

## HELIOSEISMIC HOLOGRAPHY OF SIMULATED SOLAR CONVECTION AND PROSPECTS FOR THE DETECTION OF SMALL-SCALE SUBSURFACE FLOWS

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### ABSTRACT

We perform helioseismic holography on realistic solar convection simulations and compare the observed travel-time perturbations with the expected travel times from the horizontal flows in the simulations computed from forward models under the assumption of the Born approximation. We demonstrate reasonable agreement between the observed and model travel times, which reinforces the validity of helioseismic holography in the detection of subsurface horizontal flows. An assessment is made of the uncertainty of the measured  $p$ -mode travel times from the rms of the residuals. From the variation of the signal-to-noise ratio with depth we conclude that the helioseismic detection of individual flow structures with spatial scales of supergranulation or smaller is not possible for depths below about 5 Mm below the surface over timescales of less than a day. The travel-time noise estimated from these simulations appears to be similar to noise in measurements made using solar observations. We therefore suggest that similar limitations exist regarding the detection of analogous subsurface flows in the Sun. A study of the depth dependence of the contribution to the travel-time perturbations for focus depths between 3 and 7 Mm is made, showing that approximately half of the observed signal originates within the first 2 Mm below the surface. A consequence of this is a rapid decrease (and reversal in some cases) of the travel-time perturbations with depth due to the contribution to the measurements of oppositely directed surface flows in neighboring convective cells. This confirms an earlier interpretation of similar effects reported from observations of supergranulation.

*Subject headings:* Sun: helioseismology — Sun: interior

### 1. INTRODUCTION

The understanding of solar interior dynamics, as manifested in mass flows with a variety of temporal and spatial scales beneath the surface of the Sun, has been considerably advanced by both global and local helioseismic observations. Some recent reviews of the progress in helioseismology of flows (and other properties of the solar interior) include Christensen-Dalsgaard (2002), Thompson et al. (2003), Gizon & Birch (2005), Miesch (2005), and Thompson (2006). In addition, progress has been made with increasingly improving numerical modeling procedures and computational resources. Examples of numerical models and simulations of wave propagation relevant to helioseismic studies include Birch et al. (2001), Jensen et al. (2003), Tong et al. (2003), Mansour et al. (2004), Benson et al. (2006), Hanasoge et al. (2006), Khomenko & Collados (2006), Shelyag et al. (2006), Parchevsky & Kosovichev (2007), Shelyag et al. (2007), and Zhao et al. (2007). In addition to providing physical insights into the origin and evolution of flows inferred through helioseismic observations and analysis, numerical models such as those cited above are being used to validate the observational and analysis tools themselves. The application of analysis methods to numerical (or “artificial”) data can potentially lead to improvement in both theory and observations of the phenomena being explored (e.g., Werne et al. 2004).

The mean solar rotation, and its variation with depth, latitude, and time, is detectable by both global and local helioseismic techniques, and comparisons between methods provide checks on the reliability of the analysis and modeling (e.g., Giles et al. 1998; Basu et al. 1999; Basu & Antia 2000; Haber et al. 2000, 2002; Zhao & Kosovichev 2004). Confidence in local helioseismic methods is also obtained by comparing models of low-amplitude flows such as meridional circulation derived using different procedures and assumptions (e.g., Giles et al. 1997, 1998; Braun & Fan 1998; Haber et al. 2002; Hughes & Thompson 2003; Zhao & Kosovichev 2004; Chou & Ladenkov 2005; González Hernández et al. 2006; Švanda et al. 2007). Intercomparisons of methods and data sets can sometimes lead to the discovery of important systematic effects or artifacts (González Hernández et al. 2006).

The helioseismic study of “small-scale” structures within the Sun, defined here by wave speed, flow, or magnetic perturbations in the solar interior with spatial scales less than a few tens of Mm is a particularly challenging pursuit. This is because the structures being probed have sizes that are of the order of, or perhaps only a few times larger than, the wavelength of the  $p$ - or  $f$ -modes used in the analysis. In this work we consider small-scale flows representative of solar supergranulation. The effects on mode frequencies and travel times of acoustic waves caused by even smaller scale flows (e.g., granulation) have also been explored (e.g., Murawski & Roberts 1993; Murawski & Goossens 1993; Petrovay et al. 2007), but are not considered here. Our simulations do not include magnetic fields, and consequently we are unable to assess

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the effects of magnetic fields on the modeling or interpretation of helioseismic measurements of these flows. This important issue, however, continues to be extensively explored in the context of global helioseismology, asteroseismology, and local helioseismology (e.g., Bogdan 2000; Christensen-Dalsgaard 2002; Gizon & Birch 2005; Thompson 2006).

Supergranulation has been a focus of local helioseismology for over a decade (e.g., Duvall et al. 1997; Kosovichev & Duvall 1997; Duvall & Gizon 2000; Gizon et al. 2000, 2003; Braun & Lindsey 2003; Zhao & Kosovichev 2003; Braun et al. 2004). However, as noted by Gizon & Birch (2005), there is no definitive consensus on even the depth of the supergranulation phenomenon. Some inversions of  $p$ -mode travel times indicate the presence of a “return flow” (oppositely directed flows from what is observed at the solar surface) at various depths, but typically  $\sim 10$  Mm, below the photosphere (Duvall et al. 1997; Zhao & Kosovichev 2003). Braun et al. (2004) have suggested that an observed change in sign in travel-time perturbations with increasing depth of the penetration of the modes may represent a contamination of the signal from neighboring supergranules. How this might affect the inversions, however, is not clear. Zhao et al. (2007) find that travel-time inversions on simulated supergranular-sized convection do not reproduce the simulation flow fields at depths below the photosphere greater than 3 Mm. To what depth we are able to reliably measure subsurface flows due to supergranulation and other small-scale patterns remains an important, but unanswered, question.

In this paper, we explore the prospects for helioseismic probing of small-scale flows in the solar interior by applying helioseismic holography (Lindsey & Braun 1997) to recent numerical simulations of solar convection. To do this we compare the “observed” signatures of flows (in this case, maps of the perturbations to  $p$ -mode travel times) obtained by applying helioseismic holography to the surface of realistic solar simulations to the expected (“model”) signatures obtained directly from the simulated flows. This comparison facilitates an assessment of both the expected helioseismic signals and the uncertainties in the observations (i.e., departure of the observed signatures from the expected values). While the degree of similarity between observed and model signatures provides a critical validation of our specific helioseismic analysis, our principle goal is more general than a test of a single method of observation or modeling procedure. In particular, the observational uncertainties estimated from this *forward-modeling* experiment are used to assess the general prospects for the seismic detection of small-scale flows in the solar interior. This is possible to the extent that (1) our simulation measurements have uncertainties characteristic of (or at least no worse than) local helioseismic observations in general, and (2) flows in the solar interior have properties similar to those in the simulations. The validity of the second condition is maximized by using realistic solar convection simulations (Benson et al. 2006) as described in § 2. To ensure the validity of the first of these conditions, we employ helioseismic holography in the “lateral vantage,” which (as explained below) is designed to utilize most of the  $p$ -modes propagating through a specific target point (focus) in the solar interior. We also compare the uncertainties estimated in the simulation observations with results obtained from the Michelson Doppler Imager (MDI) onboard the *Solar and Heliospheric Observatory* (SOHO). Our analysis and forward modeling procedures are described in more detail in § 3 and § 4, respectively. The results of the comparison are shown in § 5, followed in § 6 by a discussion of the implications of these results.

