The Effects of Limited Resolution, from the Subtle to the Supreme.

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- Sources and effects of limited resolution on polarization spectra:
    - spatial, temporal, spectral.
- Down-stream effects: What is the flux? What is the current? Are Maxwell's equations even *satisfied*?

Definition: **"limited resolution"**

Unresolved observations in:

- **Time**, where $t_{observe} > t_{evolve}$
  - Higher the spatial sampling, the faster the temporal sampling must be for temporal resolution.
  - Temporal sampling includes acquisition of *all* spectral/polarization information.
  - 1" → Δt < 1min.
  - Scales linearly with spatial resolution, generally.
- **Space**, in the plane of the sky, where telescope resolution > target object
- **τ/height**, where spectra are influenced by Δz > 1 photon m.f.p.
  - Effectively, all spectro-polarimetric observations are unresolved in τ/height.
  - Information is still available, but limited.

For all dimensions, limited resolution results in signal mixing.

* I am limiting this discussion to Stokes polarization data from Zeeman splitting. Many of the effects are the same for scattering/Hanlé/etc effects, but I do not treat those specifically here.

How, in practice, can you tell?

Q: is there a large percentage change between the resolution elements (pixels)?

Q: is there a large percentage change between observations?

Q: Is the spectral resolution coarser than the width of the spectral feature?

Q: Is the inferred magnetic fill fraction < 1.0 anywhere?

If answer to any of these is "yes" at any point in the (space, time,spectral) parameter space, then ***observations are unresolved.***

To investigate, rely *not* on observations, but on *synthetic* data, where

- resolution is *known*,
- underlying model is *known*,
- spectra can be *manipulated*.

Synthetic Data

- Start with a valid magnetic field, fully resolved.
- In a manner as simple or complex as desired, generate desired emergent polarization spectra.
- Combine spectra (e.g., Stokes spectra) to create “observed” light.
- Manipulate as per the tests you want to do.

For the demonstrations here:

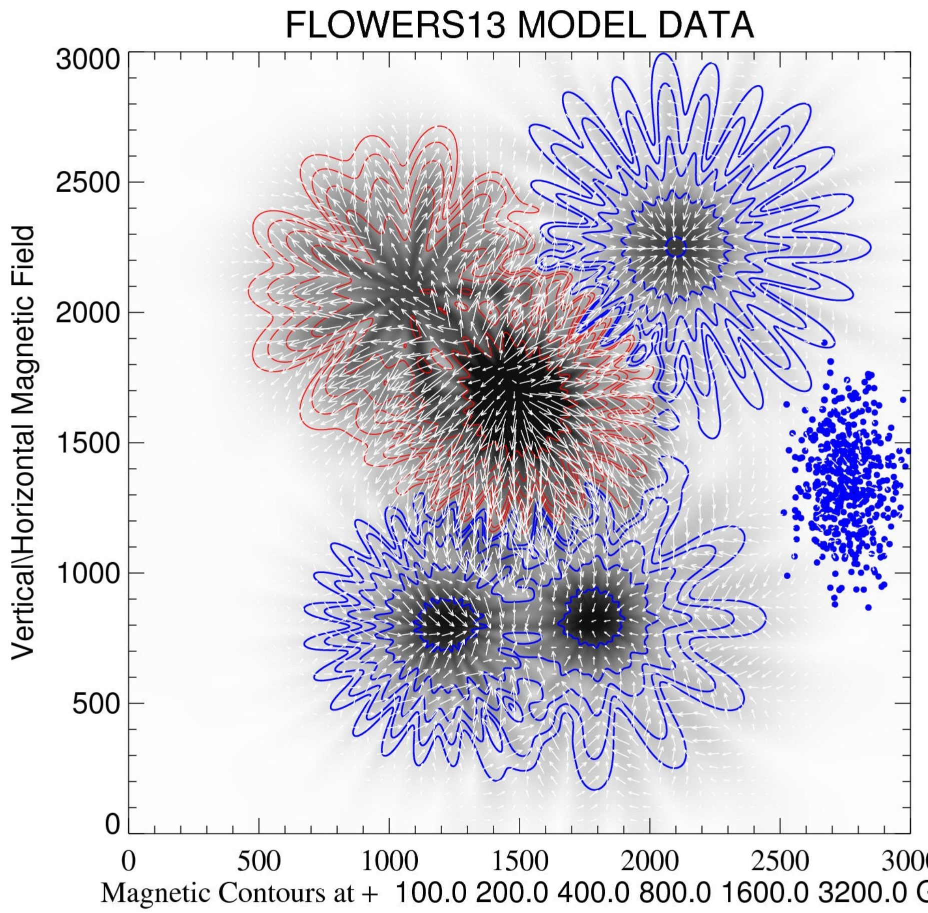
- Milne-Eddington approximation, Unno-Rachkovsky spectra were generated from boundary field
  - Either 630.25nm FeI or 617.3nm FeI line was used.
  - I am *not* testing resolution in τ
- Where needed, M-E/U-R least-squares inversion used to re-invert.
  - model field or similar used as an initial guess.

