H.N. Wang: The Huairou Solar Observatory Method

- Uses an acute-angle method with a linear force-free field computed using a Fourier Transform Method on the line-of-sight field, excluding low signal regions.
  - An initial force-free factor α is determined according to the length scale of the field.
  - The force-free parameter $\alpha_{\rm best}$ is chosen to maximize S:

$$S = \int\int \frac{|\boldsymbol{B}^{\rm obs} \cdot \boldsymbol{B}^{\rm lff}|}{B^{\rm obs} B^{\rm lff}}$$

    by iteratively changing the value of α.
  - The ambiguity resolution is revised based on this value of $\alpha_{\rm best}$.
- The field of view can be split into subwindows, with a different value of α used in each.

Y.J. Moon: The Uniform Shear Method

The Uniform Shear Method (USM) chooses the direction of the transverse field to make an acute angle with the potential field *plus a constant "shear angle offset"*.

- The ambiguity is first resolved using an acute angle method, with the potential field calculated using the Fourier Transform of the longitudinal field.
- The shear angle is defined as the angular difference between the observed transverse field and the transverse component of the potential field.
- The shear angle offset, $\Delta\theta_{\rm mp}$, is initially estimated as the most probable value for the magnetic shear angle, and the ambiguity is resolved a second time by requiring that

(1) $$-90^\circ + \Delta\theta_{\rm mp} \leq \theta^{\rm obs} - \theta^{\rm pot} \leq 90^\circ + \Delta\theta_{\rm mp}$$

- The shear angle offset is estimated a second time by fitting a Gaussian to the distribution of shear angles, and the ambiguity resolution is repeated.
- The final shear angle offset is determined shifting the second estimate through $\pm 20^\circ$ to maximize the number of pixels in the range

$$-80^\circ + \Delta\theta_{\mathrm{mp}} \leq \theta^{\mathrm{obs}} - \theta^{\mathrm{pot}} \leq 80^\circ + \Delta\theta_{\mathrm{mp}} \tag{2}$$

  and the ambiguity resolution is repeated.
- Finally, the result is effectively smoothed by forcing the transverse field to be in the same direction as the average transverse field of neighboring pixels.

Comments from Y.J. Moon

- Transverse fields showing rotating patterns in flowers seem to be too artificial. At all sunspot areas, the transverse fields have very strong shear (shear angle is around $90°$).
- To determine a most reasonable potential field method is important since the potential field method is used as a first step for several methods.
- In a physical sense, a comparison of physical quantities such as vertical current density and magnetic shear (or transverse-weighted shear) is preferred. For example, if vertical density distributions are similar to one another, there would be no significant difference in interpreting underlying physics.
- The application to real magnetograms is much more important. We may select several typical active regions (from simple to complex). Even though we have no idea of real fields, we can compare physical quantities and their differences. To check whether a physical understanding is different according to ambiguity resolution is important. In this case, we can think of some criteria to select which method is reasonable. The potential candidates are: which method gives the minimum sum of absolute vertical current density or the sum of divergence transverse field etc.

Summary of Algorithms

Method	Quantity minimized	Minimization scheme
Acute Angle	$\lvert\theta_{\rm o} - \theta_{\rm e}\rvert$	local
USM	$\lvert\theta_{\rm o} - \theta_{\rm e} - \Delta\theta_{\rm mp}\rvert$	local
Mag. Press. Grad.	$\partial B^2/\partial z$	local
NPFC	$\lvert J_z\rvert$	iterative
Pseudo-Current	$\int d^2a\, J_z^2$	conjugate gradient
UH Iterative	$\int d^2a\, J_z^2$	iterative
Minimum Energy	$\int d^2a\,(\lvert J\rvert + \lvert\nabla\cdot B\rvert)^2$	simulated annealing
AZAM	$\angle$ between neighboring pix.	interactive