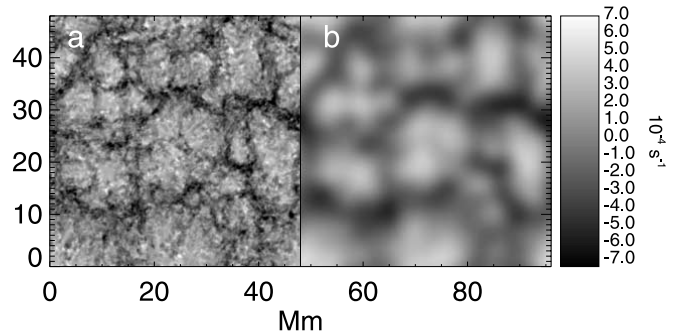


FIG. 1.— (a) Horizontal divergence of the time average of the simulated flows at a depth of 4 Mm below the surface. Positive values of the divergence, identified by bright regions, correspond to outflows. (b) The same flow divergence smeared with a two-dimensional Gaussian function with a FWHM of 4 Mm.

## 2. SIMULATIONS

The three-dimensional (3D) simulations of solar convection on supergranular scales employed in this study were performed and described by Benson et al. (2006) and have already been used to validate surface and subsurface flow diagnostics (Georgobiani et al. 2007; Zhao et al. 2007). The simulations are carried out in a box  $48 \times 48$  Mm horizontally and 20 Mm deep. The conservation equations for density, momentum, and internal energy are solved in conservative form, on a 3D staggered mesh, using sixth order finite difference spatial derivatives and a low memory, third order Runge-Kutta time advance. The grid is uniform in horizontal directions and stretched in the vertical (stratified) direction. Horizontal boundary conditions are periodic, while top and bottom boundary conditions are open. Inflows at the bottom boundary have constant pressure, specified entropy, and damped horizontal velocities. Outflow boundary values are obtained by extrapolation. The code uses a tabular equation of state, which includes local thermodynamic equilibrium (LTE) ionization of the abundant elements as well as hydrogen molecule formation, to obtain the pressure and temperature as a function of log density and internal energy per unit mass using the Uppsala atmosphere package (Gustafsson et al. 1975). Radiative heating/cooling is determined by solving the radiation transfer equation in both continua and lines using the Feautrier method, assuming LTE (Nordlund 1982; Stein & Nordlund 2003).

The acoustic waves ( $p$ -modes) generated by convective motions in these simulations have properties remarkably similar to observed solar  $p$ -modes (Georgobiani et al. 2007). One notable exception is an enhancement of acoustic power at higher temporal frequencies than observed solar values. A fortunate consequence of this is the ability to perform and test helioseismic procedures with modes extending to higher frequencies than normally used with solar data.

While power spectra of the convective motions in these simulations demonstrate the presence of structures over a broad range of spatial scales (Benson et al. 2006; Georgobiani et al. 2007), we expect that local helioseismic techniques are best suited for probing structures larger than the wavelength of the  $p$ -modes employed in the analysis. Figure 1a shows the horizontal divergence of the 8.53 hr time average of the simulated flows at a depth of 4 Mm below the surface. Figure 1b shows the same flow divergence after applying a Gaussian smoothing with a full width half-maximum (FWHM) of 4 Mm. This width is of the order of the wavelength of  $p$ -modes with a temporal frequency of 3 mHz just below the photosphere (or equivalently, the wavelength at 5 mHz of modes at a depth of about 4 Mm). Evident in both

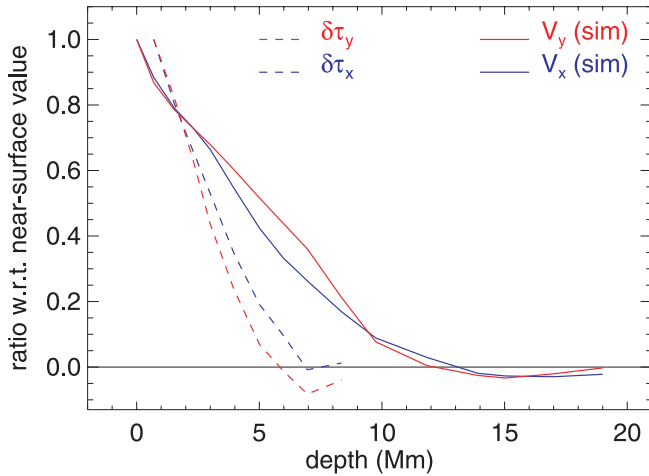


FIG. 2.—Ratio with respect to near-surface values of the simulated horizontal flows (*solid lines*) and the measured holographic travel-time perturbations (*dotted lines*) as a function of depth. The blue and red lines indicate the measurements for the  $x$ - and  $y$ -components of each quantity, respectively. For the travel-time perturbations, the depth corresponds to the depth of the focus in the lateral vantage. For both flows and travel-time perturbations, the ratio is defined to be the slope of a linear least-squares fit between the relevant quantity at the indicated depth with the quantity evaluated at a “near-surface” depth. For the simulated flows this depth is the surface of the simulations. For the travel-time perturbations this depth is 0.7 Mm below the surface.

panels (but especially in the smoothed panel) are cellular structures, the most prominent having diameters on the order of 20 Mm. Presumably these structures are the most amenable to helioseismic detection with mode wavelengths greater than 4 Mm, and serve as a proxy for solar supergranulation on a somewhat larger scale ( $\sim 30$  Mm). The simulated flow structures are coherent to depths of about 12 Mm, below which appear weak counterflows (see Fig. 2).

We use 8.53 hr of the simulations in our analysis, which is typical of solar helioseismic observations and is less than the lifetimes of individual solar supergranules ( $\sim 25$  hr). We apply helioseismic holography (see § 3) to the vertical component of velocity sampled at a height of 200 km above the continuum optical depth of unity in the background stratification (hereafter referred to as the “surface” of the simulations). We use flows sampled only every 30 s in time, yielding 1024 total time samples. The original simulations are computed in a  $500 \times 500$  horizontal grid (giving a horizontal resolution of 96 km) and are for our purposes degraded in resolution by a factor of 2 in each direction by pixel averaging.

### 3. ANALYSIS

Helioseismic holography (HH) is a method based on the phase-coherent imaging of the solar interior acoustic field. It computationally extrapolates the surface acoustic field into the solar interior (Lindsey & Braun 1997, 2000) to estimate the amplitudes of the waves propagating into and out of a focus point at a chosen depth and position in the solar interior. These amplitudes, called the ingression and egression, are estimated by a convolution of the surface oscillation signal with appropriate Green’s functions (Lindsey & Braun 2000). Here, HH is performed in the wavenumber-frequency (Fourier) domain using a set of fixed frequency bandpasses with a width of 1 mHz and centered at frequencies,  $\nu_0$ , of 3, 4, 5, and 6 mHz. While previous applications of HH to study solar flows have been limited, by