Two sets of synthetic data used here:

Model #1: a ***potential*** field

Courtesy G. Barnes

- Critically-sampled 3k×3k grid
- structures computed to resemble *penumbra* and *plage*.
- Mininum structure size assigned to be < 0.015"



"Spot/Penumbrae" model:

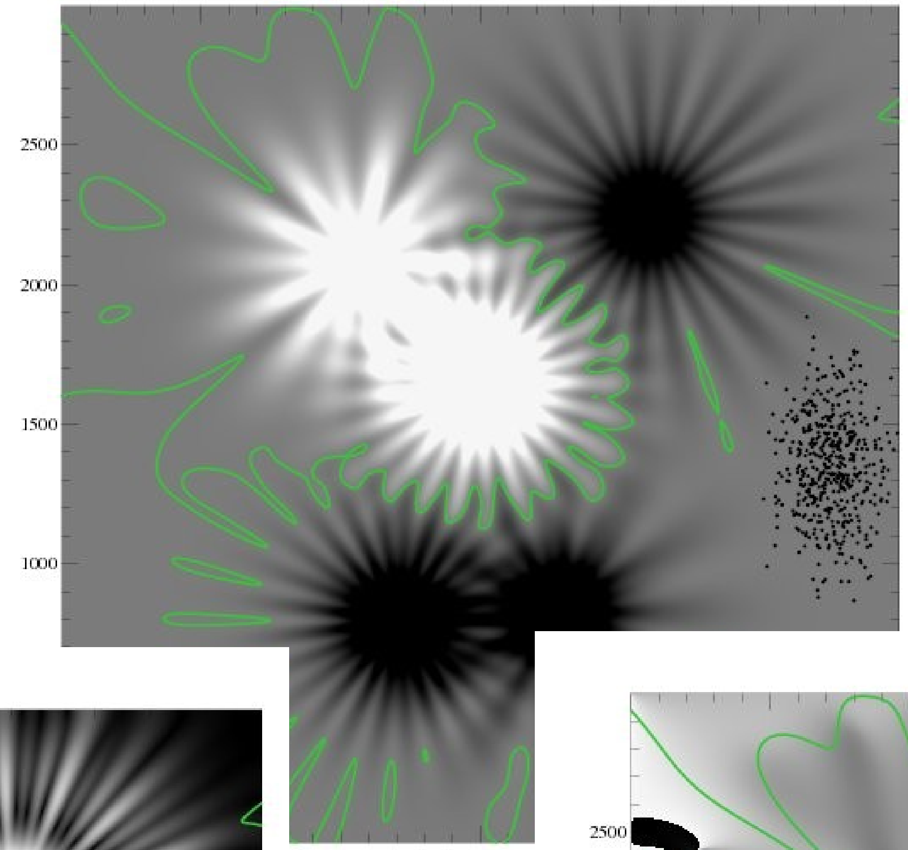
$$B_z = B_0 e^{-(\mathbf{x}-\mathbf{x}_0)^2/(a_0^i)^2} + B_1(\mathbf{x}-\mathbf{x}_0)^2 e^{-(\mathbf{x}-\mathbf{x}_0)^2/(a_1^i)^2}\left[1+\cos(n_1\theta+\phi_1^i)\right] + B_2(\mathbf{x}-\mathbf{x}_0)^2 e^{-(\mathbf{x}-\mathbf{x}_0)^2/(a_2^i)^2}\left[1+\cos(n_2\theta+\phi_2^i)\right]$$

