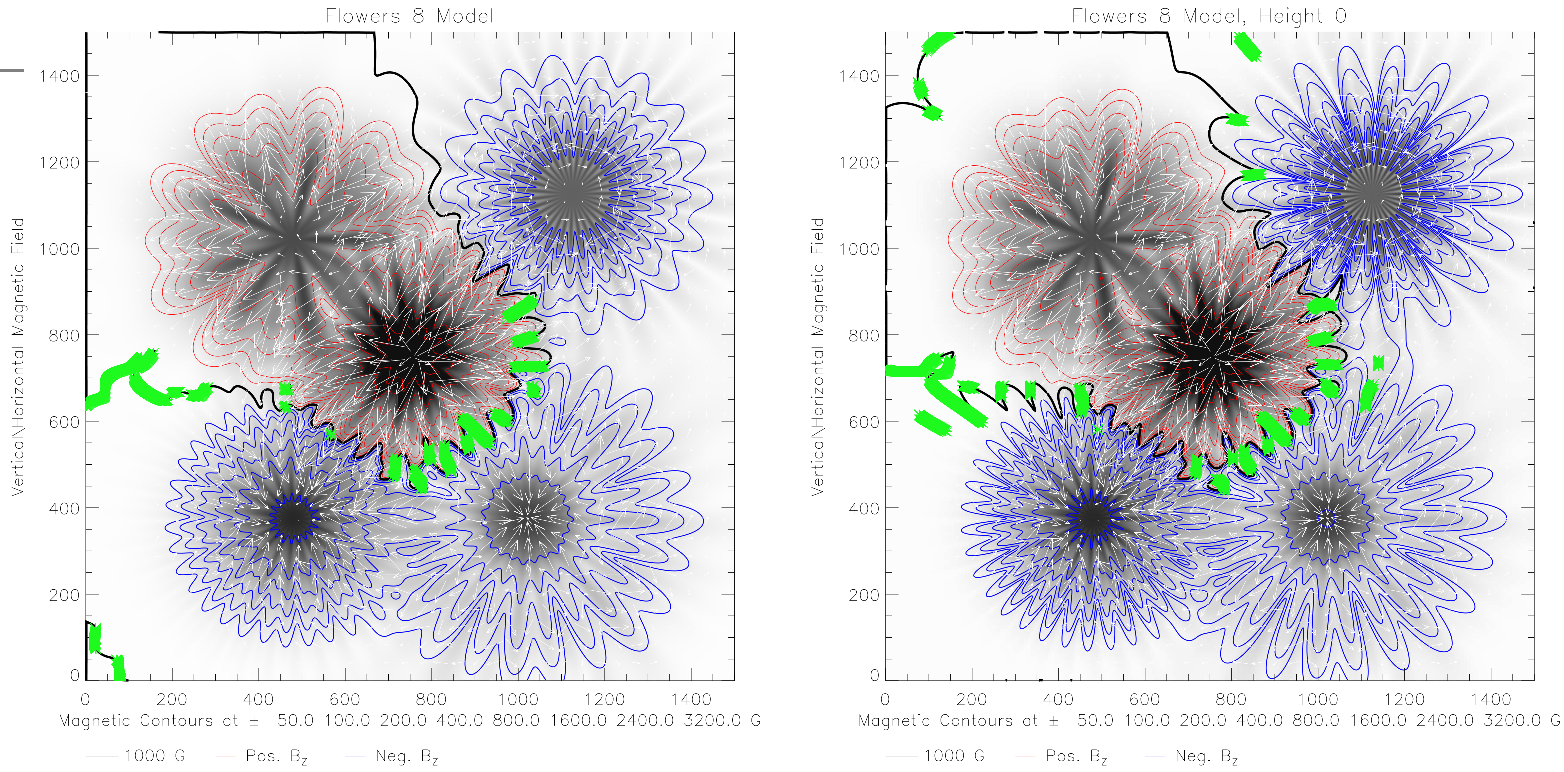
The “Flowers” Test Case

Believe it or not, this is a potential field!

- The normal (vertical) component of the field was specified on two horizontal squares a small distance apart.
  - By placing the two surfaces close together, the magnitude and direction of the horizontal field can be controlled to some extent.
- The normal component of the field was required to vanish on the sides.
- This uniquely determined the potential field in the volume between the planes. (Ask me if you really want to see what the solution looks like!)

