the generation of compact vertical vorticity anomalies, presumably through tilting and stretching of vortex tubes [e.g., Montgomery et al. 2006]. Over time, positive vorticity anomalies build up near the center of the system, whereas negative vorticity anomalies (created partly in association with diverging cold pools near the surface) become largely relegated to outer radii [e.g., Nguyen et al. 2008]. At this point, the system is prepared for the main intensification event that will be described below.

Figures 3a-3d are select time series that summarize the development of the vortex. Figure 3a shows the maximum $\varphi$-averaged azimuthal velocity $\bar{v}_{max}$, the maximum horizontal wind speed at $z = 25$ m, and the root-mean-square (rms) 25-m wind speed in a $534 \times 534$ km$^2$ box whose center coincides with that of the vortex. The start of deep convection at $t \approx 13$ h is marked by a sharp growth of the maximum 25-m wind speed and a modest decay of $\bar{v}_{max}$. The main event is defined to begin with the rapid intensification of $\bar{v}_{max}$ after approximately one day of priming. The onset of the main event is indicated by the pink circle. Figures 3b and 3c show that the radius and elevation where $\bar{v}_{max}$ occurs ($r_{max}$ and $z_{max}$) decrease precipitously at this time. The abrupt change corresponds to a small region of enhanced vorticity at the center of the system becoming the dominant vortex.\(^3\) While the\(^3\)The radius of maximum $\bar{v}$ near the surface becomes small (much less than $r_c$) at an earlier time. The magnitude of $\bar{v}$ near the surface then gradually amplifies, with some interruptions along the way. The abrupt
probability of such an event in nature is uncertain, similar phenomenology has been reported in a number of other simulations of tropical cyclogenesis using the Weather Research and Forecasting Model [Nolan 2007] and the Regional Atmospheric Modeling System [Nicholls and Montgomery 2013]. The azimuthally averaged relative vorticity $\bar{\zeta}$ near the surface of the compact central vortex (at $t = 24$ h) has a peak value of $4 \times 10^{-3}$ s$^{-1}$. Such vorticity is an order of magnitude greater than the initial midlevel maximum, and two orders of magnitude greater than $f$. Central values of $\bar{\zeta}$ exceeding $1 \times 10^{-3}$ s$^{-1}$ extend up to $z = 7$ km.

Figure 3d shows a time series of the convective asymmetry (CA) defined by

$$
CA \equiv \frac{\int_0^{z_d} \int_0^{3r_{max}}dzdr\rho_a w'^2}{\int_0^{z_d} \int_0^{3r_{max}}dzdr\bar{\rho}_a \bar{w}^2},
$$

in which $\rho_a$ is the density of moist air. CA essentially measures the kinetic energy of the asymmetric component of the vertical velocity field ($w'$) relative to the kinetic energy of the symmetric component ($\bar{w}$) within a cylinder having thrice the radius of maximum wind [cf. Schecter 2013]. The value of CA is much greater than unity during the early stage of sporadic convection, but drops to order unity by the time the compact central vortex becomes stronger than the broader midlevel vortex.

Figure 4 depicts the azimuthally averaged structure of the system shortly after the onset of rapid intensification becomes apparent in $\bar{v}_{max}$. Figures 4a and 4b portray the azimuthally averaged velocity field. The mean secondary circulation near the surface of the compact central vortex resembles that of a corner-flow. There are clearly regions in the vicinity of $\bar{v}_{max}$ and at larger radii in the lower-to-middle troposphere where the mean secondary flow is down the gradient of absolute angular momentum, defined by $M \equiv r\bar{v} + fr^2/2$. Such down-gradient flow acts to induce local spinup. Note, however, that the actual sign of $\partial \bar{v}/\partial t$ depends on the local frictional and eddy forcings as well. Details of the spinup mechanism changes in the time-derivative and location of $\bar{v}_{max}$—referred to throughout this paper as the “onset of (rapid) intensification” —occur when the near surface maximum of $\bar{v}$ first exceeds the midlevel maximum.
are of interest, but will not be examined in this study [cf. Persing et al. 2013 and references therein]. Figures 4c and 4d illustrate the thermodynamic state of the system. The updraft region of the vortex core is deeply humidified as expected. The incongruence of saturated pseudoadiabatic entropy contours and absolute angular momentum contours in the updraft region stands in notable contrast to the slantwise convective neutrality associated with axisymmetric equilibrium. Although CAPE has diminished from the start of the simulation, it has maintained values between 1000 and 1600 J kg\(^{-1}\) for \(r \leq r_{max} = 11.7\) km.

There are several aspects of the main intensification event worth noting. The orange curves in Fig. 3a show the evolution of \(\bar{v}_{max}\) in two experiments where the surface exchange coefficients are abruptly reset at \(t = 25\) h. In one experiment, \(C_d\) is unchanged but \(C_k\) is reduced to zero. In the other experiment, \(C_k\) is unchanged but \(C_d\) is reduced from 0.002 to \(10^{-4}\). While eliminating the surface flux of moist enthalpy decelerates the amplification of \(\bar{v}_{max}\), it does not immediately quench the process. Moreover, reducing \(C_d\) by a factor of 20 has negligible impact on the growth of \(\bar{v}_{max}\) for approximately ten hours. The preceding results suggest that the contemporaneous surface fluxes do not strongly control the initial burst of rapid vortex intensification [cf. Montgomery et al. 2015]. A greater sensitivity to \(C_k\) and \(C_d\) is apparent later on. Note that setting \(C_k\) to zero at \(t = 0\) ordinarily proves much more detrimental to intensification, as it inhibits recovery from low entropy downdrafts into the boundary layer during the opening act. Substantially reducing \(C_d\) at the outset is also commonly found to slow down the intensification in three-dimensional cloud-resolving numerical simulations [Montgomery et al. 2010; Schecter 2011; Persing et al. 2013; Smith et al. 2014]. A definitive theory for how the intensification rate should quantitatively depend on \(C_k\) and \(C_d\) under various circumstances may not yet exist, but should eventually emerge as understanding of the intensification process continues to evolve.

Figure 3d shows furthermore that the amplification of \(\bar{v}_{max}\) involves the development of strong supergradient flow at the location of maximum azimuthal wind speed. The dashed
curve corresponds to the supergradient flow parameter defined by

\[ SG \equiv \left. \frac{\bar{v}^2}{r} + \frac{f \bar{v}}{\rho_a^2 \partial \bar{p}/\partial r} \right|_{r_{\text{max}}, z_{\text{max}}}, \]  

(5)

in which \( p \) denotes total pressure. The value of SG measures the combined outward accelerations of the centrifugal and Coriolis forces (associated with the mean flow) normalized to the inward acceleration that is enforced by the radial pressure gradient at \( r = r_{\text{max}} \) and \( z = z_{\text{max}} \). A value of unity implies local gradient balance. During the initial burst of rapid intensification, SG attains values greater than 3. Evidently, any theory on the amplification of \( \bar{v}_{\text{max}} \) that assumes local gradient balance would not accurately apply to the simulations at hand. This result is consistent with the importance of supergradient flow emphasized in a recent review paper on tropical cyclone intensification [Montgomery and Smith 2014].

Figure 3e shows \( r-t \) Hovmöller plots of \( \bar{v} \) and mean vertical vorticity \( \bar{\zeta} \) near the surface \((z = 148.7 \text{ m})\). Once the vortex attains sufficient strength, the radius of gale force winds near the surface increases continually with time. After the vortex achieves peak intensity, the radius of maximum wind grows gradually with time as well. Attending the growth of the radius of maximum wind is a radial broadening of the vertical vorticity distribution. In principle, such broadening could facilitate the merger of neighboring vortices that are initially too compact relative to their separation distance to satisfy a merger criterion.

Figures 3f-3h illustrate how the development of an isolated vortex depends on its initial midlevel wind speed and the environmental RH in the four scenarios relevant to this study. Figure 3f shows the evolution of \( \bar{v}_{\text{max}} \) minus its initial value. Figure 3g shows the evolution of \( r_{\text{max}} \). In all cases, the transition to rapid intensification apparent in \( \bar{v}_{\text{max}} \) coincides with an abrupt reduction of the radius of maximum wind speed. As before, the abrupt reduction indicates that the near-surface velocity field of a compact central vortex has become stronger than the midlevel circulation. Figure 3h shows the evolution of average RH in a cylindrical region defined by \( 0 \leq r \leq 23 \text{ km} \) and \( 1.5 \leq z \leq 5.3 \text{ km} \). The onset of rapid intensification
generally coincides with this measure of RH exceeding 90%.

It is seen that the time-scale for the onset of rapid intensification increases with decreasing values of the initial RH and midlevel wind speed. A delayed onset at lower RH is consistent with the conventional notion that local pre-humidification is required in the middle troposphere to limit weakening of the deep convection involved in stimulating and sustaining significant vortex amplification. A delayed onset with weaker vorticity above the surface is intuitively reasonable but not thoroughly understood at this time. Previous studies have suggested that the attendant reduction of inertial stability may limit the conversion of latent heat released through convection into the kinetic energy of the azimuthally averaged circulation [Hack and Schubert 1986; Nolan et al. 2007]. Subtle changes to the local thermodynamic sounding [Figs. 1c-1h] could also be relevant [cf. Raymond et al. 2014].

4. Overview of Binary System Evolution

The discussion of binary vortex interaction begins here with an overview of the system evolution in the parameter space of this study. Of primary concern is whether or not the system produces a tropical cyclone.

4a. Evolutionary Pathways

Figure 5 summarizes how the separation distance $D$ controls the evolution of two strong midlevel vortices in relatively humid and dry environments. First consider the high RH scenario. Section 3 showed that a deeply humidified atmosphere facilitates the fast onset of rapid vortex intensification. It therefore seems reasonable that all of the binary vortex simulations at high RH produce hurricane-strength tropical cyclones, having warm-core structure with maximal winds near the surface. If $D$ is sufficiently small, the vortices promptly merge and only one hurricane develops. If $D$ has an intermediate value in the range of 200-300 km,
both of the original vortices have time to spawn hurricanes. However, the binary state is transient in that the storms eventually coalesce. Section 6 will describe the manner in which the cores of the moist convective vortices come together. At larger separation distances, $D = 400$ and 600 km in particular, two hurricanes develop and then slowly drift apart. The outward drift appears to be consistent with guidance from large-scale gyres that emerge as the vortex cores stir low-amplitude background PV. Similar drift was found in ideal 2D simulations initialized with the vertically averaged tropospheric relative vorticity distribution existing shortly after hurricane formation.\(^4\)

At low RH, there is an extended range of $D$ below 400 km where quasi-adiabatic vortex merger occurs before the diabatic processes involved in tropical cyclone formation substantially develop. For $D \leq 200$ km, a single hurricane eventually arises from the merged system. However, there exists an intermediate interval of $D$ at low RH (including $D = 250$ and 300 km) where midlevel merger occurs but no tropical cyclone is produced. Sections 4c and 5 will elucidate this intriguing result. For $D \geq 400$ km, two hurricanes develop and drift apart as in the corresponding simulations at high RH. Although double hurricane formation followed by merger was not observed at low RH, it seems possible in the data-gap between $D = 300$ and 400 km.

A limited number of simulations initialized with two weak midlevel vortices produced no major surprises, but did not entirely conform to Fig. 5. Weak systems with $D = 100$ or 250 km were found to follow the same evolutionary pathways as their stronger counterparts. This includes nondevelopment at low RH when $D = 250$ km. On the other hand, a weak system with $D = 200$ km and high RH generated only one hurricane after a lengthy delay involving a midlevel merger event. Further discussion of the weak vortex simulations is deferred until section 5d.