Parameters *a*, *φ* determine morphology of various features

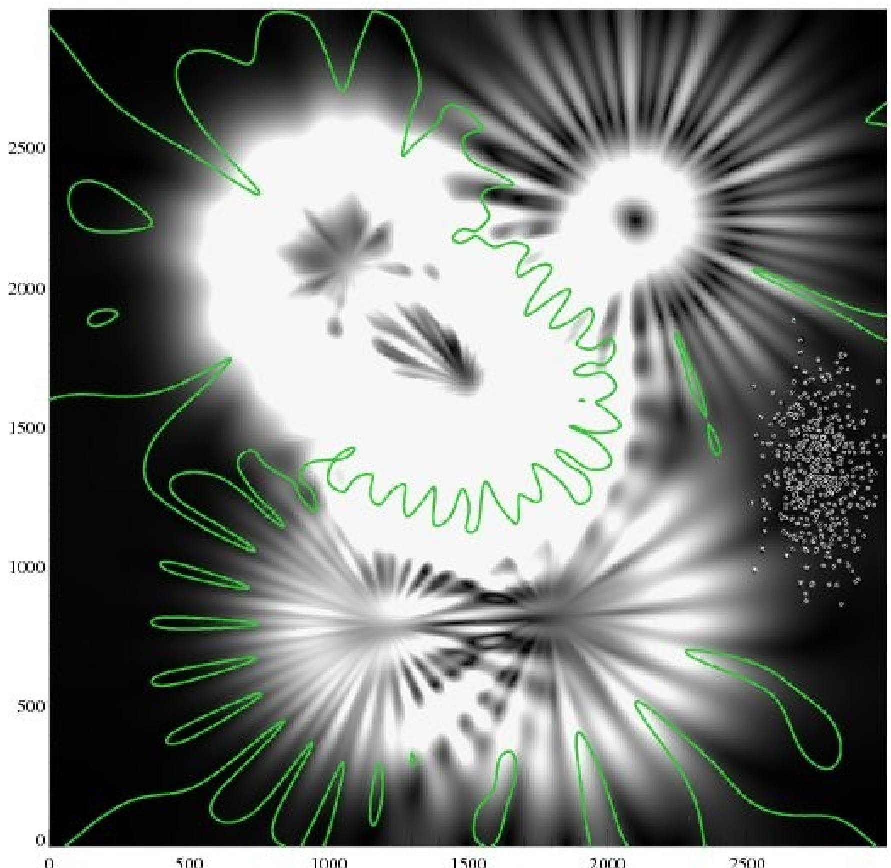
"Plage area": random distribution of same-polarity, potential, fully resolved "flux tubes", each described by $B_z = B_0 \exp(-r^2/a_0^2)$ B_0, a_0 allowed to vary. Probability distribution is Gaussian, but a minimum size between centers is imposed to avoid pile-up.

Additional views:
See Leka et al 2009; these data were used in Hare & Hound exercises for ambiguity-resolution algorithms.