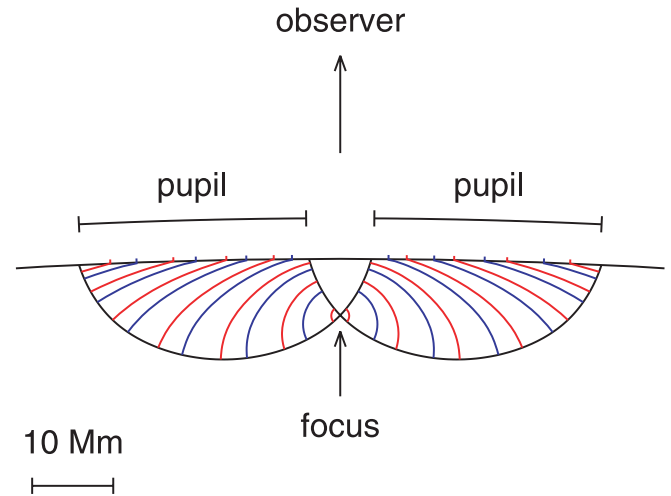


FIG. 3.—Lateral-vantage holography with the focus placed at a depth of 7 Mm below the surface. Solid black curves that pass through the focus at angles inclined  $\pm 45^\circ$  from the horizontal direction indicate the propagation of acoustic rays, which when reaching the surface (*horizontal line*) define the inner and outer radii of the pupil. The colored curves indicate wave fronts (separated by a wavelength) of sample components of the egression (*red*) and ingression (*blue*) for this focus depth. The wavelength of the wave components shown here is 5.9 Mm, corresponding to  $p$ -modes with a temporal frequency of 5 mHz. If set in motion, the red wave fronts would diverge away from the focus, while the blue wave fronts would converge toward the focus.

signal-to-noise ratio (S/N) considerations, to frequencies equal to and below 5.5 mHz (e.g., Braun & Lindsey 2003; Braun et al. 2004), we are motivated by the presence of high frequency waves in the simulations to include a 6 mHz bandpass in our analysis here.

The HH analysis is performed in the wave-mechanical formulation (Lindsey & Braun 2004). The method employed for horizontal flow diagnostics is based on the egressions and ingressions computed in the *lateral vantage* (Lindsey & Braun 2004) employing pupils spanning four quadrants extending in different directions (east, west, north, and south) from the focus (Braun et al. 2004). In the lateral vantage, the  $p$ -modes sampled by the pupil propagate through the focal point in directions inclined up to  $\pm 45^\circ$  from the direction parallel to the surface (Fig. 3). The antisymmetric phase shift, the difference in the phase perturbation of waves traveling from one pupil to its opposite and the phase perturbation of waves traveling in the reverse direction, is sensitive to horizontal flows near the focus. In general, the phase perturbation  $\delta\phi^A$  is related to an equivalent travel-time perturbation by  $\delta\tau = \delta\phi^A / 2\pi\nu_0$ . The travel-time perturbations,  $\delta\tau_x$  and  $\delta\tau_y$ , derived from antisymmetric phase shifts computed between the east-west and north-south quadrant pairs, respectively, provide the HH signatures sensitive to the two components of the horizontal flow. The sign of the travel-time perturbations are such that a positive velocity component will produce a negative value of  $\delta\tau$  (i.e., a reduction in the mode propagation time). The lateral-vantage geometry samples more than 70% of the wave modes which pass through the focus. The remaining waves, propagating more vertically than the waves appearing in the pupil, are substantially less sensitive to horizontal flows.

Table 1 lists the focus depths and the pupil radii used in lateral-vantage HH. The pupil radii are defined from ray theory. The range of (spherical-harmonic) mode degrees ( $\ell$ ) at 4 mHz, selected by each pupil, is listed in the table. The lower  $\ell$  value denotes

TABLE 1  
PUPIL RADII

Depth (Mm)	Pupil Radii (Mm)	$\ell$ at 4 mHz
0.7.....	0.7–11.1	980–1370
1.5.....	1.2–12.5	850–1190
2.3.....	1.8–13.9	750–1060
3.0.....	2.1–14.6	680–960
4.0.....	2.8–16.7	590–840
5.0.....	3.5–18.8	530–740
6.0.....	3.5–24.4	470–670
7.0.....	4.2–29.9	420–600
8.3.....	4.9–39.0	370–520

the modes propagating at  $\pm 45^\circ$  from the horizontal direction, which propagate through the focus and reach the surface at either the inner or outer pupil radius. The highest  $\ell$  value listed indicates modes propagating horizontally through the focus. The mode degrees selected by the other frequency bandpasses scale approximately with  $\nu_0$ . Dispersion effects predict slight variations in the pupil radii with wave frequency. However, experimentation shows that the measured travel times do not vary with similar adjustments of the pupil radii sufficiently to affect the comparisons and conclusions discussed here. Consequently, for convenience and simplicity, we use the same pupil for all frequency bandpasses for a given depth focus.

Comparisons of travel-time maps made with and without the use of phase-speed filters show that these filters are of marginal utility when applied to data subsequently sampled over wide pupils of the sort employed in lateral-vantage HH. We note that this is not the case for much of time-distance helioseismology (e.g., Duvall et al. 1997; Zhao et al. 2001, 2007) or for HH performed with narrow pupils (Braun & Birch 2006), where the use of phase-speed filters appears to provide a dramatic improvement in the measurements. While a reduction in scatter is sometimes evident with the use of these filters in lateral vantage HH, we find that, for both solar data and the present simulated data, the effect is comparable to what can be achieved by spatially smearing the unfiltered travel-time perturbation maps. In fact, an understanding of the consequences of smearing the travel-time maps is critical for interpreting our comparisons between observed and modeled results. With these considerations in mind, our strategy is to use observed travel-time maps made without phase-speed filters, and explore explicitly the direct consequences to our comparisons of spatial averaging of the unfiltered maps by varying amounts.

The focus depths chosen to perform the “observations” extend down to about 8 Mm below the simulation surface. At focus depths larger than this, egression-ingression correlations cease to exhibit meaningful results, even though reliable correlations are routinely observed at these depths in the Sun. This failure may be due to the lower boundary of the simulations at 20 Mm. Even though the lower boundary of the simulation is twice the depth of the focus depth where problems appear to arise, we note that the lateral-vantage geometry samples  $p$ -modes that penetrate a considerable depth below the focus, and any reflection or absorption of these waves at the lower boundary will likely adversely affect our observations. The horizontal periodic boundaries used in the simulations ensure that HH travel-time measurements, performed in the Fourier domain, are usable over the full horizontal extent of the simulations. Thus, the data is “infinite” (but periodic) horizontally, and travel-time perturbation maps are free from any detrimental effects caused by an abrupt termination of data at the

edges of the domain, such as typically experienced with helioseismic observations of the Sun.

#### 4. FORWARD MODELS

In this section we use the Born approximation approach of Gizon & Birch (2002) to estimate the HH signatures that would be expected from the flows in the simulations. The range of validity of the Born approximation for 3D time-dependent flows in stratified models is not known.

Birch & Felder (2004) studied the validity of the Born approximation as applied to time-distance helioseismology of simple steady flows in two-dimensional (2D) unstratified models. In these models, the Born approximation is typically seen to be valid when travel-time shifts are a small fraction of the wave period. The validity of the Born approximation for scattering from steady sound-speed inhomogeneities has been studied extensively in the context of earth seismology and helioseismology (e.g., Fan et al. 1995; Hung et al. 2000; Birch et al. 2001; Baig et al. 2003). In these studies, it is again typical that the Born approximation is valid when travel-time shifts are small compared to the mean wave period (although it can sometimes be valid well past this limit). The extension of this general conclusion to the current work, which involves strong time-dependent small-scale flows, is not at all certain.

Comparison between the Born approximation estimates described in this section and the actual travel times (§ 5) will provide an important estimate of the validity of the Born approximation for complicated solar-like flows. In particular, we will be able to study the extent to which the HH signatures can be predicted using only the time average of the flow.

As described in detail by Gizon & Birch (2002), the computation of the linear sensitivities of local helioseismic measurements requires, in general, two steps. The first step is the computation of the linear sensitivity of the measurement procedure (e.g., the measurement of travel times) to small changes in covariance of the wavefield. The second step is the computation of the change in the wavefield covariance introduced by small changes in the model of the solar interior (e.g., the introduction of flows or sound-speed variations). This general procedure has been applied by Birch & Gizon (2007) to the case of time-distance measurements of flows and by Birch et al. (2007) to the case of ring-diagram measurements of flows.

