NOTES ON FORWARD CALCULATION

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The forward calculation generates travel-time maps (τ) from sensitivity kernels (K), and a velocity vector of flows ($\boldsymbol{v}$). Sometimes, the velocity vector is called a model (q). The added noise is represented by n. From Švanda et al. [2011], the forward calculation is defined as:

$$\tau^a(\boldsymbol{r}) = (2\pi)^2 \int K^a(\boldsymbol{r'} - \boldsymbol{r}, z) * \boldsymbol{v}(\boldsymbol{r'}, z) d^2\boldsymbol{r'} dz + n^a(\boldsymbol{r}) \tag{1}$$

where, a is a particular measurement or travel-time map.

Applying Fourier transforms to the convolution:

$$\tau^a(\boldsymbol{k}) = (2\pi)^2 \int K^{*a}(\boldsymbol{k}, z) \boldsymbol{v}(\boldsymbol{k}, z) d^2 dz + n^a(\boldsymbol{r}) \tag{2}$$

is the forward calculation in wavespace.

Considering the integral, in wavespace and ignoring the noise, for now, the integral becomes

$$\int \left[\sum_{i'} K_{ii'} \phi_{i'}(z) \right] \left[\sum_{j'} \boldsymbol{v}_{jj'} \phi_{j'}(z) \right] dz = \sum_{i'j'} K_{ii'} \boldsymbol{v}_{jj'} \int \phi_{i'}(z) \phi_{j'}(z) dz \tag{3}$$

where the overlap matrix, Θ is

$$\int \phi_{i'}(z) \phi_{j'}(z) dz = \Theta_{i'j'} \tag{4}$$

Theta is an input file to the forward calculation. It has FITS file format and dimension $N_z \times N_z$.

$$\tau(\boldsymbol{k}) = \int \left[\sum_{i'} K_{i'} \phi_{i'}(z) \right] \left[\sum_{j'} \boldsymbol{v}_{jj'} \phi_{j'}(z) \right] dz = \sum_{i'j'} K_{i'} \Theta_{i'j'} \boldsymbol{v}_{j'}^T = K \Theta \boldsymbol{v}^T \tag{5}$$

Adding noise involves a scale parameter, s, a matrix containing noise amplitudes, A, and a matrix with random numbers, R. The amplitude matrix is an input file and it must have the same dimensions as the travel-time maps. The input parameter, s, scales the noise amplitude matrix A. The random matrix is also the same size as the travel-time maps.

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The random matrix is multiplied element by element with the scaled amplitude matrix, as in $n^a(\boldsymbol{r}) = sA.*R$. Then $\tau^a = \tau^a + (sA).*R$.

The input parameter file for the forward calculation is very similar in style to those for the inversions. The parameter file contains:

- The dimension of the problem (3D scalar or 3D vector). The dimension determines the number of kernel files expected on each line of the parameter file. If the problem is 3D scalar, there is one kernel on each line. If the problem is 3D vector, there are three kernel files expected on each line. A line of the paramter file that contains a kernel defines an additional travel-time map. For example, if there are five lines in the parameter file that contain kernels, then there will be five travel-time maps in the output directory with names like 1_d˜1.fits, 1_d˜2.fits, 1_d˜3.fits, 1_d˜4.fits, 1_d˜5.fits, where d˜ is the notation for a travel-time map, instead of τ.
- One noise amplitude file corresponding to each map. (Noise amplitude files take the place of the map files normally in a parameter file for inversions.)
- Theta file to define the overlap.
- A model data file.
- Parameters to scale the noise amplitude; a value of zero means no added noise. (Log steps for range of noise scale factors.)
- Seed for random number generator (optional).

Travel-time maps with the same scale parameter for added noise will have the same leading number in their output file name. For example, all the travel-time maps with 1_d˜*.fits have the same noise scale, and a different noise scale from those maps with output file names 2_d˜*.fits.

References

M. Švanda, L. Gizon, S. M. Hanasoge, and S. D. Ustyugov. Validated helioseismic inversions for 3D vector flows. *Astronomy & Astrophysics*, 530:A148, June 2011. doi: 10.1051/0004-6361/201016426.