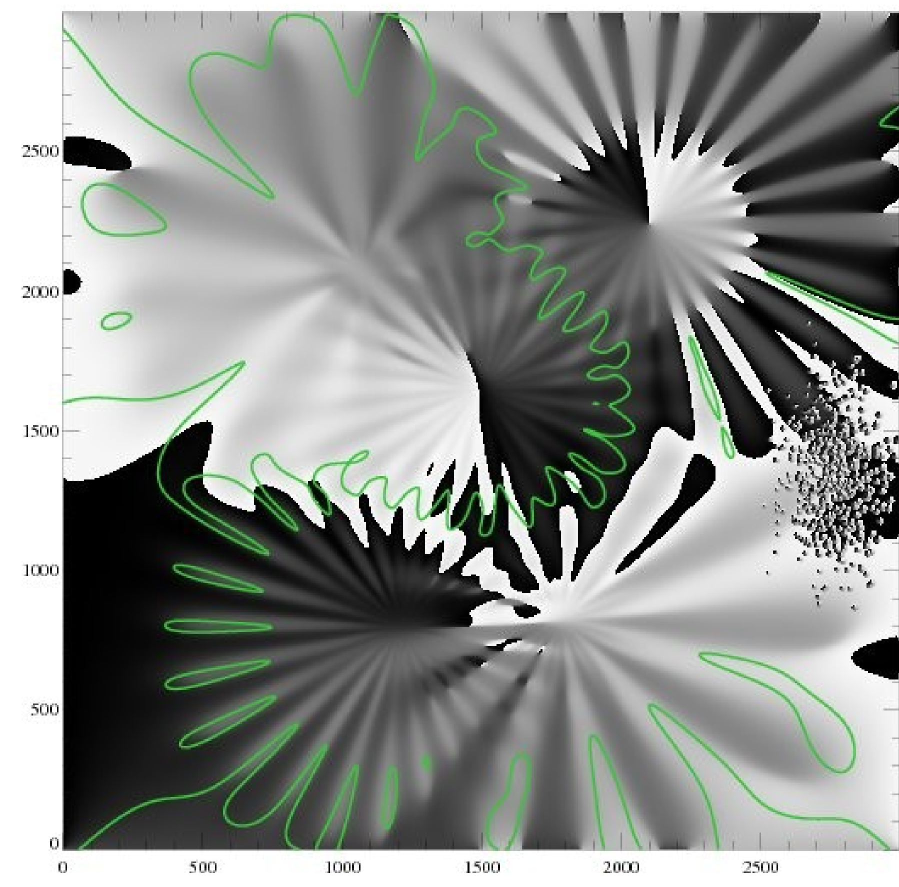
Vertical field



Horizontal Field

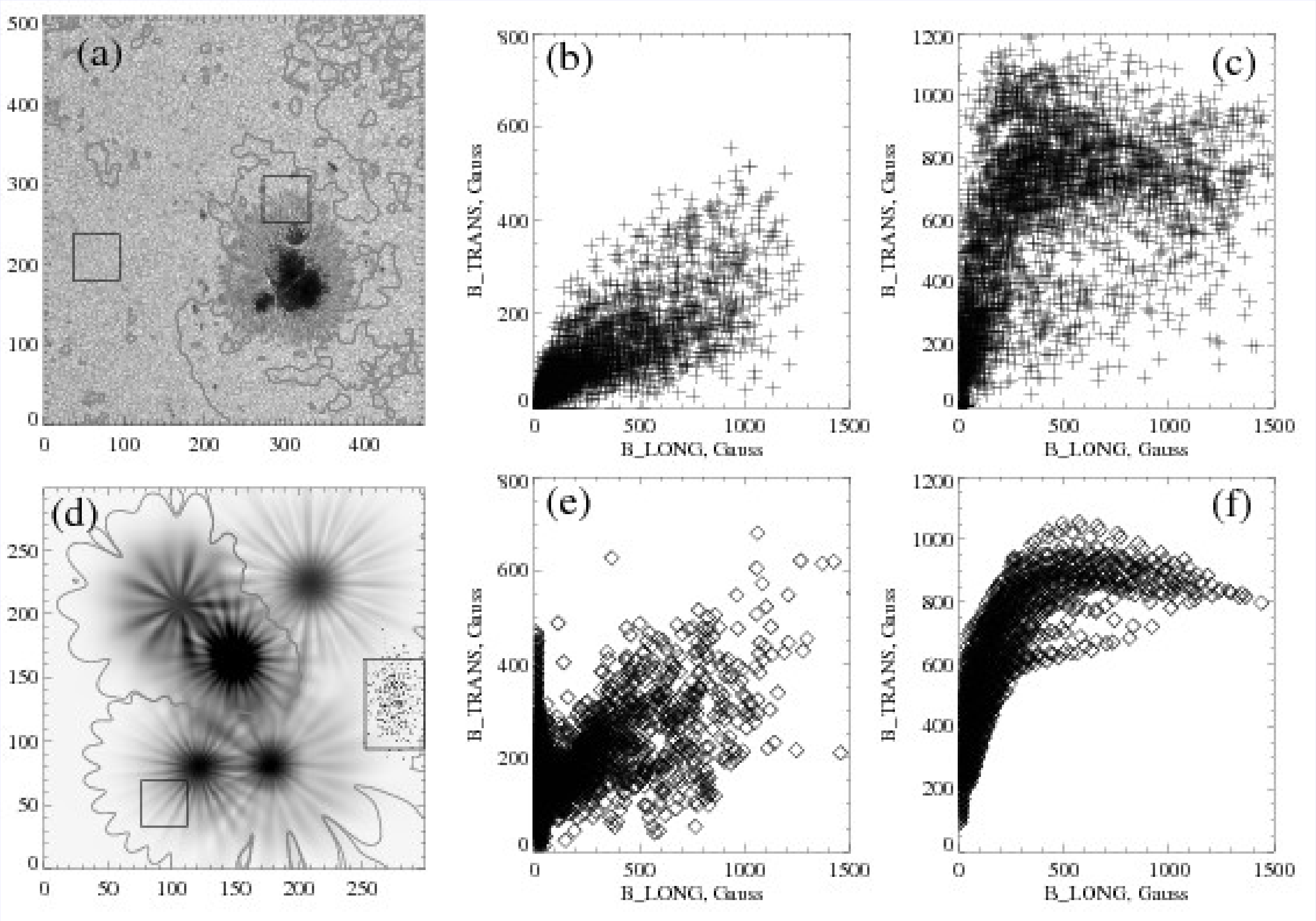


Azimuth



Test of spatial resoluton: model vs. Hinode/SP data of penumbra/plage