The application of the Gizon & Birch (2002) recipe to the case of HH is relatively straightforward. The result is a set of linear sensitivity functions (kernels),  $\mathbf{K}$ , which give the linear sensitivity of the antisymmetric phase  $\phi^A$  to small-amplitude steady flows  $\mathbf{v}(\mathbf{x})$ ,

$$\delta\phi^A = \int_{\odot} d\mathbf{x} \mathbf{K}(\mathbf{x}) \cdot \mathbf{v}(\mathbf{x}), \quad (1)$$

where  $\mathbf{x}$  denotes 3D position in the model, and the integral is taken over the entire 3D domain of the simulation. In the computation of the kernel functions we use the normal-mode Green’s functions, source model, and damping model described by Birch et al. (2004). The background stratification is given by model S (Christensen-Dalsgaard et al. 1996)

One limitation of the normal-mode Green’s functions of Birch et al. (2004) is that they employ the upper boundary condition that Lagrangian pressure perturbation vanishes at the top of model S. As discussed by Birch et al. (2004), this boundary condition is reasonable for waves at frequencies well below the acoustic cutoff frequency. For frequencies near the acoustic cutoff

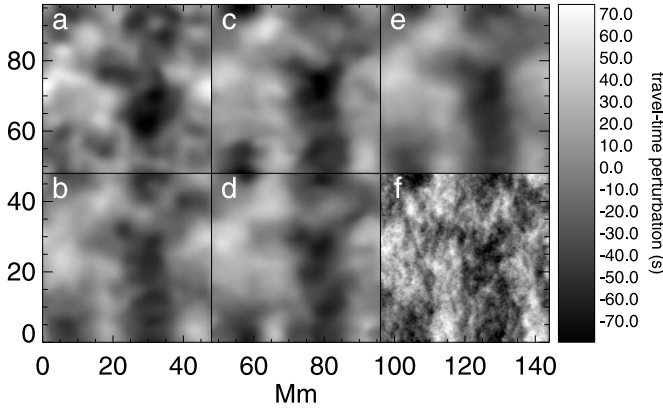


FIG. 4.— Maps of the  $p$ -mode travel-time perturbations determined from lateral-vantage HH using the east/west quadrant pair and a focus depth of 0.7 Mm below the simulation surface for the following frequency bandpasses: (a) 2.5–3.5 mHz, (b) 3.5–4.5 mHz, (c) 4.5–5.5 mHz, (d) 5.5–6.5 mHz, (e) the average of all four frequency bandpasses, and (f) the time average of the  $x$ -component of the horizontal velocity,  $v_x$ , at a depth of 0.7 Mm. The color bar gives the units of the travel-time perturbation for panels  $a$ – $e$ , while the values for the velocity ( $f$ ) range from  $1.1 \text{ km s}^{-1}$  (black) to  $-1.1 \text{ km s}^{-1}$  (white). A positive value of  $v_x$  corresponds to a flow toward the right.

frequency, this boundary condition is likely not appropriate. As a result, the kernels described in this section are not intended to model HH measurements at or above 5 mHz.

## 5. RESULTS

### 5.1. Frequency and Depth Variations of Observed Travel-Time Perturbations

An examination of the observed travel-time perturbation maps reveals a fair degree of correlation among different frequency bandpasses for a given focus depth, particularly for the shallower depths ( $\leq 5$  Mm) selected for analysis. An example, at the 0.7 Mm focus depth, is shown in Figure 4. Correlation coefficients between perturbation maps at different frequencies were computed after applying Gaussian smoothing with varying FWHM to the maps. As expected, the correlations improve with increased smoothing. With a Gaussian FWHM of 4 Mm, the correlation coefficients are of the order of 0.7 for the shallowest depths and decrease to about 0.5 at a depth of 6 Mm. At depths of 7 and 8 Mm, the 1 mHz bandpass maps are essentially uncorrelated with each other. As with analogous travel-time observations in the Sun, the measurement noise from these simulations is most likely due to realization noise caused by stochastic excitation of the  $p$ -modes. In the solar case, the properties of the measurement noise have been extensively explored (e.g., Gizon & Birch 2004).

At depths where comparisons between frequencies are meaningful, we do not detect any clear systematic variation of the travel-time perturbations with frequency. Typically, the slope of a linear fit of the travel times between any two frequencies differs from unity by less than 10%. A somewhat surprising result is that the variance of the 1 mHz bandpass maps, defined as the average over all pixels of the squared difference between each map and an average of maps over all four bandpasses (hereafter referred to as a frequency-averaged map), is essentially the same for all frequencies between 3 and 6 mHz.

As expected, there is a good correspondence, particularly at shallower depths, between the travel-time maps and the relevant component of the horizontal velocity. This is particularly evident in comparisons between the frequency-averaged perturbation maps and the simulation flows (cf., Figs. 4e and 4f). However, it

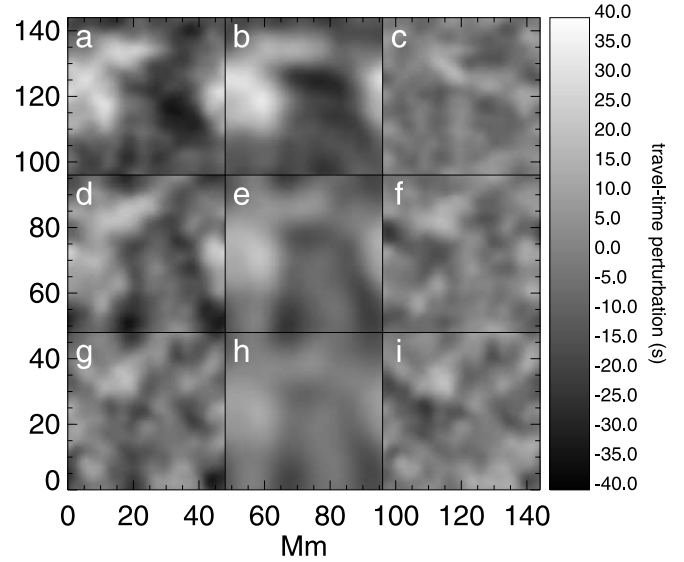


FIG. 5.— Comparisons of the observed  $p$ -mode travel-time perturbations determined from lateral-vantage HH with the perturbations computed from a forward model for several focus depths: (a) the observed, (b) model, and (c) residual (observed-model) perturbations at a focus depth of 3 Mm, (d–f) the corresponding perturbations at a focus depth of 5 Mm, (g–i) the corresponding perturbations at a focus depth of 7 Mm below the surface.

is also apparent that the ratio of the magnitude of the travel-time perturbations to their values at the shallowest focus depth (Fig. 2, dotted lines) falls off considerably faster with focus depth than a similar ratio of the simulation flow magnitudes (solid lines).