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\(^4\)The 2D Euler equations were integrated using a vortex-in-cell method similar to that described in Schecter et al. 1999, but with doubly periodic boundary conditions.
4b. Time-Scale for the Onset of Rapid Vortex Intensification

Figure 6 shows the “transition time” $\tau_t$ versus the separation distance $D$ in all simulations initialized with strong midlevel vortices. The working definition of $\tau_t$ is the time at which the altitude $z_{max}$ of $\bar{v}_{max}$ drops below 1 km. In all cases considered, $t \approx \tau_t$ corresponds to when $\bar{v}_{max}$ begins rapid amplification. For systems in which two tropical cyclones develop, the two vortices may have slightly different values of $\tau_t$. The plotted error bars are partly attributable to this, and partly attributable to finite time sampling of the data.\(^5\)

Figure 6 verifies that initializing the middle-to-upper troposphere with high RH expedites the early stage of tropical cyclone formation at any $D$. More notably, if $D = 2r_c$ (100 km) in either setting, the transition time $\tau_t$ is considerably smaller than the value found when the model is initialized with only one vortex. Smaller $\tau_t$ occurs here in association with immediate core merger at all vertical levels and faster initiation of deep convection. Perhaps the main contributing factor to smaller $\tau_t$ is the greater initial kinetic energy within (say) a 100-km radius of the system-center. At high RH, increasing $D$ to $4r_c$ and beyond brings $\tau_t$ closer to its single-vortex value. Here the transition from having midlevel to surface dominant winds occurs in both vortices amid their mutual corotation and shearing. Changing the magnitude of this interaction by changing $D$ appears to have only a subtle effect on $\tau_t$. At low RH, $\tau_t$ becomes indefinitely large in an interval that includes $D = 5r_c$ and $D = 6r_c$. Nondevelopment (for at least 380 h) in this particular interval was noted earlier and will be explained shortly. For $D = 8r_c$ and $D = 12r_c$, $\tau_t$ at low RH is between its minimum and single-vortex values.

\(^5\)Depending on the simulation, the data was archived once every 0.5, 1 or 6 h.
4c. PV Evolution Leading to Tropical Cyclone Formation and Nonformation

Figure 7a illustrates the evolution of PV for a marginal development scenario, initialized with two strong vortices separated by 200 km in a low RH environment. Here the PV is defined by \((\zeta_\theta + f)/\sigma\), in which \(\zeta_\theta\) and \(\sigma\) are respectively the isentropic relative vorticity and density. The initial evolution involves the merger of midlevel PV anomalies and the outward motion of low-level PV anomalies. The process leaves a residual amount of lower tropospheric PV at the periphery of the merged midlevel vortex. In due time, deep convection becomes active and brings about the formation of a tropical cyclone.

Figure 7b illustrates the evolution of PV for a nondevelopment scenario, initialized with two strong vortices separated by 250 km in a low RH environment. Here the complete expulsion of the lower tropospheric PV anomalies appears to decouple the merged midlevel vortex from the sea-surface and nullify its ability to stimulate deep convection. Following merger, the near-surface circulation is barely discernible within a \(2.5r_c\) radius of the midlevel vortex center. (At \(t = 60\) h, \(-0.1 < \bar{v} < 0.002\) m s\(^{-1}\) for \(z = 25\) m and \(r \leq 125\) km.) There is no sign of development in the vicinity of the midlevel system for at least 380 h with the configuration of CM1 used for this study. Furthermore, the surface winds of the ejected low-level vortices gradually decay with time. It would seem that the detached surface vortices are too weak to exploit the oceanic moisture source and independently undergo convective intensification in their local environments [cf. Rotunno and Emanuel 1987].

It is found that incomplete expulsion of surface PV in the developing system [Fig. 7a] coincides with certain statistical properties of the lower troposphere that could slightly improve the chances of convective invigoration underneath the midlevel vortex. Consider an arbitrary time after midlevel merger and before the appearance of persistent deep convection. At \(t = 96\) h, for example, the near-surface horizontal wind speed has an rms value of 0.7 m s\(^{-1}\) within a \(3r_c\) radius of the midlevel vortex center. While seemingly modest, this rms wind speed exceeds that found at \(t = 96\) h in the nondeveloping system [Fig. 7b] by
49%. The enhanced surface winds provide extra kinetic energy and amplify the dimensional surface enthalpy-flux coefficient, defined as $C_k$ times the local wind speed. Coincident with such benefits, the average RH below $z = 2$ km within a $0.5r_c$ ($2r_c$) radius of the midlevel vortex center is 91% (89%), compared to 85% (85%) in the nondeveloping system. The corresponding averages of RH between $z = 2$ and 5.5 km are less distinct between the two systems, in differing by only 1-2%.

5. Mechanism Responsible for Inhibiting Development

The shutting down of tropical cyclone formation at low RH in a specific interval of $D$ merits further consideration. The preceding flow-visualizations suggested that the mechanism requires the merger of midlevel PV anomalies and complete expulsion of surface vortices before convective intensification has a chance to fully activate. In principle, such merger and expulsion may involve the cooperation of a number of fundamental processes. This section explains the rationale for believing that one such process – the excitation of rotating misalignments – is especially important. In addition, this section presents evidence that the merger-expulsion dynamics can operate and inhibit development at high RH if the initial vortices are moderately weakened in the middle troposphere.

5a. Free Rotation of Misalignments in Isolated Mesoscale Vortices

One may reasonably assume that the vertical shear of horizontal wind associated with the outer velocity field of one vortex acts to misalign the other. Figure 8a shows how a similar misalignment would evolve in an isolated vortex. The initial vorticity distribution is given by Eq. (1) with $r \rightarrow \sqrt{(x - x_c)^2 + y^2}$, in which $x$ and $y$ are Cartesian coordinates, $x_c \equiv \epsilon[Z(z) - (Z_m + Z_s)/2]$, $Z_m = 0.65$ is the maximum of $Z$ occurring at $z_m = 5.5$ km, and $Z_s \equiv Z(0) = 0.15$. The value of $\epsilon$ corresponding to the figure is 213.3 km. The intensity
and vertical structure parameters ($\zeta_c = 0.001 \text{ s}^{-1}$ and $\delta h = 1.75 \text{ km}$) correspond to those of a strong vortex. All other fields are obtained from the equations of nonlinear balance [cf. appendix B1 of Schecter 2011]. Moisture and surface fluxes are eliminated throughout the simulation.

The misaligned vortex divides into lower, middle and upper sections. The lower and upper sections are defined to be those parts of the vortex in the lower and upper troposphere that initially move in the negative $y$-direction. By contrast, the entire midsection initially moves in the positive $y$-direction. As time progresses, the upper and lower sections of the misaligned vortex rotate counter-clockwise about the midsection.

Let $\bar{x}_\theta$ denote the average horizontal coordinate vector weighted by PV$^4$ over the area of positive PV within a 350-km radius of $(x, y) = 0$ on the isentropic surface with potential temperature $\theta$ (in K). The coordinates of $\bar{x}_\theta$ give the nominal center-point of the vortex at the altitude corresponding to $\theta$. The coordinates of $\bar{x}_{322}$ specifically give the midlevel center-point of the vortex. Figure 8b shows the orientation angle $\varphi(\theta, t)$ of the horizontal displacement vector $d(\theta, t) = \bar{x}_{322} - \bar{x}_\theta$ for two lower tropospheric values of $\theta$. To be precise, $\varphi$ is the angle between $d$ and the positive-$x$ unit vector. Different curves in each plot correspond to different magnitudes of the initial perturbation strength $d_0 \equiv |d(300, 0)| \propto \epsilon$. The rotation frequency $\dot{\varphi} \equiv \partial \varphi / \partial t$ is seen to increase with $\theta$ (and therefore $z$) in the lower troposphere. By contrast, greater perturbation strength decreases $\dot{\varphi}$ by reducing the coupling between the midsection and vertical extremities of the vortex. Note that the rotation frequencies in the lower and upper sections of the vortex are not perfectly symmetric. The asymmetry is likely introduced by vertical variation of density and static stability, not to mention the unequal distances between $z_m$ and the two vertical boundaries.

To construct a simplified model for the behavior of a mesoscale vortex exposed to the vertical shear of its binary companion, the differentially rotating misalignment will be approximated by a single normal mode. The small-amplitude characteristics of the mode are obtained from the experiment with $\epsilon = 32 \text{ km}$ and $d_0/2r_c = 0.26$. Here one finds that
the midsection of the misaligned vortex is bounded below at \( z \approx 2.5 \) km and bounded above at \( z \approx 8.6 \) km. In addition, one finds that \( \omega = (3.0 \pm 1.3) \times 10^{-5} \) s\(^{-1}\) may serve as a reasonable estimate of the mode frequency. The preceding estimate is a vertical average of \( \pi/2 \tau_y \) plus-or-minus the standard deviation of the same quantity over the intervals \( 0 \leq z \leq 2.5 \) km and \( 8.6 \leq z \leq 11 \) km. The “quarter wave-period” \( \tau_y(z) \) is defined as the time required for the \( y \)-coordinate of \( \bar{x}_\theta \) to decrease from zero to its maximum negative value, with \( \theta \) corresponding to the ambient potential temperature at height \( z \).

5b. Midlevel Merger Caused by the Excitation of Misalignment Modes

Figure 9 is an idealized illustration of the early interaction between two symmetric vortices in the absence of moist convection. In this picture, the growth and rotation of a misalignment mode decreases the midsectional orbital radius \( \rho_m \) of a vortex and increases the surface orbital radius \( \rho_s \). In other words, the excitation of misalignment modes converges midlevel PV toward the center \( O \) of the binary system while ejecting surface (and upper) PV. The following appeals to a well-known heuristic theory of vortex motion in ambient vertical wind shear to estimate the upper bound of \( D \) for which the depicted inward motion of midlevel PV can cause merger.

Let \( d \) denote the distance of the horizontal line segment connecting the midlevel and surface centers of each vortex, and let \( \varphi \) denote the angle between that line segment and the orbital tangent as shown in Fig. 9. Suppose for the moment that \( d \) is sufficiently small to permit the neglect of curvature and horizontal variation of the local shear-flow. The preceding supposition requires that the vertical shear is sufficiently weak for the vortex to remain intact. The simplified vortex-in-shear theory of RMG04 (appendix A; cf. appendix B)
can then be used to estimate that

\[ d \approx \frac{2\Delta V}{\omega} \sin \varphi \]  

and

\[ \varphi \approx \text{mod}(\omega t/2, \pi), \]  

in which \( \omega \) is the natural angular rotation frequency of the misalignment mode, \( \Delta V \) is the difference between the midlevel maximum and surface value of the azimuthal velocity field produced by one vortex at the location of the other, and \( \text{mod}(a, b) \) is the difference between \( a \) and the nearest integer multiple of \( b \) less than \( a \). Equations (6a) and (6b) suggest that a weak local shear-flow causes cyclic motion of the misalignment mode, with \( d \) maximized when \( t = \pi/\omega \) and \( \varphi = \pi/2 \). Note that slow damping (neglected here) would cause the misalignment to settle into a stationary state as \( t \) tends toward infinity [RMG04].