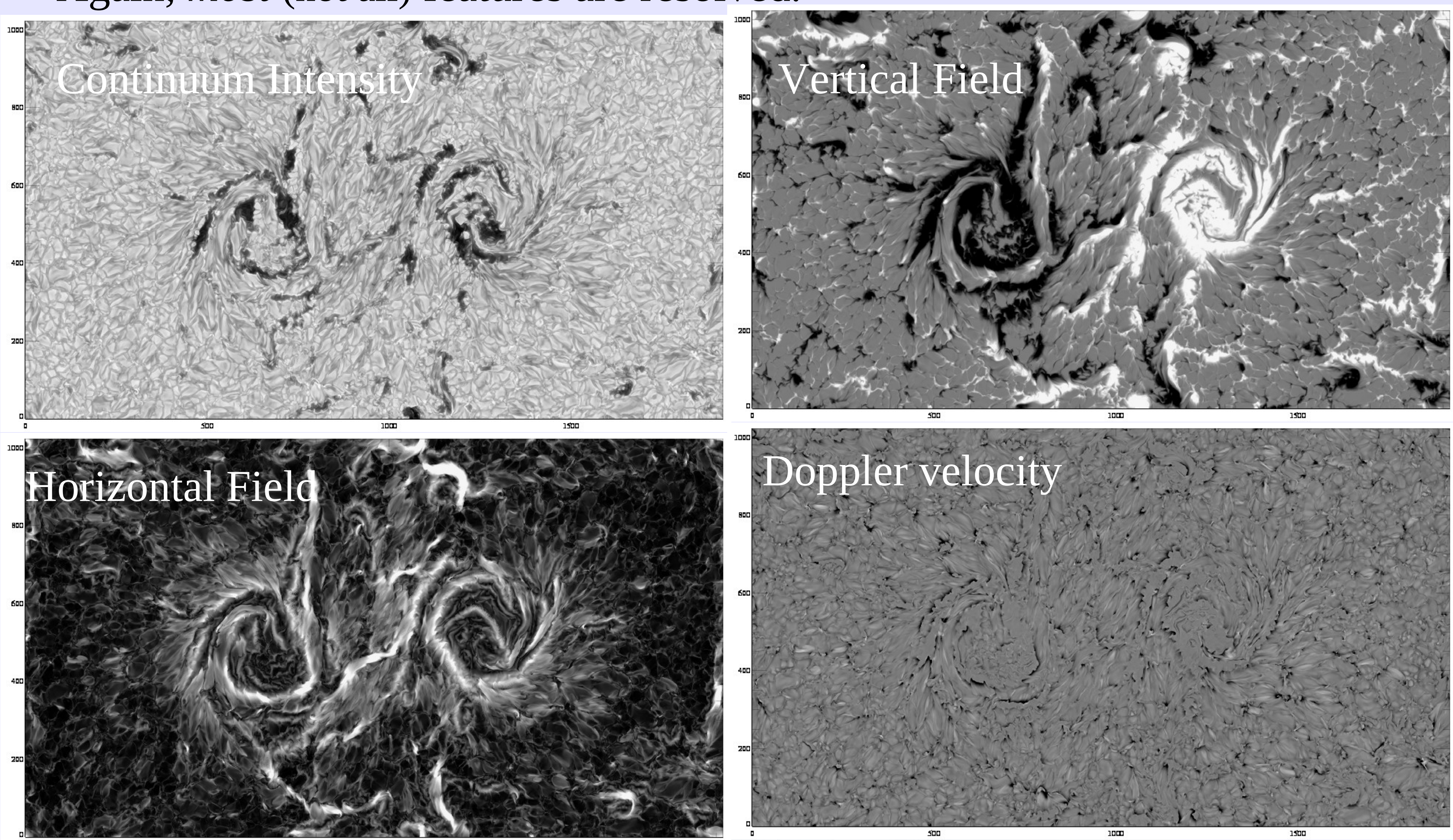
Hinode/SP continuum, boxes showing plage/penumbral areas for reference.
Blos vs Btrans plots for (b) observed plage, (c) observed penumbra