### 5.2. Comparisons with Model Travel-Time Perturbations

Using the methods described in § 4 we derive the model travel-time perturbations ( $\delta\tau_{\text{mod}}$ ) for comparisons with the observed values. Because of the restriction to low-frequency modes described in § 4 we compute  $\delta\tau_{\text{mod}}$  only for the lowest frequency bandpass ( $\nu_0 = 3$  mHz). Figure 5 shows a comparison between the frequency-averaged travel-time perturbations and the 3 mHz model for three focus depths, 3, 5, and 7 Mm below the surface. Also shown are the residuals after subtracting the model perturbations from the observations. There is good agreement between the model and observations for the shallower depths. At greater depths the similarity becomes less apparent. The patchy appearance of the residual maps is likely due to the properties of the realization noise, which is coherent over distances of about half of a  $p$ -mode wavelength (Gizon & Birch 2004). The similarity of the residuals at different focus depths follows from the highly overlapping set of modes used in the analysis for the different depths (see Table 1). We define the square of the deviation from the model,  $\sigma_{\text{mod}}^2$ , of each observed travel-time perturbation map ( $\delta\tau_{\text{obs}}$ ) as

$$\sigma_{\text{mod}}^2 = \frac{1}{N} \sum (\delta\tau_{\text{obs}} - \delta\tau_{\text{mod}})^2, \quad (2)$$

where the summation is over all  $N$  pixels in the maps.

To see how  $\sigma_{\text{mod}}$  depends on smoothing, we spatially convolve each observed perturbation map by 2D Gaussian functions with varying FWHM. Figure 6 shows the resulting  $\sigma_{\text{mod}}$  as a function of FWHM. The different colors indicate different focus depths, and the results are shown for both the 3 mHz bandpass (dotted lines) and the average over the four frequency bandpasses (solid lines). The deviation in the frequency-averaged perturbation maps

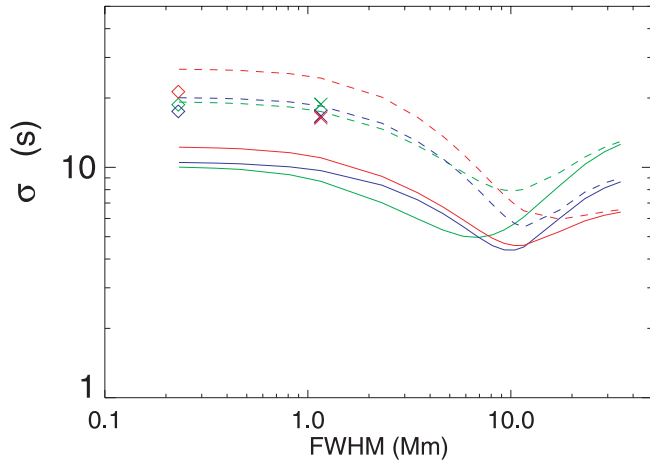


FIG. 6.—Measurements of the deviation from the forward models of the travel-time perturbations as functions of the degree of spatial smoothing of the observed perturbations. The FWHM indicates the full width at half-maximum of the two-dimensional Gaussian used to smooth the travel-time maps. The dashed lines indicate the results using 1 mHz wide frequency filters centered at 3 mHz for focus depths of 3 Mm (green), 5 Mm (blue), and 7 Mm (red) below the surface. The solid lines show the results for the frequency-averaged measurements. The diamonds indicate the standard deviations of the travel-time perturbations across the four independent frequency filters. The cross indicates similar standard deviations (across only three filters) for MDI observations (see text). The colors of the symbols indicate the same focus depths as the lines.

is reduced from the single 3 mHz bandpass by a factor of 2 (i.e., the decrease from the dotted to solid lines) as expected from independent measurements with similar uncertainties (although this improvement declines with more smoothing).

If each pixel represented an independent sample of the travel-time perturbation with an uncertainty that is normally distributed we would expect  $\sigma_{\text{mod}}$  to decrease with the inverse of the FWHM. Figure 6 shows, however, a considerably weaker decrease of the deviation, most likely due to the finite wavelengths of the modes used to produce the maps (Gizon & Birch 2004). In other words, the maps are already “smoothed” by finite-wavelength effects, which can be readily seen in the raw perturbation maps. With values of FWHM more than about 10 Mm, the effects of smoothing increase the departure from the model signatures, since the level of smoothing compromises the ability to resolve the dominant flow structures (e.g., Fig. 1). When “oversmoothed” in this way, there is less (and sometimes no) improvement achieved by frequency averaging. The values of  $\sigma_{\text{mod}}$  are generally similar for all of the depths shown, although the deeper two measurements can apparently withstand slightly greater smoothing to obtain smaller deviations from the model.

We have examined the statistics of the travel-time measurements in the independent 1 mHz-wide frequency bandpasses by computing the standard deviation of the measurements in each pixel from the mean over the four frequency bandpasses. Some caution is warranted in directly comparing the pixel average of this quantity, which we denote  $\sigma_{\nu}$ , with  $\sigma_{\text{mod}}$  when the data is smoothed. For example, substantial smoothing can reduce the difference between frequency bandpasses to an arbitrarily small value, even while both differ substantially from the model. A correspondence between these two quantities is expected, however, in the limit of little or no smoothing, if there are no systematic differences between frequency bandpasses of the perturbations or their uncertainties. The values of  $\sigma_{\nu}$  from the maps of  $\delta\tau_{\text{obs}}$ , smoothed with a Gaussian FWHM only minimally larger than the pixel size, are indicated by the diamonds in Figure 6. The

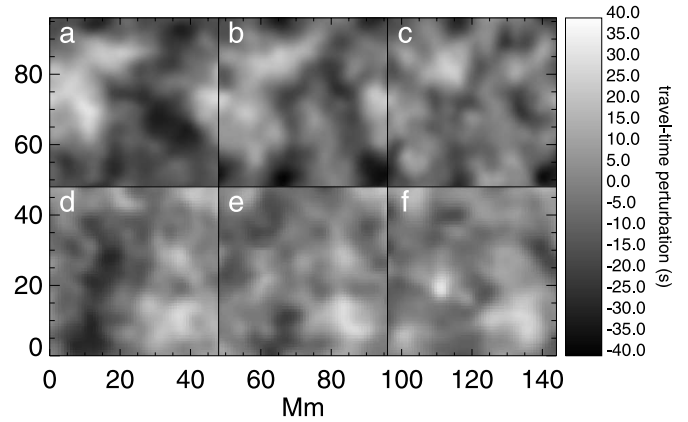


FIG. 7.—Maps of frequency-averaged travel-time perturbations measured from the simulations and from a cropped region of the Sun observed with *SOHO*/MDI. The top three panels show the results for the simulations at focus depths of (a) 3 Mm, (b) 5 Mm, and (c) 7 Mm, and the bottom three panels show the same depths for the MDI observations. Only frequency bandpasses centered at 3, 4, and 5 mHz were used to make the simulation and MDI averages displayed here.

correspondence is particularly striking for depths of 3 and 5 Mm. At a depth of 7 Mm, the ratio  $\sigma_{\nu}/\sigma_{\text{mod}}$  is about 0.8.

### 5.3. Comparisons with MDI Observations

It is of interest to compare the properties of the simulation observations with results obtained with actual solar observations. An 8.53 hr time series, with 60 s cadence, of solar images was processed identically to the simulated data reported here. The data set consists of 512 Dopplergrams obtained by *SOHO*/MDI (Scherrer et al. 1995) on 1999 June 1. The MDI Dopplergrams were interpolated onto Postel’s projections, tracked with a Carrington rotation rate and centered on longitude  $L = 306^\circ$  and latitude  $B = 7^\circ$ . The projected frames consist of a grid of  $512 \times 512$  pixels with a resolution of  $828 \text{ km pixel}^{-1}$ , obtained after a  $2 \times 2$  pixel averaging of the original high-resolution pixels. Travel-time perturbations were computed, with the same HH analysis as the simulations, for focus depths of 3, 5, and 7 Mm below the surface over 1 mHz temporal bandpasses centered at 3, 4, and 5 mHz. In general, HH travel-time measurements from solar data, unlike the simulations, are not reliable at higher frequencies. Figure 7 shows cropped ( $48 \times 48$  Mm) travel-time perturbation maps, averaged over all frequency bandpasses, of a region close to the center of the solar disk, compared with maps obtained from the simulated data at the same focus depths. It would be difficult to identify, from general appearances alone, which measurements came from the artificial data and which came from solar data (except perhaps for the finer pixel size noticeable in the simulations).