Figure 10 shows the initial orbital radius of a vortex \((D/2)\) minus the maximum value of \( d/2 \) estimated from Eq. (6a). Evaluation of Eq. (6a) requires knowledge of \( \omega \) and \( \Delta V \). The isolated vortex experiment of section 5a suggests that \( \omega = (3.0 \pm 1.3) \times 10^{-5} \text{ s}^{-1} \) is a reasonable parameterization for the strong vortices considered in this study. Equation (1) implies that an appropriate formula for the vertical shear is

\[ \Delta V = \frac{\zeta_c(Z_m - Z_s)r_c^2}{2D} \left[ 1 - \left( \frac{D}{r_h} \right)^2 \right] H(r_h - D). \]  

Different curves in Fig. 10 correspond to different values of \( \omega \) within the limits of uncertainty. The lowest curve is viewed as an approximation of the minimum value of the midsectional orbital radius \( \rho_m \) that is achievable through the excitation of a misalignment mode. The formal restriction of Eq. (6a) to small \( d \) is temporarily ignored.

For midlevel merger to occur, the rotating misalignment modes must bring \( \rho_m \) down to a certain critical value \( \rho_* \). It seems reasonable to assume that \( \rho_* \approx 1.6r_c \), based on studies of symmetric vortex merger in ideal 2D hydrodynamics [Rossow 1977; Saffman and Szeto 1980; Overman and Zabusky 1982; Fine et al. 1991]. With this assumption, Fig. 10 suggests
that midlevel merger is possible through the growth and rotation of misalignment modes only if $D \lesssim D_u = 300$ km. The preceding result is in fair agreement with the transition from merger to non-merger dynamics between 300 and 400 km in the binary vortex experiments initialized with strong vortices and low RH.

As noted earlier, the midlevel merger process prevents tropical cyclone formation only when the attendant growth of $\rho_s$ is sufficiently large for the surface PV anomalies to end up detached from the midlevel system. If the initial orbital radius of the binary system ($D/2$) is close to $\rho_*$, merger shortly terminates the misalignment cycle. Premature termination of the misalignment cycle limits the growth of $\rho_s$ and may therefore deactivate the mechanism that would otherwise stop convective vortex intensification. A need for progression of the misalignment cycle would seem to explain why $D$ must exceed a threshold greater than $2\rho_*$ while remaining less than $D_u$ to prevent tropical cyclone formation in the low RH binary vortex experiments under consideration.

5c. Predictions of a Nonlinear Analogue Model

Section 5b overlooked a number of dynamical factors in discussing how a baroclinic vortex reacts to the vertical shear imposed by its binary companion. One such factor is curvature of the effective shear-flow. Another is the inequality of midlevel and surface displacements from the axis of a rotating misalignment. A third is weaker coupling between the midsection and vertical extremities of the vortex with increasing $d$.

The following compares the behavior of vortices in strong binary systems simulated with CM1 to the predictions of an analogue model that includes all of the aforementioned elements. The model is presented below and expounded in appendix B. Note that the equations constituting the model are similar to those considered elsewhere for baroclinic vortex motion in slightly different contexts [Walsh and Pratt 1995; Smith et al. 2000; Vandermeirsch et al. 2002].
5c.1 Synopsis of the Analogue Model

Let the index $\alpha \in \{1, 2\}$ refer to (1) the lower section and (2) the midsection of an individual baroclinic vortex in a symmetric binary system. The motion of the upper section is assumed to be the same as that of the lower section and is not considered explicitly. Let $x_\alpha$ denote the horizontal coordinate vector of the PV centroid at level $\alpha$, defined such that $x = 0$ corresponds to the center of the binary system. The equation of motion for $x_\alpha$ is here approximated by

$$\frac{d}{dt} x_\alpha = V_\alpha(\rho_\alpha; D) \hat{z} \times x_\alpha \rho_\alpha + Q_\beta F \left[ \left( \frac{d}{\lambda} \right)^2 \right] \hat{z} \times (x_\alpha - x_\beta),$$  \tag{8}$$

in which $\rho_\alpha \equiv |x_\alpha|$, $d \equiv |x_1 - x_2|$, $\hat{z}$ is the vertical unit vector, and $\beta \neq \alpha$. It is assumed that $V_\alpha$, $Q_\beta$ and $F$ are positive. Each of these factors are discussed below.

The first term on the right-hand side of Eq. (8) is intended to mimic the primary influence of the companion vortex. It has the form of a cyclonic circular shear-flow that varies with both radius from its center ($\rho$) and height (indicated by $\alpha$). Appendix B2 gives the specific magnitude and functional form of $V_\alpha$ deemed to provide an adequate representation of the companion vortex, when the two vortices of the binary system are initially separated by a distance $D$. The second term on the right-hand side of Eq. (8) arises from the interaction between the midsection and the phase-locked vertical extremities of the vortex whose motion is being modeled. It specifically accounts for the intrinsic tendency of the PV centroids at levels 1 and 2 to rotate around each other. The interaction parameters $\{Q_\alpha\}$ determine the maximum rotation frequency and the ratio of PV-centroid displacements from their center of corotation. The function $F$ determines how the rotation frequency decays with the inter-centroid displacement $d$ normalized to the characteristic decay length $\lambda$. Appendix B2 provides the specific functional form of $F$. In addition, appendix B2 derives specific param-
eterizations for \( \{Q_\alpha\} \) and \( \lambda \) (170 km) from the isolated vortex experiments of section 5a. In what follows, the PV centroid positions are generally given in a polar coordinate system \((\rho, \phi)\) whose origin coincides with the center of the binary system.\(^6\) Note that reflectional symmetry about the origin eliminates the need for a distinct set of equations for the PV centroids of the companion vortex.

The vertical shear imposed in the analogue model to mimic the influence of the companion vortex acts to separate centroids 1 and 2. The azimuthal separation excites a rotating misalignment mode that initially decreases \( \rho_2 \) and increases \( \rho_1 \) [cf. Fig. 9 with \( \rho_m \to \rho_2 \) and \( \rho_s \to \rho_1 \)]. For the parameter settings in appendix B2, two distinct scenarios follow. Figure 11 depicts the PV-centroid trajectories in both cases. If \( D \) is sufficiently small, as in Fig. 11a, \( \rho_2 \) drops to the estimated critical radius for midlevel merger \( (\rho_* = 1.6r_c) \) before or shortly after \( \delta \phi_{12} \equiv \phi_2 - \phi_1 \) reaches \( \pi/2 \). The behavior of the system afterwards does not seem relevant to the binary vortex experiments. Nevertheless, it can be described as the cyclic reunification and severing of the PV centroids. If \( D \) exceeds a well-defined threshold, as in Fig. 11b, the PV centroids remain closely bonded in a state of shear-modulated corotation. Most importantly, \( \rho_2 \) remains greater than \( \rho_* \) forever.

The centroid motion predicted by the analogue model is consistent with the existence of an interval of \( D \) where midlevel merger occurs and the surface PV expulsion is comparable to that found in nondeveloping systems at low RH.

Figure 12a shows the orbital radius \( \rho_1 \) of the low-level centroid and the orbital radius \( \rho_2 \) of the midlevel centroid at the time \( \tau_e \) marking one of two possible events that may occur depending on the value of \( D \). The solid curves under present consideration show results from the analogue model parameterized to predict the behavior of the strong binary vortex experiments. The dotted curves and symbols will be discussed later. For \( D \) less than about 330 km, \( \tau_e \) is essentially the time of midlevel merger. Specifically, \( \tau_e \) is the time at which \( \rho_2 = \rho_* \) or (if earlier) \( \delta \phi_{12} = \pi/2 \). The latter time is the presumed applicability limit of the

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\(^6\)This should not be confused with a vortex-centered coordinate system denoted in this paper by \((r, \varphi)\).
analogue model. It so happens that when $\delta \phi_{12} = \pi/2$ before $\rho_2 = \rho_*$, the coincident value of $\rho_2$ is very close to $\rho_*$. For $D$ greater than 330 km, the centroids remain bonded and $\rho_2$ never drops close to the critical merger radius. In this regime, $\tau_e$ is the time at which $\rho_2$ reaches its absolute minimum (and $\rho_1$ is simultaneously maximized). The transition between the merger scenario and the bonded centroid scenario is evident in the discontinuities of the orbital radius curves.

For $D$ between 210 and 330 km, $\rho_1$ is greater than $3r_c$ at the time $\tau_e$ of imminent midlevel merger. The CM1 experiments suggest that $\rho_1$ of this magnitude is sufficient to prevent the onset of convective vortex intensification at low RH. The misalignment dynamics of the analogue model therefore provides a plausible explanation for the thorough surface PV expulsion attending midlevel merger that seems responsible for preventing tropical cyclone formation when $D$ is in the vicinity of 250 and 300 km. At smaller $D$, the model predicts merger but insufficient expulsion. At larger $D$, the model predicts that the vortices will remain intact, with only modest misalignment [Fig. 12c]. To facilitate the following discussion, the special range of $D$ between 210 and 330 km will be called the merger-expulsion (ME) interval.

One caveat of the preceding discussion is that to inhibit tropical cyclone formation, the merger-expulsion process should reach an advanced stage before the time $\tau_t$ required for the onset of intensification. Figure 12b compares values of $\tau_e$ predicted by the analogue model (solid curves) to the ranges of $\tau_t$ found in developing binary systems initialized with strong vortices (colored rectangles). Consider first intensification under conditions of low RH. For values of $D$ in the lower part of the ME interval, the predicted merger time $\tau_e$ is less than any low-RH value of $\tau_t$. In the upper part of the ME interval, $\tau_e$ breaches the lower limit of $\tau_t$. Nevertheless, the PV centroids typically exhibit severe displacement at earlier times. For example, for $D = 300$ km, the inter-centroid displacement $d$ is $2.3r_c$ when $t$ reaches the

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7When $\delta \phi_{12}$ increases beyond $\pi/2$, the neglected radial motion caused by interaction between the midlevel centroid and the vertical extremities of a companion vortex may become significant in the actual binary system.
minimum $\tau_t$. It is not unreasonable to speculate that such a large misalignment would itself
considerably delay intensification. Accordingly, for most $D$ in the ME interval, the analogue
model seems to predict a sufficiently fast merger-expulsion process to disable convective
intensification at low RH. The same cannot be said at high RH, where all observed values
of $\tau_t$ are less than the merger times over most of the ME interval.\(^8\)

Providing some limited insight into parameter sensitivity, the dotted curves in Fig. 12
show results from the analogue model when the decay length $\lambda$ of the centroid interaction
is doubled from 170 to 340 km. With the greater value of $\lambda$, the transition from sever-
able to bonded centroids with increasing $D$ occurs somewhat sooner, and the maximum
inter-centroid displacement at $\tau_e$ is not quite as large. Otherwise, there are no remarkable
differences with the primary results. The basic picture is also the same when $\lambda$ is dropped
to 130 km, or when the exponent $\mu$ prescribing the radial variation of the circular shear
flow (see appendix B2) is raised from 2 to 3 (not shown). One minor difference in either
case is that $\rho_2$ (at $t = \tau_e$) deviates slightly farther from $\rho_s$ toward $2r_c$ as $D$ increases to
the transition value.