The Boundary Fields

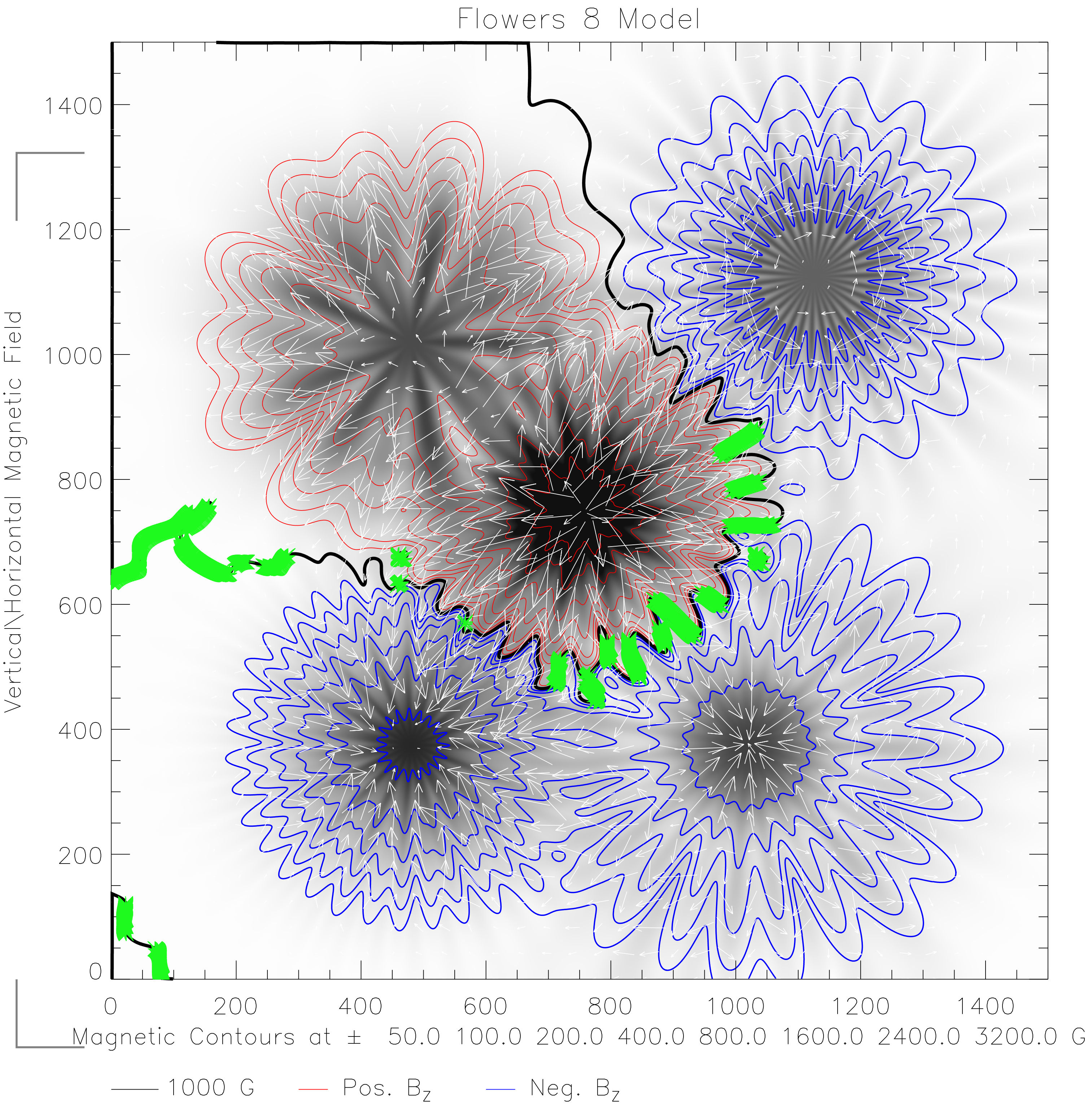


- Each sunspot is constructed from three components at each height:

$$B_z = B_0 e^{-(\boldsymbol{x}-\boldsymbol{x}_0)^2/(a_0^i)^2} + B_1(\boldsymbol{x}-\boldsymbol{x}_0)^2 e^{-(\boldsymbol{x}-\boldsymbol{x}_0)^2/(a_1^i)^2}\left[1+\cos(n_1\theta+\phi_1^i)\right]$$
$$+B_2(\boldsymbol{x}-\boldsymbol{x}_0)^2 e^{-(\boldsymbol{x}-\boldsymbol{x}_0)^2/(a_2^i)^2}\left[1+\cos(n_2\theta+\phi_2^i)\right]$$

with (slightly) different values of some of the parameters at the two heights.

  - Choosing $a_j^0 < a_j^1$ results in expanding field with height.
  - Choosing $\phi_j^0 \neq \phi_j^1$ results in an apparent “twist” of the field.


Flowers 8 Model
Vertical\Horizontal Magnetic Field
Magnetic Contours at ± 50.0 100.0 200.0 400.0 800.0 1600.0 2400.0 3200.0 G
1000 G
Pos. B_z
Neg. B_z

Potential Field Solution in a Box

Solve for the scalar potential Φ, which determines the field as $\boldsymbol{B} = -\boldsymbol{\nabla}\Phi$, and satisfies $\nabla^2\Phi = 0$.

- Construct a scalar potential whose normal derivative vanishes on the “side” walls, but not on the top and bottom:

$$
\begin{aligned}
\Phi(\boldsymbol{x}) &= \frac{1}{\pi}\sum_{m=0}^{N_x-1}\sum_{n=0}^{N_y-1}\left[\frac{m^2}{L_x^2}+\frac{n^2}{L_y^2}\right]^{-1/2}\cos\left(\frac{m\pi x}{L_x}\right)\cos\left(\frac{n\pi y}{L_y}\right) \\
&\quad\times\left\{A_{mn}^{+}\cosh\left[\pi\sqrt{\frac{m^2}{L_x^2}+\frac{n^2}{L_y^2}}\,z\right]+A_{mn}^{-}\cosh\left[\pi\sqrt{\frac{m^2}{L_x^2}+\frac{n^2}{L_y^2}}\,(z-L_z)\right]\right\}
\end{aligned}
$$

with

$$
A_{mn}^{\pm} = \frac{\mp(2-\delta_{m0})(2-\delta_{n0})}{L_x L_y \sinh\left[\pi L_z\sqrt{\frac{m^2}{L_x^2}+\frac{n^2}{L_y^2}}\right]}\int_0^{L_x}dx\int_0^{L_y}dy\,B^{\pm}\cos\left(\frac{m\pi x}{L_x}\right)\cos\left(\frac{n\pi y}{L_y}\right)
$$

- This can be expressed in terms of FFTs (or fast cosine transforms) of the “horizontal” directions, and so is reasonably fast.