To assess the noise in the MDI observations, we compute the pixel-averaged standard deviation across the three frequency bandpasses. For these measurements, only a minimal smoothing (FWHM  $\approx 1.2$  Mm) is applied. The results are shown by the crosses in Figure 6. These values agree very well with the single-bandpass model deviations of the simulation measurements after smoothing with the same Gaussian function. We conclude that the magnitude of the uncertainties in HH travel-time perturbation measurements made with 8 hr of solar observations are very similar to those obtained with the simulated data analyzed here.

### 5.4. Depth Contribution of Signal

We use the forward models to examine the relative contribution with depth to the expected travel-time perturbations from the flow

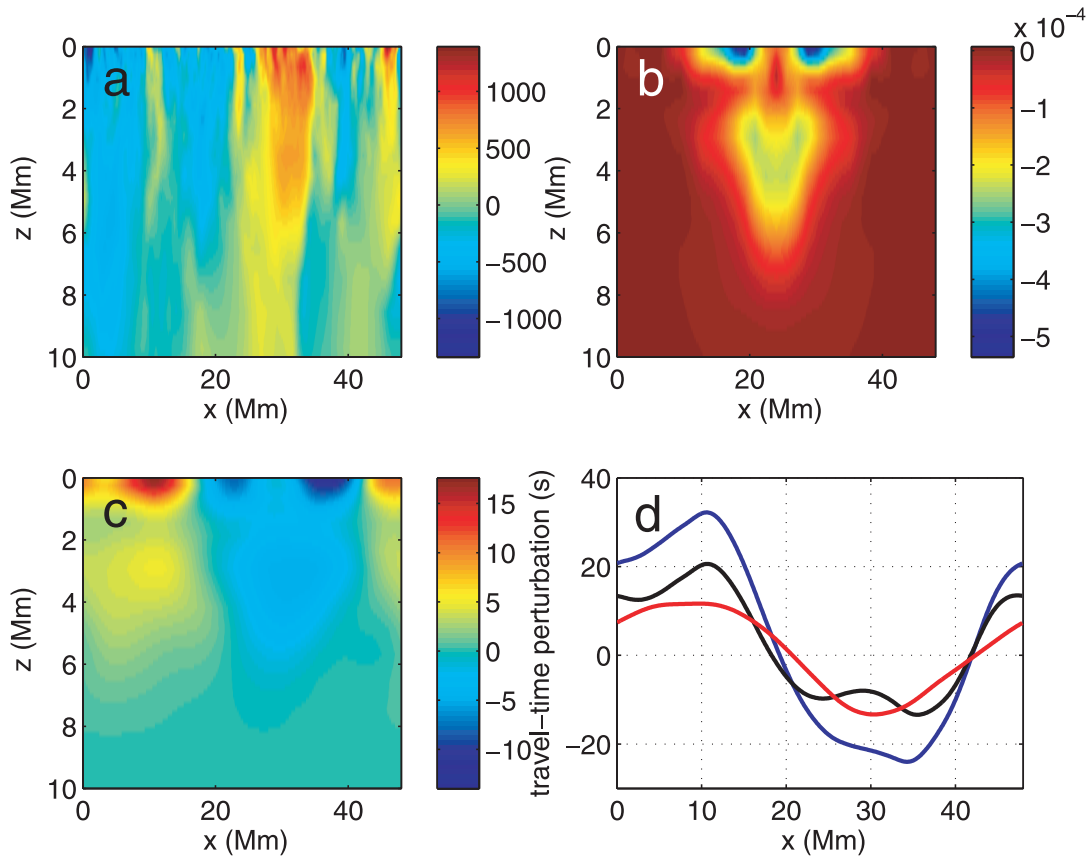


FIG. 8.—(a) Vertical slice at  $y = 24$  Mm through the time-averaged  $x$ -component of the flow from the simulation. (b) A slice through the travel-time kernel for a focus depth of 3 Mm, the color shows values in units of  $\text{s Mm}^{-3}$  ( $\text{m s}^{-1}$ ). The travel-time kernel is defined as  $\mathbf{K}/2\pi\nu_0$ , where  $\mathbf{K}$  is given by eq. (1) for the antisymmetric phase shift computed between the east and west pupil quadrants, and  $\nu_0 = 3$  mHz. The travel-time kernel is negative and relates a positive (negative) flow in the  $x$ -direction to a travel-time decrease (increase). (c) A vertical slice, at  $y = 24$  Mm, through the horizontal convolution of the kernel with the  $x$ -component of the flow field; the color bar has units of  $\text{s Mm}^{-1}$ . (d) The total travel-time perturbation (blue), the target signal (red), and the remaining (total minus target) signal (black) as functions of the  $x$ -coordinate of the focus position. The target signal is integrated over a range of depth from 2 to 6 Mm.

structures. To successfully infer the magnitude of subsurface flows from the travel-time observations, we will very likely require that the expected perturbations due to flows at a desired focus depth exceed the noise present in the observations. Thus, we need to extract the relevant portion of the expected signatures (i.e., due to the flow near the target focus) from the total travel-time perturbations (which result from flows over the entire depth range sampled by the modes used in the measurements).

Assessing the contribution to the travel-time signatures of flows near the surface are of particular interest. This is motivated by the desire to understand the rapid decrease in the observed travel-time perturbations with focus depth, as exhibited in Figure 2. Previous HH analyses and modeling of supergranulation observed with *SOHO*/MDI have indicated the possibility of a rapid decrease, and eventual reversal, of the travel-time perturbations with depth (Braun & Lindsey 2003; Braun et al. 2004). An explanation was offered by Braun & Lindsey (2003) that the results were consistent with an increase with focus depth of the near-surface contribution to the travel-time perturbations from oppositely directed flows in neighboring supergranule cells. These near-surface contributions increased with focus depth, because the pupils needed for lateral-vantage HH increase in radii with depth (e.g., Table 1) eventually exceeding the size of the supergranular cells. Forward models assuming simple, shallow, cellular flows were able to reproduce this effect (Braun et al. 2004),

and it is of considerable interest to see if such behavior exists for the flow patterns present in the simulations of Benson et al. (2006).

To assess the depth dependence of the contribution to the travel times due to subsurface flows, we perform a horizontal convolution of the kernels (§ 4) with the  $x$ -component of the flow fields. Figure 8 shows a vertical slice of the  $x$ -component of the flows, a slice through the kernel functions (for a focus depth of 3 Mm), and a slice through the horizontal convolution of the two. Figures 9 and 10 show the same for focus depths of 5 and 7 Mm, respectively. What is readily apparent in these figures is that, while the flows fall off gradually with depth (Fig. 8a), the contribution to the travel-time perturbations (Figs. 8c, 9c, and 10c) due to near-surface layers (with depths from 0–1 Mm) is disproportionately large. This results from the strong near-surface sensitivity of the travel-time kernels (Figs. 8b, 9b, and 10b). It is noteworthy that the rms of the travel-time perturbation integrated from the surface to a depth of 2 Mm is, for all three depths shown, roughly the same as the rms of the perturbation integrated from 2 Mm to the bottom of the simulation, where the rms is computed over the full horizontal domain.