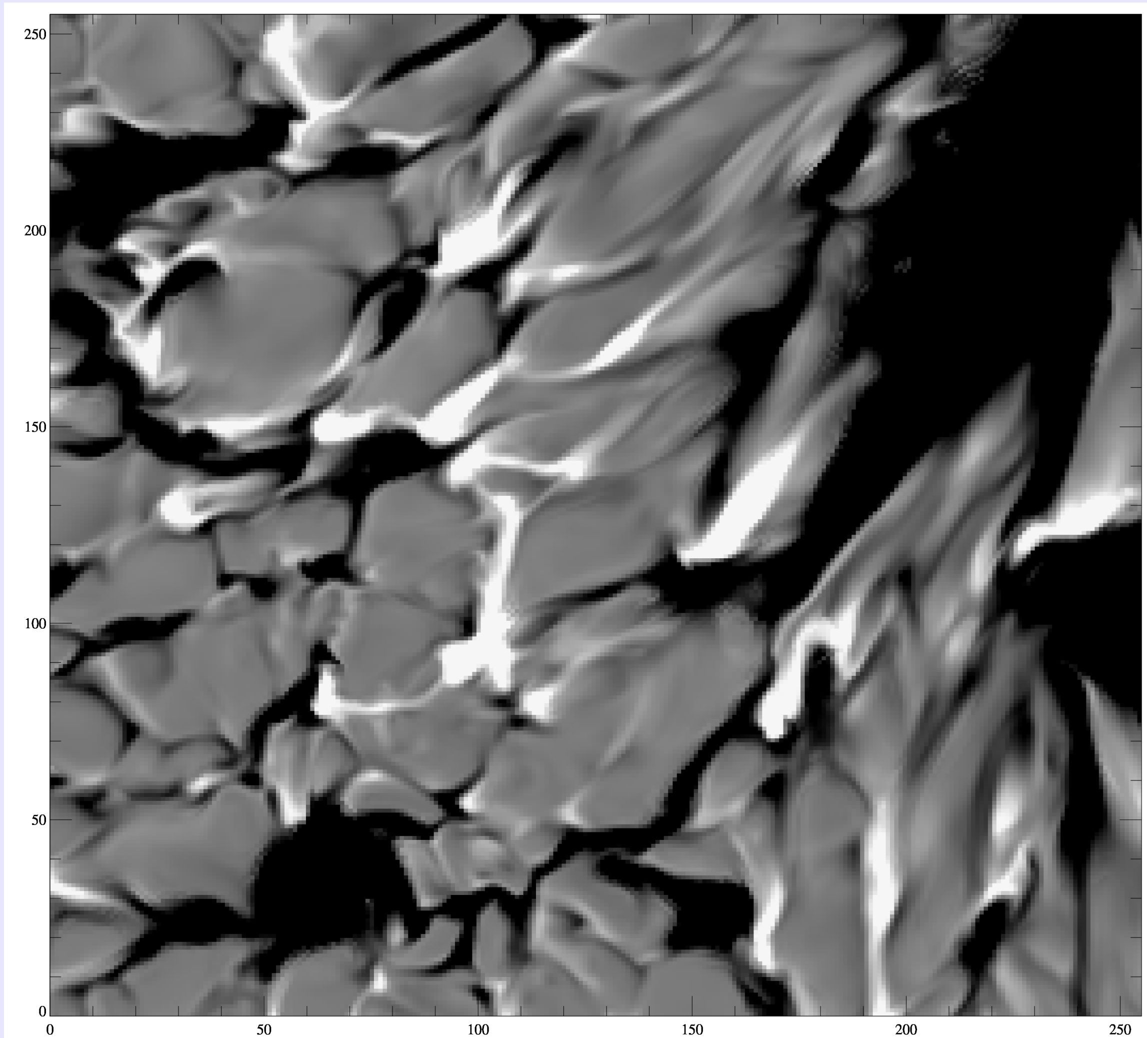


Bin-10, ~0.3" model data continuum, boxes showing plage/penumbra.
Blos vs Btrans plots for (e) model plage, (f) model penumbra

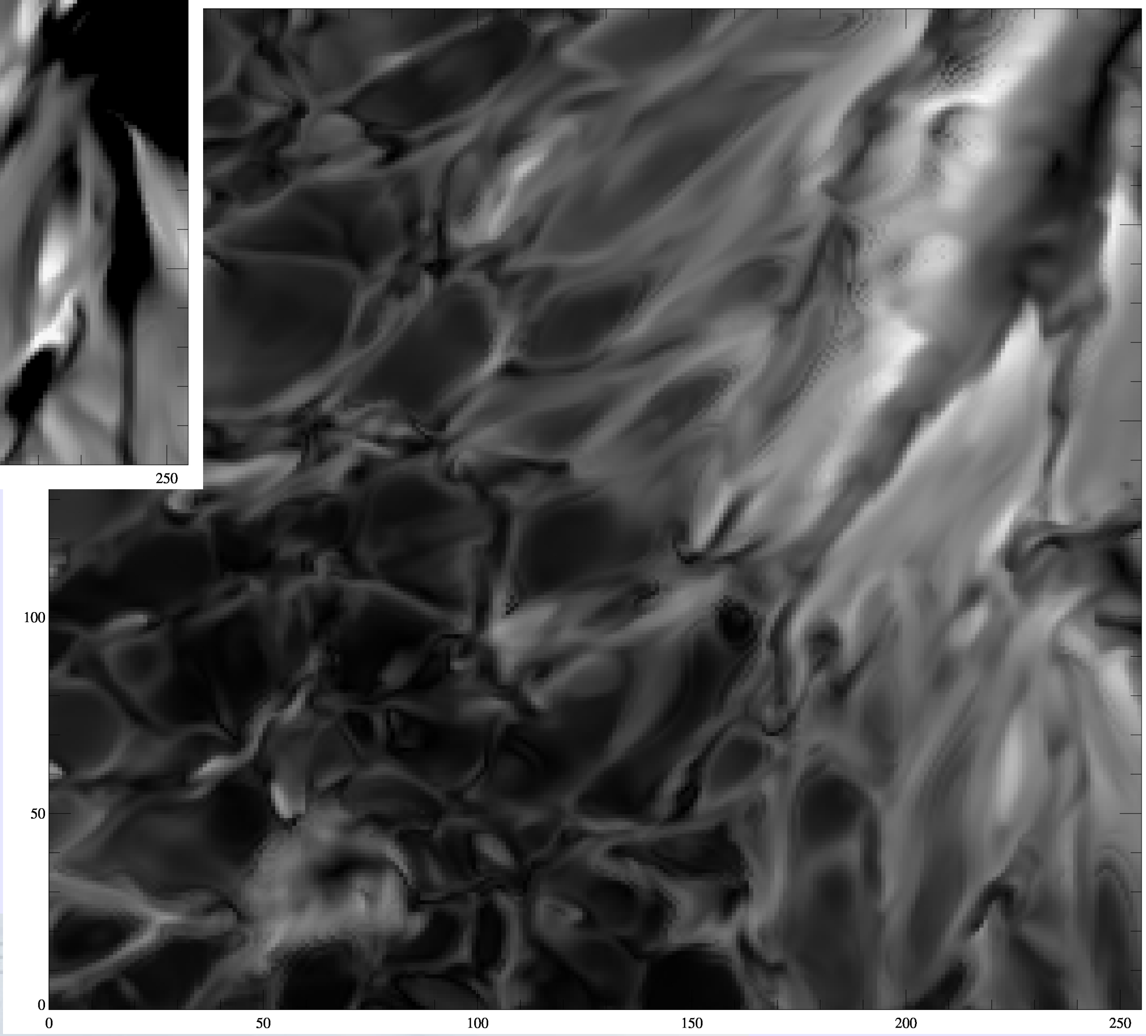
Model # 2:

