Resolving the 180 Degree Ambiguity: The Minimum Energy Solution

T. Metcalf

The Minimum Energy Solution

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- The goal is to simultaneously minimize J and $\nabla \cdot B$
- Minimizing the divergence gives a physically meaningful solution. (A linear or pseudo non-linear force free solution gives $\partial B_z / \partial z$.)
- Minimizing the current gives a locally smooth solution.
- Aly (1988) showed that the free energy is bounded above by a limit proportional to $\alpha^2 = J^2 / B^2$. Since B^2 is unambiguous, by minimizing J^2, we are minimizing the upper limit to the free energy. It is in this sense that we find the 'minimum energy solution'.
- There are 2^N possible solutions, so we use simulated annealing to find the global minimum.

Simulated Annealing

- There are many local minima in this problem, but we only want the global minimum. The simulated annealing algorithm is very good at finding a global minimum in the presence of many local minima (Metropolis et al., 1953).
- The minimization is treated as the slow freezing or annealing of a liquid. In analogy with this, there is a 'temperature' and 'energy' associated with the minimization problem,
- Randomly flip vectors: if the merit function $E = (|J| + |\nabla \cdot B|)^2$ improves, accept the flip. If the merit function is increased, randomly accept with a probability $p = \exp[-(E' - E)/kT]$
- As the temperature decreases, *p* is reduced and the solution 'freezes' into the global minimum.

Advantages and Disadvantages of the Method

- Advantages
  - The algorithm is objective and reproducible and requires no human intervention.
  - The algorithm is robust. It usually gives the answer that one intuitively expects.
  - All methods require some assumption to operate. The assumptions here are clear and physically well motivated.
- Disadvantages
  - The simulated annealing is slow.

Example

- What to look for in the example:
  - The vectors are initially randomized when the 'temperature' is high.
  - As the solution 'cools', the strong field regions, where J from an incorrect orientation is high, 'freeze' first.
  - Then the regions with weaker field will 'freeze', until convergence.
  - The 'specific heat' is defined as dE/dT and gives the rate of convergence. The specific heat is small when we reach convergence.
- The results for the four preferred test cases follow the example.