For what follows we define the “target signal” to be the contribution to the travel-time perturbation over a “target range” of depths about a given focus depth. This range is intended to be proxy for the depth resolution of any modeling of the observations (through either forward or inverse methods). For a depth of

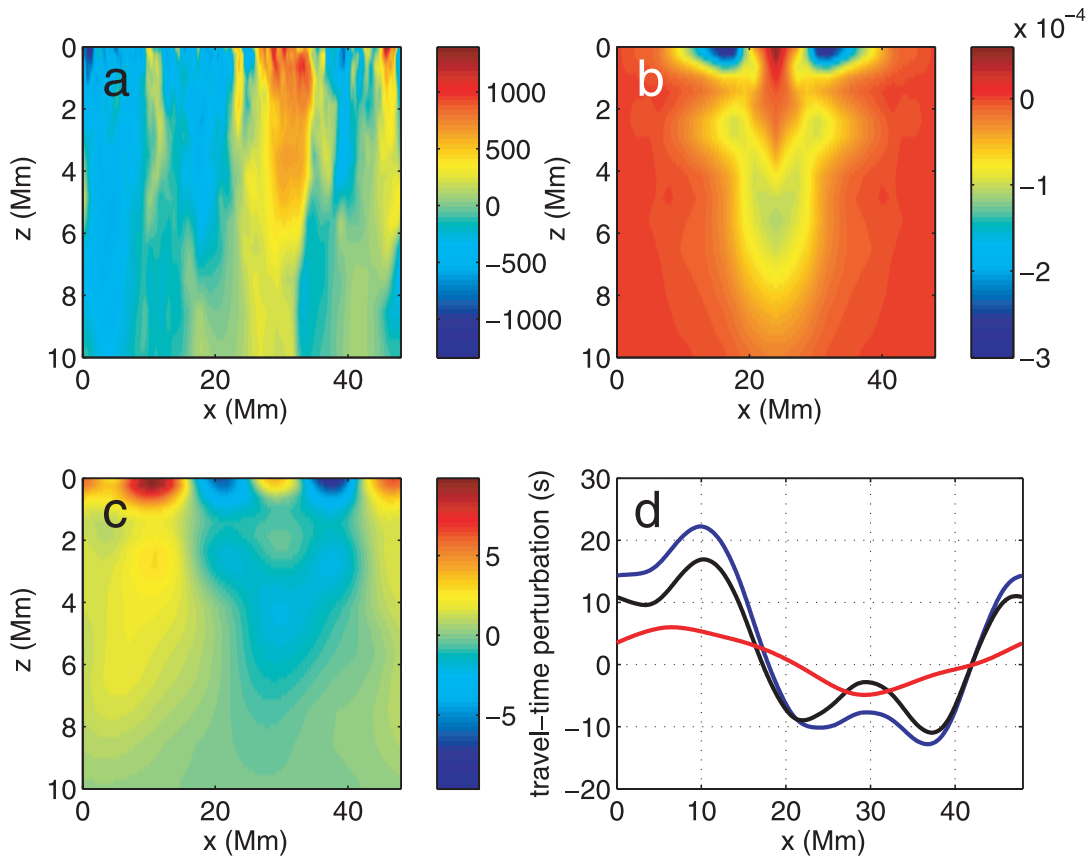


FIG. 9.— Same as Fig. 8, but for a focus depth of 5 Mm. The target signal is integrated over a range of depth from 4 to 8 Mm.

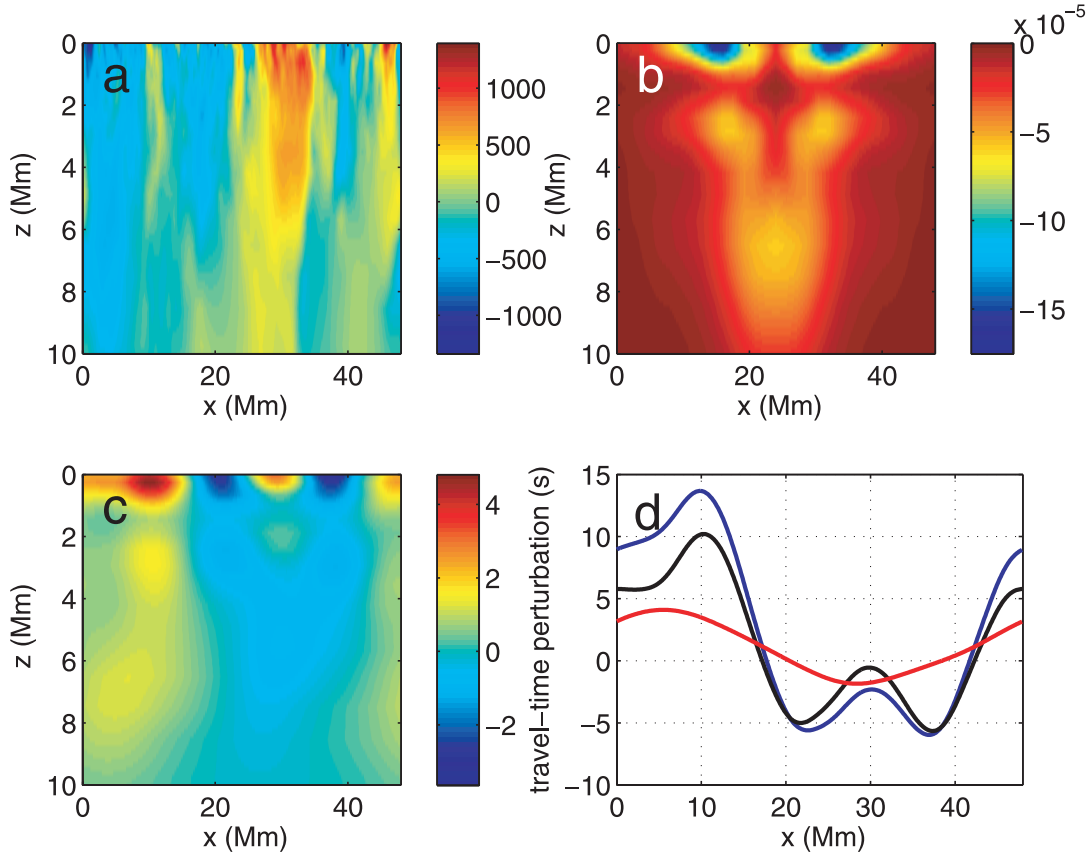


FIG. 10.— Same as Fig. 8, but for a focus depth of 7 Mm. The target signal is integrated over a range of depth from 5 to 10 Mm. In panel *c* at  $x \approx 30$  Mm, notice the positive contribution to the travel-time perturbation from the near-surface. This contribution is due to flows located in the near-surface lobes of the kernel and with opposite sign to the flow at the focus point.



TABLE 2  
SIGNAL-TO-NOISE RATIO ESTIMATES

Depth (Mm) (1)	rms Total Signal (s) (2)	rms Target (s) (3)	S/N 8 hr Simulations (4)	S/N 24 hr Sun (5)
3.0.....	15.6	8.7	1.7	2.6
5.0.....	10.5	3.6	0.8	1.2
7.0.....	7.4	1.7	0.4	0.6

3 Mm below the surface, the integration is carried out from 2 to 6 Mm. For the focus of 5 Mm, the target range is from 4 to 8 Mm and for 7 Mm, the target range used is 5–10 Mm. These ranges were selected by a visual inspection of the sensitivity functions (e.g., Figs. 8*b*, 9*b*, and 10*b*). Figures 8*d*, 9*d*, and 10*d* show plots of the total travel-time perturbations (*blue lines*), the target signal (*red lines*), and the contribution away from the target (*black lines*), over a slice of the data cube. Note that Figures 9 and 10 indicate a positive contribution to the travel-time perturbations at  $x \approx 30$  Mm from the near surface, which is opposite in sign to what is expected for the rightward-directed flow at this location (Fig. 8*a*). This contribution is due to flows located in the near-surface lobes of the kernel, which have opposite sign from the flow at the focus. These measurements offer support to the suggestion that the rapid decrease (and potential reversal in sign) of the total travel-time perturbation with increasing depth results from the near-surface contribution of nearby (and oppositely directed) flows.

