

Notes on Forward Calculation

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February 27, 2012

The forward calculation generates travel-time maps (τ) from sensitivity kernels (K), and a velocity vector of flows (\mathbf{v}). Sometimes, the velocity vector is called a model (q). Added noise component is represented by n . [?]

- Not sure about the $(2\pi)^2$.

$$\tau^a(\mathbf{r}) = (2\pi)^2 \int K^a(\mathbf{r}' - \mathbf{r}, z) * \mathbf{v}(\mathbf{r}', z) d^2\mathbf{r}' dz + n^a(\mathbf{r}) \quad (1)$$

where, a is a particular measurement or travel-time map. Considering the overlap, taking the Fourier transform, and ignoring the noise, for now, the integral becomes

$$\int \left[\sum_{i'} K_{ii'} \phi_{i'}(z) \right] \left[\sum_{j'} \mathbf{v}_{jj'} \phi_{j'}(z) \right] dz = \sum_{i'j'} K_{ii'} \mathbf{v}_{jj'} \int \phi_{i'}(z) \phi_{j'}(z) dz \quad (2)$$

The overlap matrix, Θ is

$$\int \phi_{i'}(z) \phi_{j'}(z) dz = \Theta_{i'j'} \quad (3)$$

Theta is an input file in FITS format and has dimension $N_z \times N_z$.

$$\tau(\mathbf{k}) = \int \left[\sum_{i'} K_{i'} \phi_{i'}(z) \right] \left[\sum_{j'} \mathbf{v}_{jj'} \phi_{j'}(z) \right] dz = \sum_{i'j'} K_{i'} \Theta_{i'j'} \mathbf{v}_{j'}^T = K \Theta \mathbf{v}^T \quad (4)$$

The input parameter file for the forward calculation is very similar in style to the parameters files for inversions.

- The dimension of the forward problem (3D scalar or 3D vector) determines the number of kernels expected in each line of the input parameter file. If the problem is 3D scalar, there is one kernel on each line. If the problem is 3D vector, there are three kernel files in each line. Each line of the parameter file that contains a kernel defines an additional travel-time map. For example, if there are five lines in the parameter file that contain kernels, then there will be five travel-time maps in the output directory with names like 1_d~1.fits, 1_d~2.fits, 1_d~3.fits, 1_d~4.fits, 1_d~5.fits.

- Noise amplitude files take the place of map files in a parameter file for inversions.
- There is one amplitude file for each kernel.
- Theta file has the same meaning as in inversions.
- the model data file takes the place of the covariance file.
- Looping parameters scale the noise amplitude; a zero loop parameter means no noise.
- travel-time maps with the same scale for the noise have the same leading number in their file name. For example, all the travel-time maps with 1_d~*.fits have the same noise scale, and the noise scale different than all maps with file names 2_d~*.fits.
- random number generator seed
- log steps for range of noise scale factors.