Time-series of photospheric boundary data from MHD simulation of emerging flux system, *courtesy M. Cheung*. Again, *most* (not all) features are resolved.





Δt = 53 seconds,
Δx = 48km

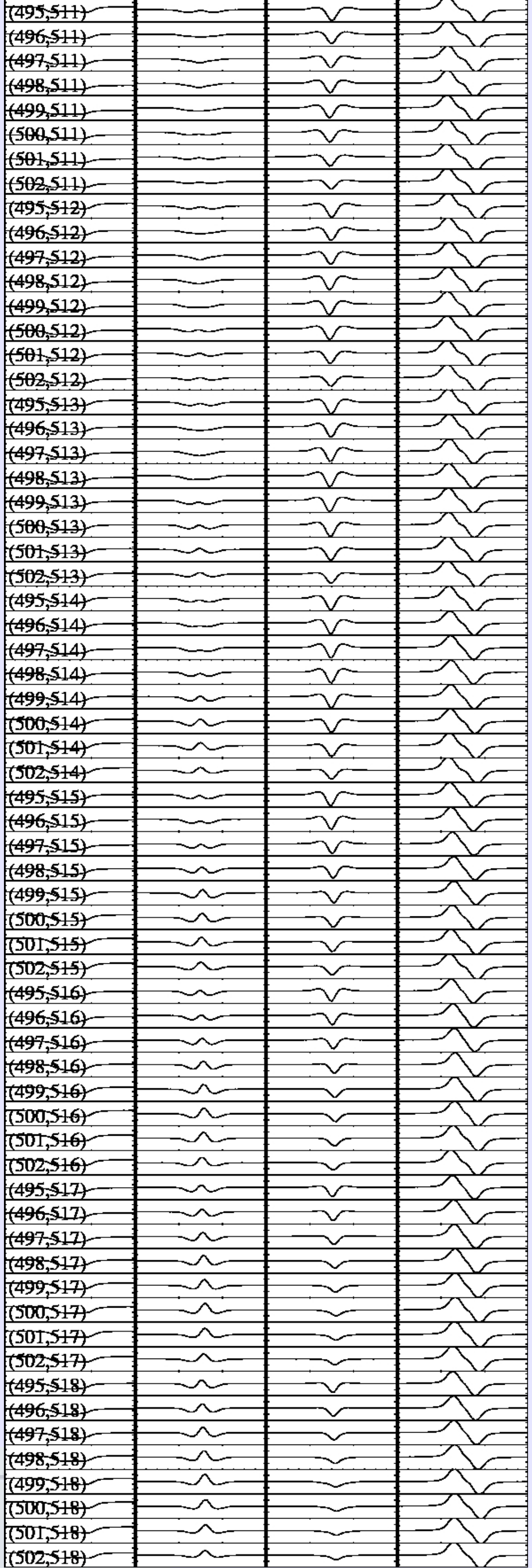


(These data are also being used for Hare & Hound challenges regarding feature tracking and plasma-velocity determination).