To estimate the S/N required for the detection and modeling of the subsurface flows in the simulations we use the rms of the target signals as defined above. For the noise levels, we use the optimal (i.e., minimum) values of  $\sigma_{\text{mod}}$  from Figure 6 for each depth. For 3 Mm, this is 5 s, and for 5 and 7 Mm, this is 4.5 s. Table 2 shows the values of the rms for the total travel-time perturbation (col. [2]), the rms of the target signal (col. [3]), and the resulting S/N (col. [4]) for the three depths listed in column (1). These results do not substantially change if the “target” is adjusted to include flows at deeper layers. For example, extending the lower depth limit of each target range to the bottom of the simulation box increases the rms target signal and the resulting S/N by approximately 5%–15% for the depths listed in Table 2. It is clear that the S/N is too small for detecting subsurface flows in these simulations at and below 5 Mm with 8 hr of data. Note that a S/N of one implies the observed travel times are consistent, within the noise, with no flows. A S/N greater than unity is required for an unambiguous detection of a flow.

## 6. DISCUSSION

A major finding of this work is that lateral-vantage holography recovers the model travel times about as well as one should expect. In other words, the rms of the residuals (observed minus model) is very close to the noise levels as deduced by the variance between independent frequency bandpasses. We can also infer from the similarity of results across the different bandpasses that the validation of the method applies across a wide range of temporal frequencies. In addition, we have gained a fair degree of confidence in the use of the Born approximation for modeling  $p$ -mode travel-times for flows similar to the ones simulated here. We note, however, that models have only been computed in one frequency bandpass, and that further effort is needed to extend the modeling to higher frequencies. One important issue not addressed here is the potential improvement in the spatial resolution of flows (or other perturbations) deduced with observations

made with increasing temporal frequency of the  $p$ -modes employed in the analysis. This is plausible if the resolution scales with the mode wavelength which, for a mode propagating through a fixed focus, varies inversely with the temporal frequency.

We have made a quantitative assessment of the contribution of near-surface layers to travel-time measurements as a whole. A major consequence of our result is a plausible explanation for the relatively rapid decrease with depth (and eventual sign reversal) of the travel-time perturbations, which has also been detected in MDI observations of solar supergranulation. This trend is not necessarily detrimental to successful modeling of subsurface flows, given a sufficient S/N in the travel-time measurement, and the assumption that the near-surface perturbations are well understood. Most modeling efforts typically include an assessment of flows at a range of depths, including the near-surface regions. In the simulations performed here, the horizontal flows decrease gradually with depth (Figs. 2 and 8*a*). However, strong shear layers immediately below the photosphere or the existence of surface perturbations due to magnetic fields (neither of which is present in these simulations) could enhance the near-surface contributions and potentially complicate the modeling of flows. Some possible complications include strong surface perturbations that are not easily modeled under assumption of the Born approximation, or that might require the use of “surface terms,” which are not routinely employed in local helioseismology (see Braun & Birch 2006). Numerical simulations provide an excellent means of testing these possibilities.

In as far as these simulations predict the types of flows likely to be present in Sun, it is appropriate to extrapolate our results to evaluate the prospects for the detection and modeling of actual solar flows. We assume similar noise levels, as confirmed by the comparisons performed in § 5.3. An increase in the temporal duration of the observations from 8 to 24 hr of observations, or roughly the lifetime of the supergranules, should produce an increase in the S/N by a factor of  $\sqrt{3}$ . On the other hand, our experience has shown that only the 3, 4, and 5 mHz frequency bandpasses can be used successfully in solar measurements, so that the nominal increase in S/N is a more modest factor of about 1.5. The resulting S/N values are shown in column (5) of Table 2. The general result, that supergranule-sized flows are essentially undetectable using current methods below depths around 5 Mm, is not substantially changed.

It is fair to point out that our pessimistic conclusions about detecting small-scale subsurface flows below depths of 5 Mm are based on idealized conditions and assumptions. To the extent that these conditions may not be representative of actual or proposed solar measurements and modeling, it is plausible that our conclusions may not be fully relevant. On the other hand, it has been our goal to err, if at all, on the side of optimism in computing the S/N values quoted here. Our noise estimates are inferred from forward models constructed directly from the subsurface flows present in the simulations, and thus represent an idealized, “best-case,” scenario. Thus, the S/N values quoted do not address possible errors introduced by modeling limitations or systematic effects encountered in solar measurements. Zhao et al. (2007) have performed inversions of time-distance measurements made from this simulation and find only weak correlations between the actual and inverted flows at depths 3 Mm and greater below the surface. This appears to be consistent with our findings regarding S/N estimates, although the methods employed are different.

While the use of lateral-vantage HH is a fairly efficient means of measuring the effects of the horizontal component of the flow field (see § 3), we recognize that additional information may be used to infer the properties of subsurface flows. These include

using helioseismic measurements of vertical flows and the assumption of mass conservation as an additional constraint on horizontal flow components, or matching subsurface flows to those observed at the surface through other means (e.g., direct Doppler measurements or feature tracking). The degree to which this additional information can be used to substantially improve our ability to measure flows deeper than a few Mm below the photosphere, however, remains to be determined. We also recognize that solar supergranulation has a spatial scale somewhat larger than the cells explored in these simulations, which may permit somewhat more spatial smearing and subsequent reduction in noise. Some improvement might be expected by increasing the temporal duration of the observations, since some supergranules last as long as several days. And obviously, many subsurface dynamic phenomena, both known and unknown, will still be accessible after temporal and/or spatial averaging. It is also important to keep in mind the spatial and temporal scales of the flows for which our findings are relevant. Clearly, S/N values far greater than unity are possible, even at substantial depths below the photosphere, for measurements of large-scale or long-lived structures such as meridional and zonal flows.

Birch et al. (2006) presented a method to combine travel-time measurements, from time-distance helioseismology, of thousands of supergranules, allowing the exploration of supergranule-averaged flows. In general, a parameterization of small-scale flows (in terms of statistical moments or Fourier components, for example) might be more amenable to modeling than the reconstruction of individual flow vectors (e.g., Woodard 2006, 2007).

Forward modeling efforts with both solar and artificial data appear to offer considerable utility for understanding both the

advantages and limitations of helioseismic methods. It would be useful to extend the analysis presented here to address a variety of additional issues. Understanding the trade-off between maximizing spatial resolution in modeling perturbations and minimizing the uncertainties in the models is a major issue that can be explored through forward modeling. For example, it should be straightforward to measure the signal and noise values for different spectral components (in the spatial domain) of flows and other perturbations to address this.

Finally, we emphasize the importance of numerical simulations for helioseismic analysis. Some improvements are now becoming available or are in preparation. These include simulations which cover larger areas and extend deeper into the solar interior and are carried out in spherical geometry. There is also a need to complement realistic simulations (which give the best current estimate of what is likely to be found in the solar interior) with the ability to prescribe and adjust perturbations (in order to fully test observing and modeling methods and to allow for surprises in the Sun). The addition of magnetic fields to numerical simulations will be of particular utility to test both their influence on convective and other flows, and their influence on local helioseismic measurements.

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