5c.2 Further Comparison to the CM1 Experiments

The analogue model is now compared more closely to the CM1 experiments initialized with
two strong vortices at low RH. A detailed comparison requires a procedure for computing
the central position vectors $x_m$ and $x_s$ of the midlevel and surface components of an experi-
mental vortex. Similar to the method used in section 5a, $x_m$ is equated to an average of
the horizontal position vector weighted by PV\(^4\) at $\theta = 322$ K. The averaging is taken over

\(^8\)Note that the merger time essentially equals $\tau_t$ at high RH when $D$ has a value (such as 200 km) just
short of the ME interval. For a strong binary system with $D = 200$ km, one finds that misalignment and
midlevel merger at high RH progress simultaneously with the development of two tropical cyclone cores in
the overlap regions between midlevel and surface PV that exist in both vortices. This stands in contrast to
the scenario at low RH and $D = 200$ km, where the development of a single tropical cyclone core within
the merged midlevel vortex does not occur until well after midlevel merger completes (without thorough
expulsion of the surface PV anomalies).
an area of positive PV within a radius \( R \) of a subjective approximation for the coordinates of \( x_m \) determined by visual inspection. The value of \( R \) is made sufficiently small to prevent overlap of the averaging region with the companion vortex, and ranges from 75-250 km. The estimated error of the orbital radius obtained from this method is 5-10\%, based on comparisons to alternative computations. The nominal surface center \( x_s \) is determined as for \( x_m \) but with \( \theta = 300 \text{ K} \).

The discrete symbols in Fig. 12a show orbital radii at key events in the experiments. Because numerical error slightly corrupts the symmetry of the binary system, the plotted radii are averages between the two constituent vortices. For \( D \leq 300 \) km, the \( \times \)s show the midlevel and surface radii (\( \rho_m \) and \( \rho_s \)) at the midlevel merger time, indicated by the condition \( \rho_m = \rho_s = 1.6r_c \). The +s at \( D = 250 \) and 300 km show \( \rho_m \) and \( \rho_s \) when the azimuthal separation between the midlevel and surface centers of each experimental vortex (\( \phi_m - \phi_s \)) reaches \( \pi/2 \). Because midlevel merger occurs shortly thereafter, the +s are close to the \( \times \)s. It is found that the plotted experimental values of \( \rho_s \) are within 25\% of the values expected from the analogue model (\( \sim \rho_1 \) at \( t = \tau_e \)).

For \( D = 400 \) km, the experimental vortices remain intact. In this case, the thick \( \times \)s in Fig. 12a show the orbital radii when \( \rho_s - \rho_m \) is maximized. The thick +s show the orbital radii at the nearly equivalent time \( \pi/\omega_o \), when \( \rho_s - \rho_m \) is theoretically maximized under the assumption that the misalignment stays very small. Although the analogue model correctly predicts bonded centroids at \( D = 400 \) km, its predictions for the orbital radii at the time of maximal misalignment (\( \rho_1 \) and \( \rho_2 \) at \( t = \tau_e \approx \pi/\omega_o \)) are slightly off. This moderate quantitative discrepancy with the experimental data may be partly attributable to the prior onset of moist convection.

The discrete symbols in Fig. 12b essentially show the times at which the events of midlevel merger or maximal misalignment occur in the experiments.\(^9\) The discrete symbols in Fig. 12c show the coincident inter-centroid displacements, given by \( d = |x_m - x_s| \).\(^9\) It is

\(^9\)The exact event corresponding to each symbol (for a given \( D \)) is the same as in Fig. 12a.
seen that each experimental event-time agrees reasonably well with that predicted by the analogue model ($\sim \tau_e$). The inter-centroid displacements are also in fair agreement with their corresponding values in the analogue model ($|x_2 - x_1| \text{ at } t = \tau_e$). The most noticeable discrepancy at $D = 300 \text{ km}$ is associated with the latest event-time. During the lead-up, factors neglected by the analogue model may become increasingly relevant (see section 5e).

While it is not claimed to be irrefutable proof, the foregoing analysis suggests that the excitation of rotating misalignments in each vortex is largely responsible for midlevel merger and low-level PV expulsion in the CM1 simulations at hand. It is worth bearing in mind that such misalignment dynamics is expected to vary with the PV distribution of each vortex and the moist static stability of its core [cf. RMG04; Schecter and Montgomery 2004,2007; Reasor and Montgomery 2015; Schecter 2015]. Structural modifications to each vortex may also change the characteristic magnitude of $\tau_t$. It is therefore reasonable to assume that details of the binary vortices factor into whether or not there exists an interval of $D$ where the merger-expulsion phenomenon can occur on a time-scale that is sufficiently fast for the process to inhibit convective vortex intensification. A thorough investigation of this matter would be a worthwhile follow-up to the present study. The discussion below merely considers the consequences of switching from strong to weak midlevel vortices of the kind defined in section 2.

5d. Inhibition of Tropical Cyclone Formation in the Weak Vortex Experiments

The preceding discussion focused on a mechanism for deactivating tropical cyclone formation in binary systems initialized with strong midlevel vortices. For these systems, the efficacy of the mechanism requires not only intermediate values of $D$, but also low RH. The same conditions can also prevent tropical cyclone formation in weaker binary systems. Moreover, for certain values of $D$, the longer time required to initiate rapid intensification in a weaker system can allow midlevel merger and surface PV expulsion to occur even at high RH. While
this reconfiguration of the flow may not prevent tropical cyclone formation at high RH, it coincides with a significant delay of intensification. The following elaborates on the preceding statements.

Figure 13a shows the temporal evolution of the maximum surface wind speeds in five experiments where the constituent vortices of the binary systems are initially diminished in the middle troposphere [Figs. 1f-1h]. Under low RH conditions, the weak system configured with $D = 100$ km transitions into a tropical cyclone faster than an individual vortex would develop in isolation [cf. Fig. 3f, dashed blue curve]. By contrast, there is no sign of development for at least 400 h when $D = 250$ km. As in the stronger systems, nondevelopment involves midlevel merger and surface PV expulsion (not shown).

Under high RH conditions, the weak system intensifies relatively fast when $D = 100$ km and 250 km. In the former case, merger occurs immediately at all levels. The onset of vigorous deep convection and the initiation of rapid vortex intensification occur in less than one day. In the latter case, persistent deep convection begins before vertical shear induces the midsection of each vortex to fully separate from its surface section. Rapid intensification is well underway after two days of simulation time. Two hurricanes develop and eventually merge (not shown).

A different scenario is found at high RH when the weak binary system is configured with $D = 200$ km. Here the system clearly exhibits midlevel merger and marginal expulsion of lower tropospheric PV [Fig. 13b]. The merger event occurs at a time between 42 and 48 hours. Only sporadic deep convection is seen before then, and for approximately one day after. Figure 13c is a typical snapshot of middle-tropospheric vertical velocity superposed on the rain field associated with this convection. Figure 13d compares the initial azimuthal velocity $\bar{v}$—measured from the center of the system—to that found 20 hours after merger. The ejection of surface vortices shifts the peak of $\bar{v}$ radially outward. Consistent with the low RH experiments, midlevel merger does not appear to stimulate the immediate amplification of cyclonic winds directly underneath the merged vortex ($\rho \lesssim 100$ km).
takes roughly one day after merger for persistent deep convection to materialize within the midlevel vortex. Rapid intensification gets well underway by $t = 3.5$ days, and the midlevel vortex eventually transforms into a single hurricane. The ejected surface vortices become sheared into spiral filaments.

5e. Additional Dynamics

Despite providing considerable insight, the analogue model of section 5c seems to have missed two notable aspects of the experimental misalignment dynamics. One missing aspect is the eventual transformation of the surface vortices into objects resembling asymmetric dipoles. Figure 14 illustrates the process for the simulation initialized with a strong binary system, low RH and $D = 250$ km. The development of dipole-like structures appears to assist the long-term expulsion of the surface vortices.\textsuperscript{10} Another unaddressed feature (considered here for simulations initialized with strong vortices and low RH) is the emergence of an azimuthally averaged radial inflow toward the center of the binary system in the lower-to-middle troposphere. For $D = 250$ km (300 km), the magnitude of the symmetric inflow is of order 0.2 m s$^{-1}$ (0.1 m s$^{-1}$) in the midsection of each vortex. Such a magnitude seems nonnegligible compared to the time-averaged radial drift velocity before merger, given by $d\rho_m/dt = -0.45$ m s$^{-1}$ ($-0.3$ m s$^{-1}$).

As a further remark, the effects of Ekman-like pumping and core diffusion by subgrid turbulence seem unessential to the experimental vortex dynamics that leaves the system unable to proceed with hurricane formation at low RH. Repeating the $D = 250$-km simulation with a stress-free surface boundary condition (and without moisture) yields very similar

\footnotesize\textsuperscript{10}The transformation of surface vortices into objects resembling asymmetric dipoles in Fig. 14 appears to be associated with the baroclinic nature of the system. It failed to occur in a nondivergent 2D simulation with the same initial horizontal distribution of vertical vorticity. One notable distinction of the baroclinic system at $t = 0$ is a slight positive growth trend of $\phi$-averaged (background) PV with increasing radius $\rho$ near the surface in the broad neighborhood of the binary vortices. Further meticulous study would be required to determine the extent to which this feature might contribute to the development of the dipole-like structures seen here [cf. Carnevale et al. 1991b; Schecter and Dubin 2001; Chan 2010].
results for midlevel merger and expulsion of surface vortices. Furthermore, the horizontal momentum mixing coefficient $K_h$ is generally less than 50 m$^2$s$^{-1}$ in the cores of the midlevel vortices in the time leading to merger. The diffusive lengthscale ($\sqrt{K_h t}$) over a 1-3 day period for such $K_h$ is merely 2-4 km.

6. Merger After the Onset of Intensification

If the time-scale for the onset of convective vortex intensification is less than or equal to that of the adiabatic misalignment and midlevel merger process, the nature of the binary vortex interaction changes considerably. The morphology of each vortex is radically altered by the emergence of a compact convective core. There are no remarkable cases in the experiments under consideration where the interaction of such transformed vortices permanently reverses the intensification process. Nevertheless, the merger of the convectively intensified cores is an essential part of system development in pre-humidified environments, when $D$ is within some intermediate range of values. A brief description of such merger is therefore appropriate.

The discussion herein centers on the experiment with two strong vortices that are initially separated by 200 km in a high RH environment [Fig. 2]. In this particular experiment, the vortices commence intensification but do not mature before their cores coalesce.

Figures 15a and 15b show the evolution of several velocity and lengthscale statistics characterizing the state of the binary system. The velocity statistics include $\bar{v}_{max}$ for the individual vortices and $\bar{v}_{max}$ for the system as a whole. The former (vortex-centric) velocity statistics are calculated with coordinate systems centered on the individual vortices. The latter (system-centric) velocity statistic is calculated with a coordinate system centered on the midpoint between the two vortices, or on the center of the merged storm. Also plotted are the maximum and rms near-surface wind speeds in a $534 \times 534$ km$^2$ box centered on the initial midpoint of the two vortices. It is seen that the individual vortices weaken after
their separation distance becomes sufficiently small. The near-surface maximum wind speed follows a similar trend. At the same time, deep convection loses strength and becomes less focused on the vortex cores [Fig. 15c]. On the other hand, the rms near-surface wind speed steadily amplifies and the growth of system-centric $\bar{v}_{\text{max}}$ accelerates.

The process of core merger in the experiment under consideration appears to be relatively complex, and not solely governed by a vortex-vortex interaction. To begin with, the cores are embedded in a deformable background PV field left behind by the parent vortices and modified by earlier diabatic processes. It is well-known that the interactions of concentrated vortices with background PV can play an important role in driving merger [Carnevale et al. 1991a; Ritchie and Holland 1993; Schecter and Dubin 2001; Soga et al. 2003]. Furthermore, the horizontal flow field has significant divergence. One way to appreciate the latter fact is to decompose the horizontal velocity field into its rotational (solenoidal) and irrotational components. By the Helmholtz decomposition, the horizontal velocity field $\mathbf{u}$ may be written

$$\mathbf{u} = \hat{z} \times \nabla_h \Psi + \nabla_h \Phi,$$

in which $\nabla_h$ is the horizontal gradient operator. The first term on the right-hand side (henceforth $\mathbf{u}_\Psi$) has zero divergence and is determined by the vertical vorticity field through the relation $\nabla_h^2 \Psi = \zeta$. The second term (henceforth $\mathbf{u}_\Phi$) has zero vorticity and is determined by the divergence of $\mathbf{u}$ through the relation $\nabla_h^2 \Phi = \nabla_h \cdot \mathbf{u}$. Both Poisson equations are solved here with the doubly periodic horizontal boundary conditions appropriate to the present numerical experiments. Figure 16 shows both $\mathbf{u}_\Psi$ and $\mathbf{u}_\Phi$ at a selected time ($t = 28$ h) during the merger process. Comparison of the rms and maximum wind speeds associated with $\mathbf{u}_\Psi$ and $\mathbf{u}_\Phi$ at various levels (printed on the plots) suggest that the irrotational flow field is non-negligible. Moreover, $\mathbf{u}_\Phi$ exhibits strong lower-tropospheric convergence in between the two vortices [Fig. 16e]. It is reasonable to speculate that such convergence (associated with deep convection) substantially assists merger. The attendant production of a local
vorticity perturbation might also influence the process.

It is readily verified that the coalescence of vortex cores in the moist convective system differs substantially from idealized dry-adiabatic vortex dynamics. Figure 17 compares the actual diabatic merger process to that found in a simplified simulation where moisture and surface fluxes are eliminated at $t = 28$ h. At the same instant, the irrotational component of the velocity field ($u_\psi$) is removed, and the pressure and potential temperature fields are reinitialized to satisfy the equations of nonlinear balance [Schecter 2011, appendix B1]. Although core merger occurs in the simplified simulation, it takes considerably longer to complete.\footnote{Note that to prevent excessive $\theta$-perturbations arising from the imposed balance conditions near the surface, $u_\psi(x, z)$ in the dry-adiabatic simulation is initially reset to $u_\psi(x, z_b)$ for $z < z_b = 1.1$ km. The time required for adiabatic merger of the dominant near-surface vorticity anomalies is found to be somewhat longer with $z_b = 0.32$ km. Note also that adding surface drag (with $C_d = 0.002$) to the dry-adiabatic simulation was not found to expedite near-surface merger.}

7. Sensitivity of Tropical Cyclone Structure to the Initial Vortex Separation Distance

Having seen the potential sensitivity of intensification to the initial vortex separation distance, one might speculate that $D$ also has considerable influence on the properties of a newly formed tropical cyclone. The properties investigated herein pertain to the azimuthally averaged vortex fields, which are indicated by overbars. The vortex-centered cylindrical coordinate system used for the azimuthal averaging is defined as in section 3. The analysis is restricted to simulations initialized with strong midlevel vortices.

Figures 18a and 18b show how two basic tropical cyclone parameters vary with $D$ at the time of peak azimuthal velocity $\tau_p$. Note that $\tau_p$ depends on both $D$ and the environmental RH. Figure 18a shows $\bar{v}_{\text{max}}$, here representing the maximum of $\bar{v}(r, z, \tau_p)$. Figure 18b shows the radius $r_{\text{max}}$ at the point in the $r$-$z$ plane where $\bar{v}_{\text{max}}$ occurs. Missing data points at low RH and intermediate $D$ are attributable to nondevelopment. Regardless of the RH setting, it is seen that $\bar{v}_{\text{max}}$ decreases roughly 20 m s$^{-1}$ as $D$ increases from $2r_c$ to $8r_c$ (100 to 400 km).
A return to greater intensity appears to occur before $D = 12r_c$. The value of $r_{\text{max}}$ increases in proportion to $D$ for cases in which merger occurs prior to peak intensity.\footnote{The high RH simulation with $D = 300$ km is the only case initialized with strong vortices where merger occurs after peak vortex intensity is achieved. At $t = \tau_p$, the two vortices are separated by 280-290 km.} Otherwise, the individual tropical cyclones of the binary system have values of $r_{\text{max}}$ between 8 and 12 km, which is not too far above the resolution threshold of the simulations. Note that $\bar{v}_{\text{max}}$ continues to decay with increasing $D$ beyond the transition from merged to binary systems, where $r_{\text{max}}$ precipitously drops. Such curious behavior is not fully understood at this time, but seems worthy of future investigation.

If the tropical cyclones were stationary and axisymmetric at peak intensity, the theoretical value of $\bar{v}_{\text{max}}^2$ according to Bryan and Rotunno 2009b (BR09b) would be

$$V_{th}^2 \equiv S + \Gamma,$$  \hspace{1cm} (10)

in which

$$S \equiv \frac{T_s}{T_o} \frac{C_k}{C_d} (\bar{T}_b - T_o) (\bar{s}_{s*} - \bar{s}_s) \text{ at } r_{\text{max}},$$  \hspace{1cm} (11)

and

$$\Gamma \equiv \frac{T_s}{T_o} r \bar{\eta}_\phi \bar{w} \text{ at } r_{\text{max}}, z_{\text{max}}.$$  \hspace{1cm} (12)

The definition of $S$ involves the saturation pseudoadiabatic entropy at the sea-surface ($\bar{s}_{s*}$) and the pseudoadiabatic entropy at the top of the surface layer ($\bar{s}_s$). It also involves the following absolute temperatures: the sea-surface temperature $T_s$, the outflow temperature $T_o$, and the temperature $\bar{T}_b$ at the location ($r_{\text{max}}, z_{\text{max}}$) of $\bar{v}_{\text{max}}$. The outflow temperature $T_o$ is here estimated by the ambient temperature at the altitude of maximum radial velocity $\bar{u}$ in the upper troposphere. The definition of $\Gamma$ involves the azimuthal vorticity $\bar{\eta}_\phi$ and vertical velocity $\bar{w}$. The $S$-component of $V_{th}$ exists whether or not the derivation assumes gradient-wind and hydrostatic balance. The $\Gamma$-component arises when these traditional balance assumptions are relaxed.
Assuming quasi-steady and nearly axisymmetric hurricane structure, Fig. 19a suggests that a significant level of imbalance affects the maximum wind speed of each simulated vortex at \( t = \tau_p \) [cf. Smith et al. 2008; Bryan 2012; Schecter 2013]. Let \( I \equiv \sqrt{(S + \Gamma)/S - 1} \) represent the fractional difference between \( V_{th} \) and the magnitude it would have neglecting the imbalance parameter \( \Gamma \). It is seen that \( 0.4 < I < 0.7 \).

Figure 19b provides additional information on how changing \( D \) influences the values of \( S \) and \( \Gamma \) in the developed vortices. The plotted data are differences between each of these variables and their reference values obtained from a single-vortex experiment at the same RH. Such differences are denoted by the prefix \( \delta_r \). To be clear, each variable and its reference value are evaluated at \( t = \tau_p \). It is seen that \( |\delta_r\Gamma| \) usually exceeds \( |\delta_rS| \). In this sense, prior and/or current interaction between neighboring vortices seems to affect \( V_{th}^2 \) more through modification of unbalanced flow (in the azimuthal mean) than through modification of air-sea disequilibrium in the vicinity of the eyewall updraft.

It is interesting that \( \bar{v}_{max}^2 \) and \( V_{th}^2 \) are found to have quantitatively consistent variations with \( D \) [Fig. 19b]. However, the actual value of \( V_{th}^2 \) (as computed here) tends to exceed \( \bar{v}_{max}^2 \) at \( t = \tau_p \) by a magnitude in the neighborhood of \( \varepsilon_o = 1.9 \times 10^3 \text{ m}^2\text{s}^{-2} \). Figure 19c verifies that subtracting \( \varepsilon_o \) from the right-hand side of the theoretical wind speed formula [Eq. (10)] leads to a good empirical formula for the present data set. A moderate discrepancy between \( V_{th}^2 \) and \( \bar{v}_{max}^2 \) is not entirely surprising, partly because discretization error is expected, and partly because the theory under consideration involves a number of simplifications. In addition to assuming steady axisymmetric flow, the derivation of \( V_{th}^2 \) is based on pseudoadiabatic thermodynamics and an idealized treatment of the boundary layer [BR09b].

With respect to the latter, the boundary layer is assumed to be well-mixed with radial advection balancing vertical turbulent transport and dissipative heating.\(^\text{13}\)

\(^{13}\)The boundary layer model essentially provides a formula for the negative radial entropy gradient at the location of maximum wind speed [BR09b, section 2b]. An overestimate of the radial gradient would cause \( V_{th} \) to overestimate \( \bar{v}_{max} \). Regarding the other assumptions, there is evidence that simulations with traditional warm-rain microphysics may produce somewhat weaker tropical cyclones than those with purely pseudoadiabatic thermodynamics [Bryan and Rotunno 2009a]. There is also evidence that non-axisymmetric simulations produce weaker tropical cyclones than their axisymmetric counterparts [Yang et al. 2007; Persing...
8. Conclusions

The preceding sections of this paper examined idealized binary mesoscale vortex interactions in settings that permit tropical cyclone formation. The initial conditions consisted of two identical midlevel vortices devoid of convection. The ambient conditions were characterized by high or low RH in the middle-to-upper troposphere. The cloud resolving model used to simulate the dynamics incorporated a basic parameterization of air-sea interaction, but neglected ice microphysics and radiation for relative simplicity. The main objective was to advance current understanding of how the interaction of neighboring vortices may influence their convective intensification. It has been found that binary interaction can inhibit the onset of intensification if the initial vortex separation distance $D$ falls within a specific interval determined by vortex properties and environmental conditions.

The inhibition interval has been shown to correspond to a parameter regime in which binary vortex interaction leads to midlevel merger and the outward expulsion of surface PV anomalies. An analogue model has been developed to explain (and predict) the range of $D$ in which this behavior is possible under quasi-adiabatic dynamics. The merger-expulsion mechanism accounted for by the model is initiated by the vertical variation of the orbital angular velocity bending each vortex in the binary system [Fig. 9]. The bending excites rotating misalignments that swing the midsections of each vortex inward while swinging their lower sections (and upper sections) outward. Details of the process are predictable from the known dominant frequency and vertical structure of a freely rotating misalignment.

The model demonstrates that if $D$ is too small, midlevel merger will occur before the lower sections of the vortices are completely ejected. If $D$ is too large, the mutual shearing of the binary vortices will be too weak to cause significant misalignment, let alone midlevel merger. et al. 2013; cf. Naylor and Schecter 2014]. It is conceivable that the weakening is partly connected to changes in $S$ and $\Gamma$, and thereby partly consistent with the “zero-order” wind speed formula [Eq. (10)]. One might also speculate that some of the weakening is connected to additional corrective terms.
The predicted interval of $D$ where complete expulsion of surface PV should attend midlevel merger compared favorably to a set of numerical experiments initialized with low RH to hinder deep convection early on. In these experiments, intensification never followed the detachment of the merged midlevel vortex from the lower tropospheric PV anomalies. The process left minimal surface winds underneath the midlevel vortex, and the ejected surface vortices were apparently too weak to exploit the oceanic moisture source and amplify on their own. The analogue model also correctly predicted that there should be no range of $D$ in which the merger-expulsion scenario could operate in simulations initialized with moderately strong vortices in high RH environments. Under such conditions, the time-scale predicted for the quasi-adiabatic merger-expulsion dynamics is appreciably greater than the time required for the onset of diabatic (convective) vortex intensification. Accordingly, moderately strong binary systems at high RH generally produce two hurricanes in the interval of $D$ that prevents tropical cyclone formation at low RH.

One might be tempted to believe that the quasi-adiabatic merger-expulsion phenomenon cannot occur in the relatively high RH parameter regime typical of potentially developing tropical weather systems. However, time scales vary considerably with details of the initial vortex parameters (among other factors). Reducing midlevel vortex intensity increases the time required for the onset of intensification, at least in experiments of the kind considered here. Consequently, midlevel merger and surface PV expulsion are possible for relatively weak binary systems with high RH and intermediate $D$. An illustrative example involving marginal expulsion [Fig. 13b] suggested that the midlevel merger process may cause substantial delay of intensification, even at high RH. Further testing of stochastic variability would be required to make a definitive conclusion on this matter [cf. Nguyen et al. 2008; Zhang and Tao 2013].

If the constituent vortices of the binary system develop persistent cumulus convection, their interaction acquires a different character. The distinct merger process was discussed for a specific simulation initialized with moderately strong midlevel vortices at high RH. The vortices promptly developed compact convective cores with maximal wind speeds near
the surface. The vortex cores weakened during their mutual approach, but the system-centric $\bar{v}_{max}$ and the integrated surface kinetic energy continued to amplify. Consistent with earlier studies, core merger was found to occur more rapidly in the convective binary system than one might expect from dry-adiabatic vortex dynamics [e.g., Wang and Holland 1996; Hendricks et al. 2004; Fang and Zhang 2011].

The final part of this study briefly examined how prior or existent binary vortex interactions may influence the structure of a tropical cyclone in a system that successfully develops. The analysis was limited to simulations initialized with two moderately strong midlevel vortices, and considered storm structure only at the time $\tau_p$ of peak vortex intensity. The size $r_{max}$ and strength $\bar{v}_{max}$ of a fully-developed tropical cyclone were found to grow and decay (respectively) with increasing values of the initial vortex separation distance $D$, provided that vortex merger occurred before $\tau_p$. Systems that were binary at $\tau_p$, here considered for $D$ between $6r_c$ and $12r_c$, had tropical cyclones among the smaller and weaker of those in the simulation set.

Needless to say, the computational configuration used for this study was highly idealized. While the symmetric problem is a good starting point for understanding binary vortex interaction, generalization to the asymmetric problem will be necessary to understand the variety of interactions that may occur in natural systems [cf. Melander et al. 1987; Dritschel and Waugh 1992; Ritchie and Holland 1993; Prieto et al. 2003]. It is also reasonable to assume that the nature of binary vortex interaction and its consequences on development will vary with the unresolved physics parameterizations of the cloud model. Whether or not this is a matter of detail or profound significance remains to be seen. The answer will come from future experiments with more complicated microphysics, more realistic air-sea interaction, better resolved boundary layer processes, and/or atmospheric radiation.

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Appendix A: Linear Response of a Stratified Vortex to Vertical Shear

Consider an axially symmetric (barotropic) Rankine-like vortex\textsuperscript{14} in a stratified atmosphere exposed to the shear-flow $V(z)\hat{x}$. As usual, let $r$ and $\varphi$ denote the radius and polar angle of a vortex-centered cylindrical coordinate system. The heuristic linear model of RMG04 suggests that the PV perturbation will evolve according to the following equation:

$$q' = -\frac{2V}{\omega} \frac{dq}{dr} \sin\left(\frac{\omega t}{2}\right) \cos\left(\varphi - \frac{\omega t}{2}\right),$$  \hspace{1cm} (A1)

in which $\omega$ is the natural oscillation frequency of the misalignment mode excited by the shear flow, and $\bar{q}(r)$ is the unperturbed PV. The derivation of Eq. (A1) assumes that $V$ is proportional to the vertical wavefunction of the mode. In the Boussinesq approximation of RMG04, this would imply that $V$ is specifically proportional to $\cos(kz)$, in which $k$ is the vertical wavenumber.

Suppose that $q'$ is the perturbation generated by displacing the original PV distribution a distance $\delta r(z)$ along the radial unit vector at $\varphi = \omega t/2$ (for $0 \leq \omega t < 2\pi$). To first order in $\delta r$, such a perturbation has the form

$$q' = -\delta r \frac{dq}{dr} \cos\left(\varphi - \frac{\omega t}{2}\right).$$  \hspace{1cm} (A2)

\textsuperscript{14}A Rankine-like vortex has nearly uniform vorticity within its core and negligible vorticity beyond a narrow transition layer at the edge of its core.
Equating the right-hand sides of Eqs. (A1) and (A2) yields

$$\delta r = \frac{2V}{\omega} \sin \left( \frac{\omega t}{2} \right).$$  \hspace{1cm} (A3)

The preceding analysis provides the basis for Eqs. (6a) and (6b) of section 5b. Note that $q'$ and $\delta r$ may be smaller in non-axially symmetric non-Rankine-like vortices. In a generic vortex, natural damping or weaker coupling to the applied forcing could inhibit the growth of a misalignment mode [cf. RMG04; Schecter 2015].

Appendix B: Details of the Analogue Model

The purpose of this appendix is to elaborate on the analogue model for the motion of PV centroids in a vortex exposed to the vertical shear of its binary companion. Such elaboration involves greater discussion of model dynamics and an explanation of the parameterization. A sensible parametrization scheme is important for using the model to assess the plausibility of the following proposition: the intrinsic rotational tendencies of the vortex misalignments are behind the radial migrations of PV centroids in the low RH CM1 experiments.

B1. Simple Solutions

The nature of the analogue model is best understood by considering its solutions. Recall that the model consists of equations of motion for PV centroids in the lower section and midsection of the vortex. The lower and middle centroids are labeled by the subscripts 1 and 2, respectively. The upper level centroid is assumed to move in concert with that of the low level centroid, and is not considered explicitly. Letting $(x_\alpha, y_\alpha)$ denote the Cartesian coordinates of the PV centroid at level $\alpha$, the vector equation of motion for that centroid
[Eq. (8)] can be rewritten as follows:

\[
\begin{align*}
\frac{dx_\alpha}{dt} &= -V_\alpha(\rho_\alpha; D) \frac{y_\alpha}{\rho_\alpha} - Q_\beta F \left[ (d/\lambda)^2 \right] (y_\alpha - y_\beta), \\
\frac{dy_\alpha}{dt} &= V_\alpha(\rho_\alpha; D) \frac{x_\alpha}{\rho_\alpha} + Q_\beta F \left[ (d/\lambda)^2 \right] (x_\alpha - x_\beta),
\end{align*}
\]  

(B1a)  
(B1b)

in which \( \beta \neq \alpha \). Analytical solutions of the complete system of equations governing both centroids are readily obtained in two special limits.

Consider first the approximation in which curvature and radial shear are neglected, and the misalignment of centroids is assumed to be small. In other words, suppose that \( x_\alpha/\rho_\alpha \to 0, \ dV_\alpha/\rho_\alpha \to 0, \) and \( d/\lambda \to 0 \). In this parameter regime, Eqs. (B1a) and (B1b) reduce to

\[
\begin{align*}
\frac{dx_\alpha}{dt} &= -\dot{V}_\alpha - Q_\beta (y_\alpha - y_\beta), \\
\frac{dy_\alpha}{dt} &= Q_\beta (x_\alpha - x_\beta),
\end{align*}
\]  

(B2a)  
(B2b)

in which \( \dot{V}_\alpha \equiv V_\alpha(D/2; D) \). Here it is assumed that both PV centroids are initially situated at \( (x_\alpha, y_\alpha) = (0, D/2) \). It is also assumed that \( F(0) = 1 \), which will be the convention throughout this appendix.

Equations (B2a) and (B2b) imply that the center of interaction coordinates obey

\[
\begin{align*}
X &\equiv \frac{Q_1 x_1 + Q_2 x_2}{Q_1 + Q_2} = -\frac{Q_1 \dot{V}_1 + Q_2 \dot{V}_2}{Q_1 + Q_2} t, \\
Y &\equiv \frac{Q_1 y_1 + Q_2 y_2}{Q_1 + Q_2} = \frac{D}{2}.
\end{align*}
\]  

(B3a)  
(B3b)

In other words, the central \( y \)-coordinate is conserved and the central \( x \)-coordinate moves at constant velocity determined by the local vertical shear-flow and the centroid interaction parameters, until the approximate equations of motion become inaccurate. Of greater
interest, the relative coordinates can be shown to satisfy

$$
\delta x \equiv x_1 - x_2 = \frac{\dot{V}_2 - \dot{V}_1}{\omega_o} \sin(\omega_o t), \quad (B4a)
$$

$$
\delta y \equiv y_1 - y_2 = \frac{\dot{V}_2 - \dot{V}_1}{\omega_o} [1 - \cos(\omega_o t)], \quad (B4b)
$$

in which

$$
\omega_o \equiv Q_1 + Q_2. \quad (B5)
$$

The displacement between the centroids is thus

$$
d \equiv \sqrt{\delta x^2 + \delta y^2} = 2 \frac{|\dot{V}_1 - \dot{V}_2|}{\omega_o} \sin[\varphi(t)], \quad (B6)
$$

in which $\varphi \equiv \text{mod}[\omega_o t/2, \pi]$. One may readily verify that $\varphi(t)$ is the angle between the relative displacement vector $\mathbf{x}_1 - \mathbf{x}_2$ and the unit vector in the positive $x$-direction; that is, $\varphi(t) = \cos^{-1}(\delta x/d)$ assuming $\dot{V}_2 > \dot{V}_1$. The preceding equations for $d$ and $\varphi$ are essentially those of the RMG04 model, provided that $\omega_o$ is the corotation frequency of slightly displaced PV centroids in the absence of imposed shear. The latter fact is verified below.

Suppose that $V_\alpha = 0$ and that the following conditions are satisfied at $t = 0$: $y_\alpha = 0$, $X = 0$ and $\delta x = d_0$. Under the preceding conditions, Eqs. (B1a) and (B1b) yield

$$
\delta x = d_0 \cos(\omega_d t), \quad \delta y = d_0 \sin(\omega_d t), \quad (B7a)
$$

$$
x_1 = -\frac{Q_2}{Q_1} x_2, \quad y_1 = -\frac{Q_2}{Q_1} y_2, \quad (B7b)
$$

in which

$$
\omega_d \equiv F \left[ (d_0/\lambda)^2 \right] (Q_1 + Q_2). \quad (B8)
$$

Equations (B7a)-(B7b) imply that the PV centroids undergo uniform circular motion about the origin at frequency $\omega_d$. Since $F$ is a monotonically decaying function of its argument, $\omega_d$ decreases with increasing misalignment $(d_0)$. The maximum frequency occurring at $d_0 = 0^+$
is $\omega_o$, defined by Eq. (B5). The rotation radius of centroid $\alpha$ is given by $r_\alpha = \sqrt{x_\alpha^2 + y_\alpha^2}$. It follows from (B7b) that

$$\frac{r_2}{r_1} = \frac{Q_1}{Q_2}. \quad (B9)$$

**B2. Parameterization**

The first step in parameterizing the model for comparison to simulations initialized with moderately strong vortices is to match its behavior with $V_\alpha = 0$ to that of the free misalignment experiments in section 5a. Based on the experiment with the smallest initial value of the inter-centroid separation distance, the following equations for the interaction parameters $Q_1$ and $Q_2$ seem reasonable:

$$Q_1 + Q_2 \equiv \omega_o \approx 3 \times 10^{-5} \text{ s}^{-1}, \quad (B10a)$$

$$\frac{Q_1}{Q_2} \equiv \frac{r_2}{r_1} \approx 0.4. \quad (B10b)$$

The rationale for the preceding estimate of $\omega_o$ was explained at the end of section 5a. The rationale for the preceding estimate of $r_2/r_1$ is that the maximum positive value of $y$ of the midlevel centroid is 0.36 times the maximum negative value of $y$ obtained by the near-surface centroid evaluated at $\theta = 300$ K. Interestingly, it is found that the vertically integrated vorticity in the symmetric upper and lower sections of the experimental vortex is 0.4 times that of the integrated midsectional vorticity. In other words, $2 \int_0^{z_l} dzZ = 0.4 \int_{z_l}^{z_l+2z_l} dZ$, in which $z_l = 2.5$ km is the upper boundary of the lower section of the vortex [cf. section 5a]. It is agreeable that the proportionality between $Q_1$ and $Q_2$ equals the proportionality between (1) the combined strength of lower and upper sections of the vortex and (2) the strength of the midsection.

The appropriate value for $\lambda$ depends partly on the functional form chosen for $F$. The form chosen here (and justified shortly) is adapted from the QG equations of motion governing two interacting potential point-vortices [cf. Walsh and Pratt 1995]. These equations are
isomorphic to (B1a) and (B1b) with $V_\alpha = 0$ and

$$F \equiv \frac{1}{[(d/\lambda)^2 + 1]^{3/2}} \quad (B11)$$

Figure B1 compares curve fits of the form $\omega_f F[(d_0/\lambda_f)^2]$ to the frequencies measured in the misalignment experiments. Here $d_0$ is the initial displacement between the PV centroids at $\theta = 322$ and 304 K. The latter level corresponds to where the rotation frequency $\dot{\varphi}$ of the displacement vector (as defined in section 5a) is closest to $\omega_o = 3 \times 10^{-5}$ s$^{-1}$. Multiple data points for a given value of $d_0$ were obtained from measurements of $\dot{\varphi}$ over different time intervals, and their spread provides a reasonable estimate of the uncertainty. The best fit is found with $\lambda_f = 144$ km, and provides a fairly good description of the data. This value is in between two characteristic lengthscales of the vortex that one might expect $F$ to depend on in a more sophisticated model. The first is the vortex diameter $2r_c = 100$ km. The second is the intrinsic deformation radius, defined by

$$l_d \equiv \frac{Nz_m}{\sqrt{(\zeta_v + f)(2\Omega_v + f)}}, \quad (B12)$$

in which $\zeta_v$ is the characteristic vorticity, $\Omega_v$ is the characteristic angular velocity, and $N = 0.011$ s$^{-1}$ is the approximate Brunt-Väisälä frequency [cf. Shapiro and Montgomery 1993]. To evaluate Eq. (B12), the characteristic vorticity and angular velocity are equated to their vertical averages at $r = r_c$. Both averages are approximated by $\zeta_c Z_{0m}/2$, in which $Z_{0m} \equiv \int_0^{z_m} dz Z/z_m$. It follows that $l_d = 170$ km. The value $\lambda = 170$ km is chosen for the parameterization of the analogue model, because it is not unreasonable (according to the dotted curve in Fig. B1) and yields slightly better agreement with the binary vortex experiments when used in combination with the other parameter settings.

The circular shear flow $V_\alpha(\rho_\alpha; D)$ is designed to mimic the tangential component of the velocity field associated with the companion vortex (and its halo) in a binary system. The proper radial dependence of $V_\alpha$ is not entirely clear for the interaction of the misaligned and
deformable mesoscale vortices under consideration. For simplicity, it will be assumed that

\[ V_\alpha = \frac{C_\alpha(D)}{\rho_\alpha^{\mu-1}}, \]  

(B13)

in which \( \mu = 2 \) unless stated otherwise for sensitivity tests. Equation (B13) implies an angular velocity proportional to \( \rho_\alpha^{\mu} \). With greater certainty, the value of \( V_\alpha \) at \( \rho_\alpha = D/2 \) should equal the initial tangential velocity field in the neighborhood of centroid \( \alpha \) in the corresponding binary vortex experiment. The preceding requirement leads to

\[ C_\alpha = \left( \frac{D}{2} \right)^{\mu-1} \frac{\zeta_c Z_\alpha r_c^2 (1 - D^2/r_h^2)}{2D} H(r_h - D), \]  

(B14)

in which \( Z_1 = 0.27 \) is obtained from the vertical average of \( Z \) in the lower section of the vortex \( (0 \leq z \leq 2.5 \text{ km}) \) and \( Z_2 = 0.56 \) is obtained from the vertical average of \( Z \) in the midsection \( (2.5 < z < 8.5 \text{ km}) \). With this parameterization, the ratio \( C_1/C_2 \) equals 0.48.

B3. Polar Coordinates and Hamiltonian Structure

In a polar coordinate system \((\rho, \phi)\) whose center-point coincides with that of the imposed circular shear flow, Eqs. (B1a) and (B1b) become

\[ \frac{d\phi_\alpha}{dt} = \frac{V_\alpha}{\rho_\alpha} + \frac{Q_\beta F}{\rho_\alpha} \left[ \rho_\alpha - \rho_\beta \cos (\phi_\alpha - \phi_\beta) \right], \]  

(B15a)

\[ \frac{d\rho_\alpha}{dt} = -Q_\beta F \rho_\beta \sin (\phi_\alpha - \phi_\beta). \]  

(B15b)

Let \((q_\alpha, p_\alpha)\) denote the following canonical coordinates [cf. Lansky et al. 1997]:

\[ q_\alpha \equiv \phi_\alpha, \]  

(B16)

\[ p_\alpha \equiv Q_\alpha \left( \frac{\rho_\alpha}{\lambda} \right)^2. \]  

It is readily verified that Eqs. (B15a) and (B15b) can be recast into the following Hamiltonian
form:
\[
\frac{dq_\alpha}{dt} = \frac{\partial H}{\partial p_\alpha} \quad \text{and} \quad \frac{dp_\alpha}{dt} = -\frac{\partial H}{\partial q_\alpha}.
\]  
(B17)

The Hamiltonian is given by
\[
H = H_S + H_I,
\]  
(B18)
in which
\[
H_S = \begin{cases} 
\sum_\alpha C_\alpha Q_\alpha^{\mu/2} p_\alpha^{1-\mu/2} / \lambda^\mu(1 - \mu/2) & \mu \neq 2 \\
\sum_\alpha C_\alpha Q_\alpha / \lambda^2 \ln(p_\alpha) & \mu = 2,
\end{cases}
\]  
(B19)
and
\[
H_I = Q_1 Q_2 \int (d/\lambda)^2 F(\xi) d\xi
\]  
(B20)
\[
= -2 \frac{Q_1 Q_2}{\sqrt{(d/\lambda)^2 + 1}} + \text{const.}
\]

It is noted here that
\[
\left(\frac{d}{\lambda}\right)^2 = \frac{p_1}{Q_1} + \frac{p_2}{Q_2} - 2 \sqrt{\frac{p_1 p_2}{Q_1 Q_2}} \cos(q_1 - q_2).
\]  
(B21)

Although Eqs. (B11) and (B13) have been used for $F$ and $V_\alpha$, their specific functional forms are not especially important to the Hamiltonian structure of the model. It follows from Eqs. (B17) and the rotational symmetry of the system that $H$ and $L \equiv p_1 + p_2$ are constants of motion.
References


Figure 1: (a,b) Initial ambient soundings of the high RH (red) and low RH (blue) simulations. (a) Vertical profiles of relative humidity. (b) Nearly equivalent temperature profiles. (c)-(e) Initial condition of a strong midlevel vortex in isolation. (c) Azimuthal velocity (solid contours; m s$^{-1}$) and virtual potential temperature (dashed contours; K) in the $r$-$z$ plane. The velocity contour interval is 2 m s$^{-1}$. (d) Actual and saturated equivalent potential temperatures at $r = 0$ (black, superscript 0) and $r = 600$ km (red, superscript a) at high RH. (e) Same as (d) but at low RH and with blue curves replacing red curves. (f)-(h) Same as (c)-(e) but for a weaker midlevel vortex in isolation. Note here that the ambient and central $\theta_e$ curves nearly overlap.
Figure 2: Evolution of binary vortices initialized with strong midlevel winds, $D = 200$ km and high RH. (top) Rain field $\sigma_r$ normalized to its instantaneous maximum. The value of $\sigma_{r,\text{max}}$ varies by less than 8% of its mean between the three snapshots at times greater than zero. The initial condition—depicted as a solid black square—is devoid of rain. (bottom) Horizontal wind speed 81 m above sea-level.
Figure 3: (a)-(e) Evolution of a strong vortex at high RH. (a) Time series of $\bar{v}_{\text{max}}$ and select 25-m horizontal wind speed ($u_{25}$) statistics. The orange curves show $\bar{v}_{\text{max}}$ in experiments where $C_d$ is reduced to $10^{-4}$ (CDR) or the surface enthalpy flux is eliminated (CK0) at the onset of rapid intensification (pink circle). (b)-(d) Time series of (b) the radius of maximum $\bar{v}$, (c) the altitude of maximum $\bar{v}$, (d; solid) the convective asymmetry, and (d; dashed) the degree of supergradient flow at the location of maximum $\bar{v}$. (e) Hovmöller plots of (left) $\bar{v}$ and (right) $\bar{\zeta}$ at an altitude of 149 m above sea-level. The velocity (vorticity) contour labels are in units of m s$^{-1}$ ($10^{-2}$ s$^{-1}$). (f)-(h) Comparison of vortex development in simulations initialized with a strong vortex and high RH (solid red; SH), a strong vortex and low RH (solid blue; SL), a weak vortex and high RH (dashed red; WH), and a weak vortex and low RH (dashed blue; WL). The time series are of (f) the difference $\Delta$ between $\bar{v}_{\text{max}}$ and its initial value, (g) the radius of maximum $\bar{v}$, and (h) the lower-to-middle tropospheric RH averaged within a 23-km radius of the vortex center.
Figure 4: Onset of rapid intensification ($t = 26 \text{ h}$) in a simulation initialized with a strong midlevel vortex and high RH. (a) Portrait of the azimuthally averaged velocity field. Vectors and colors respectively show the direction and magnitude of the secondary velocity field ($\bar{u}, \bar{w}$). The velocity vectors are scaled anisotropically to be tangent to streamlines on an anisotropic grid, and their spacing does not match the CM1 grid spacing. Contours (labeled in units of $10^5 \text{ m}^2\text{s}^{-1}$ on the edges of the plot) show the absolute angular momentum $\bar{M}$. The thick contour passes through the location ($r_{\text{max}}, z_{\text{max}}$) where $\bar{v} = \bar{v}_{\text{max}}$. (b) Same as (a) but near the surface of a small convectively generated vortex beginning to dominate the parent vortex. The colorbar shown here applies to (a) as well. (c) Superposed contour plots of $\bar{M}$ (solid), saturated pseudoadiabatic entropy $\bar{s}_*$ (dotted) and relative humidity (shading). The $\bar{M}$ contours are labeled as before, and the $\bar{s}_*$ contours are in units of J kg$^{-1}$ K$^{-1}$. The red $\bar{M}$ contour passes through the location of $\bar{v}_{\text{max}}$. (d) Radial profile of surface-based CAPE associated with the mean fields at the onset of rapid intensification (solid) and at the beginning of the simulation (dashed).
Figure 5: Diagram showing how the evolution of two strong midlevel vortices varies with the separation distance $D$ and relative humidity. Each evolutionary pathway is summarized by a simple expression, in which MV denotes a midlevel mesoscale vortex, H denotes a hurricane, the coefficient of MV or H denotes the number of vortices, and arrows point in the direction of time. The question mark above $2\text{MV} \rightarrow 2\text{H} \rightarrow 1\text{H}$ in the right column indicates that the pathway is conceivable but is not observed in the present simulation set at low RH.
Figure 6: Transition time $\tau_t$ versus the initial separation distance $D$ of two strong midlevel vortices under high RH (red diamonds) and low RH (blue circles) conditions. At low RH, the simulations with $D = 250$ and 300 km show no sign of tropical cyclone formation for $t \leq 380$ h. Note that $D = 0$ in this plot and others corresponds to a single-vortex simulation, which is theoretically equivalent to a binary vortex simulation with $D \to \infty$. 
Figure 7: Evolution of potential vorticity (PV) in low RH simulations of two strong vortices with (a) $D=200$ km and (b) $D=250$ km, respectively representing developing and nondeveloping scenarios. All plotted fields are smoothed with $5\times5$ km$^2$ boxcar averaging. The color contour plots show PV normalized to its initial maximum on an isentropic surface just above the seasurface ($\theta = 300$ K), whereas the white contours show PV in the middle troposphere ($\theta = 322$ K). Specifically, the white contours correspond to 0.1, 0.5 and 0.95 times the initial maximum PV at potential temperature $\theta = 322$ K.
Figure 8: (a) Evolution of PV in a dry CM1 simulation of an isolated strong vortex misaligned at $t = 0$. The initial displacement $d_0$ between midlevel and surface PV is 107 km, which means that $d_0/2r_c = 1.07$. The color and contour schemes are equivalent to those of Fig. 7. The white dot is the midpoint of the initial line segment connecting the vorticity centers at $z = 0$ and 5.5 km. (b) Orientation angle $\varphi$ of the horizontal displacement of midlevel PV relative to low-level PV at (left) $\theta = 300$ K and (right) $\theta = 307$ K. Midlevel PV is taken on the isentropic surface with $\theta = 322$ K. The orientation angles $\varphi = 0$ and $\varphi = \pi/2$ respectively correspond to displacements in the positive-$x$ and positive-$y$ directions.
Figure 9: Idealized illustration of how the generation and rotation of misalignments may cause midlevel convergence and surface separation in a binary system of symmetric vortices. Light and dark gray circles respectively represent the midsections and surface components of misaligned vortices. The dashed circles with arrows on the top of the right-hand figure convey the motion of an undamped misalignment mode in weak shear [cf. RMG04]. Note that the mean rotation of the binary system is subtracted out of the picture in order to simplify the cartoon.
Figure 10: Variation of $D/2 - \Delta V/\omega$ with the initial separation distance $D$ between two strong vortices. The black curve is computed with $\omega = \omega_o \equiv 3.0 \times 10^{-5} \text{s}^{-1}$. The gray curves are computed with $\omega = \omega_o \pm \delta \omega$, in which $\delta \omega = 1.3 \times 10^{-5} \text{s}^{-1}$ is the estimated uncertainty of the natural mode frequency.
Figure 11: PV centroid trajectories in the analogue model for (a) \( D = 200 \) km and (b) \( D = 400 \) km with a parameterization appropriate for the strong vortex simulations (appendix B2). The orange curves show trajectories of the low level centroid (1) and the black curves show trajectories of the midsectional centroid (2). The thick solid (thin dotted) sections of each curve correspond to \( t \leq \tau_e \) \( (t > \tau_e) \), in which \( \tau_e \) is defined in the text. Plot (a) shows one complete misalignment cycle, whereas plot (b) shows many. In (a), +s mark centroid locations when \( \delta \phi_{12} \) first equals \( \pi/2 \), whereas \( \times \)s mark centroid locations when the midsectional orbital radius \( \rho_2 \) first equals \( \rho_\ast = 1.6r_c \). In (b), \( \times \)s mark centroid locations when \( \rho_2 \) first reaches its minimum value.
Figure 12: Predicted and simulated aspects of PV centroid motion in a strong vortex exposed to the vertical shear of its binary partner. Solid and dotted curves are predictions from the analogue model with $\lambda = 170$ and 340 km, respectively. The $\times$s and $+$s are data from CM1 simulations initialized with two strong vortices, as explained in section 5c.2. (a) Orbital radius of the bottom centroid ($\rho_{1e}$, orange) and midsectional centroid ($\rho_{2e}$, black) at the time $\tau_e$ of midlevel merger or (if the centroids are bonded) maximal vortex misalignment. (b) The time $\tau_e$ along with the range of transition times $\{\tau_t\}$ that are seen when tropical cyclones form in high RH (pink) and low RH (blue) environments. (c) The displacement distance ($d_e$) between the midlevel and bottom centroids at $t = \tau_e$. 
Figure 13: Notable results from the weak vortex experiments. (a) Time series of the maximum surface wind speed at low RH (left) and high RH (right). The curve labels give the initial separation distance $D$. The red ‘m’ indicates where midlevel vortex merger occurs in the simulation initialized with $D = 200$ km and high RH. (b),(c) State of the simulation initialized with $D = 200$ km and high RH at a time ($t = 60$ h) after midlevel merger and before intensification. (b) PV distributions near the surface ($\theta = 300$ K; color) and in the middle troposphere ($\theta = 322$ K; white contours). The PV units on the colorbar are normalized to the instantaneous maximum at $\theta = 300$ K. Thick and thin white contours respectively correspond to midlevel PV equaling 12.5% and 25% of the midlevel maximum. The radius of the dashed yellow circle equals the initial orbital radius of the binary system. (c) Superposed contour plots of the rain field ($\sigma_r$ normalized to its instantaneous maximum) and vertical velocity ($w$) at $z = 6.9$ km. Values of $w$ exceeding 2.5 m s$^{-1}$ are mapped onto red, and values less than 0.31 m s$^{-1}$ in magnitude are rendered invisible so as not to cover the rain field plot. The main 12.5% midlevel PV contour in (b) is shown for reference. (d) The mean azimuthal velocity at $z = 25$ m versus the system-centric radius $\rho$ at $t = 0$ and 68 h for the simulation depicted in (b) and (c).
Figure 14: Snapshots of near-surface relative vorticity ($\zeta$) after midlevel merger in the simulation initialized with two strong vortices separated by 250 km in a low RH environment. The plotted vorticity field is smoothed with $15 \times 15$ km$^2$ boxcar averaging. The colorscale is logarithmic, with both the positive and negative halves spanning 3 decades of $\zeta$. 
Figure 15: Aspects of binary system development in a simulation initialized with strong midlevel vortices, high RH and $D = 200$ km. (a) Evolution of $\bar{v}_{\text{max}}$ measured in vortex centered coordinate systems (I,II) and the system centered coordinate system (s). Also shown are time series of the maximum and rms values of the 25-m horizontal wind speed $u_{25}$. (b) Evolution of $r_{\text{max}}$ in both vortices (I,II) and in the merged system (m). Also shown is the half-distance $\rho_{I,II}$ between the vortices prior to merger. (c) Superposed contour plots of the rain field ($\sigma_r$ normalized to the maximum at $t = 23$ h) and vertical velocity ($w$) at $z = 6.9$ km. Values of $w$ exceeding 5 m s$^{-1}$ are mapped onto red, and values less than 0.625 m s$^{-1}$ in magnitude are rendered invisible so as not to cover the rain field plots. The printed statistic $\delta MF_+$ is the percent change in the positive vertical mass flux within a 251-km radius of the system center at $z = 6.9$ km, between $t = 23$ h (left plot) and 31 h (right plot).
Figure 16: (a)-(c) Streamlines associated with the rotational component of the horizontal velocity field $u$ at $t = 28$ h in the simulation initialized with strong midlevel vortices, high RH and $D = 200$ km. Streamline segments have lengths proportional to the local wind speed and arrows pointing in the direction of the flow. The rms and maximum wind speeds in the displayed $267 \times 267$ km$^2$ box are printed on the top edges of each plot. The vertical levels corresponding to each plot are printed at the bottom of each column. (d)-(f) Same as (a)-(c) but for the irrotational component of $u$. Pink circles in (d) and (e) indicate the vortex locations deduced from (a) and (b), respectively.
Figure 17: Comparison of 3D diabatic merger (starting at $t = 28$ h in the simulation of Figs. 2,15,16) and 3D adiabatic merger. See text for details. (a) Initial vertical relative vorticity ($\zeta$) distributions in both experiments at $z = 1.1$ km (color) and 5.3 km (contours). The outermost, middle and innermost contours of a nested set respectively correspond to $\zeta = 1.3, 3.2$ and $5.8 \times 10^{-3} \text{s}^{-1}$. (b) Initial rain field (shading) and vertical velocity at $z = 6.9$ km (color) in the diabatic experiment. Values of $w$ exceeding 5 m s$^{-1}$ are mapped onto red, and values less than 0.625 m s$^{-1}$ in magnitude are rendered invisible. (c),(d) As in (a) but at different times in the diabatic experiment. (e),(f) As in (a) but at different times in the adiabatic experiment and with white contours corresponding to $\zeta = 1.1$ and $2.3 \times 10^{-3} \text{s}^{-1}$.
Figure 18: (a) Maximum azimuthal wind speed $\bar{v}_{max}$ at the time of peak intensity $\tau_p$ versus the initial vortex separation distance $D$. The system may have one hurricane ($D \leq 250$ km) or two hurricanes ($D \geq 300$ km) at $t = \tau_p$, depending on whether or not vortex merger has occurred. (b) Radius of maximum wind speed $r_{max}$ at the time of peak intensity versus $D$. Red and blue symbols respectively correspond to high RH and low RH simulations. Data at $D = 0$ are from simulations initialized with only one midlevel vortex. The dotted reference line in (b) corresponds to a radius of 5 times the horizontal grid spacing.
Figure 19: Summary of how $S$, $\Gamma$ and select functions of the two vary with $D$ at the time of peak vortex intensity. (a) Nondimensional function of $S$ and $\Gamma$ used to help assess the theoretical importance of imbalance to the maximum azimuthal wind speed. Values much greater than 0.1 suggest major importance. (b) Dimensional changes of $S$, $\Gamma$ and $S + \Gamma$ from their reference values (defined in the text). Also shown is the corresponding change of $\bar{v}_{\max}^2$. (c) Empirically adjusted expression for the squared wind speed $(S + \Gamma - \varepsilon_o)$ normalized to $\bar{v}_{\max}^2$. In all plots, red curves and diamonds represent high RH simulations, whereas blue curves and circles represent low RH simulations.
Figure B1: Angular rotation frequency versus the initial displacement of PV centroids at \( \theta = 322 \text{ K} \) and \( \theta = 304 \text{ K} \) in the isolated misalignment experiment of section 5a. The solid curve is a best fit to a function of the form \( \omega_f F[(d_0/\lambda_f)^2] \), in which \( \omega_f \) and \( \lambda_f \) are the fit parameters. The dotted curve is \( 3F[(d_0/170)^2] \times 10^{-5} \text{ s}^{-1} \).