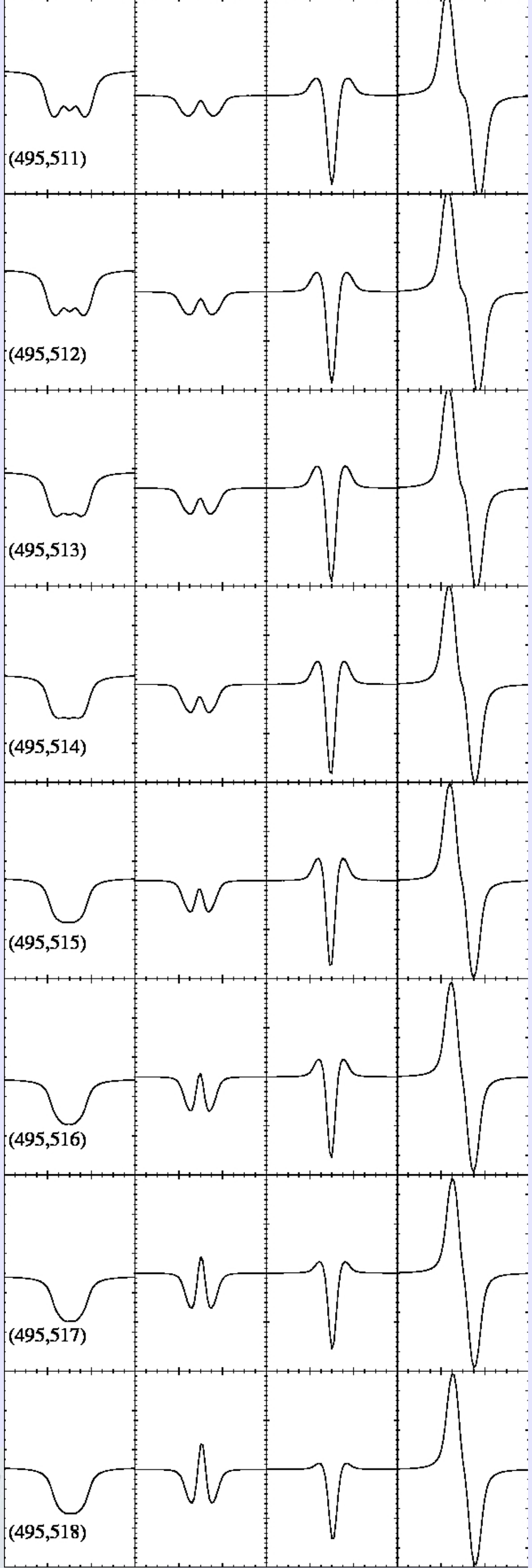
Signal Mixing:

- Observed quantities are ***intensity-weighted mixes of polarization***.
- What enters a telescope is the total light
- demodulation to Stokes spectra occurs after spatial/temporal mixing.
- demodulated(<I+/-P>) ≠ <I> + <P>

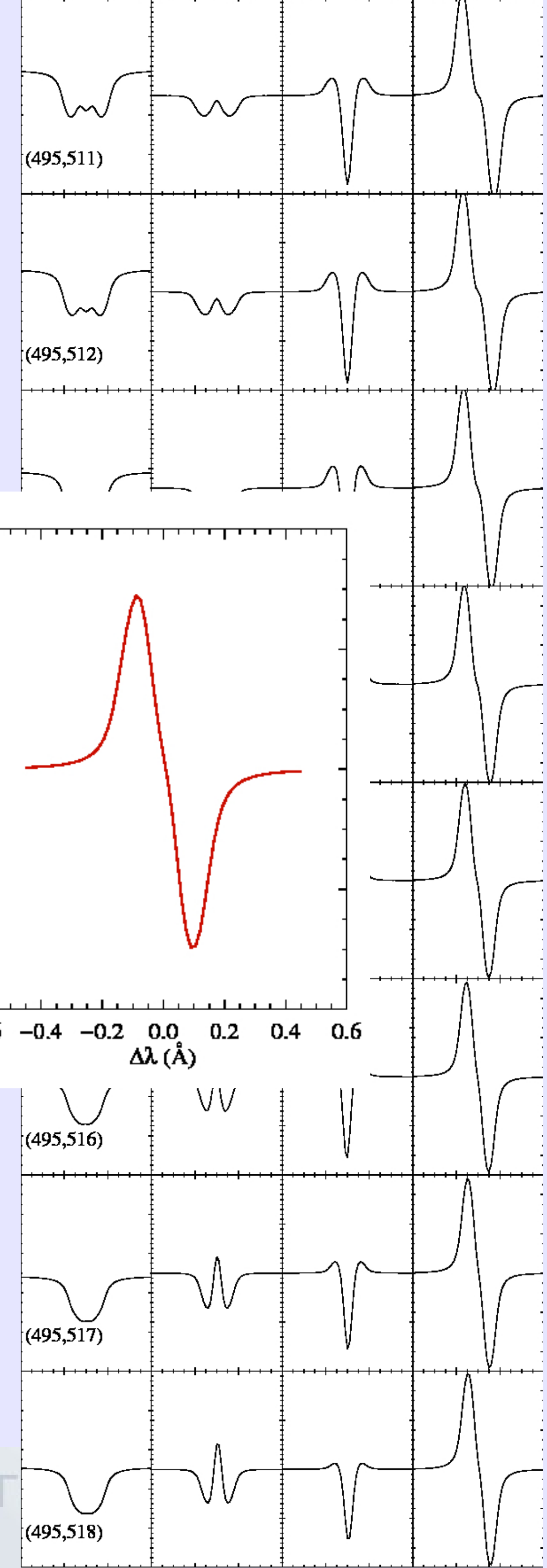
Consider an example, 64 spectra from a spatially-resolved area of the synthetic data:

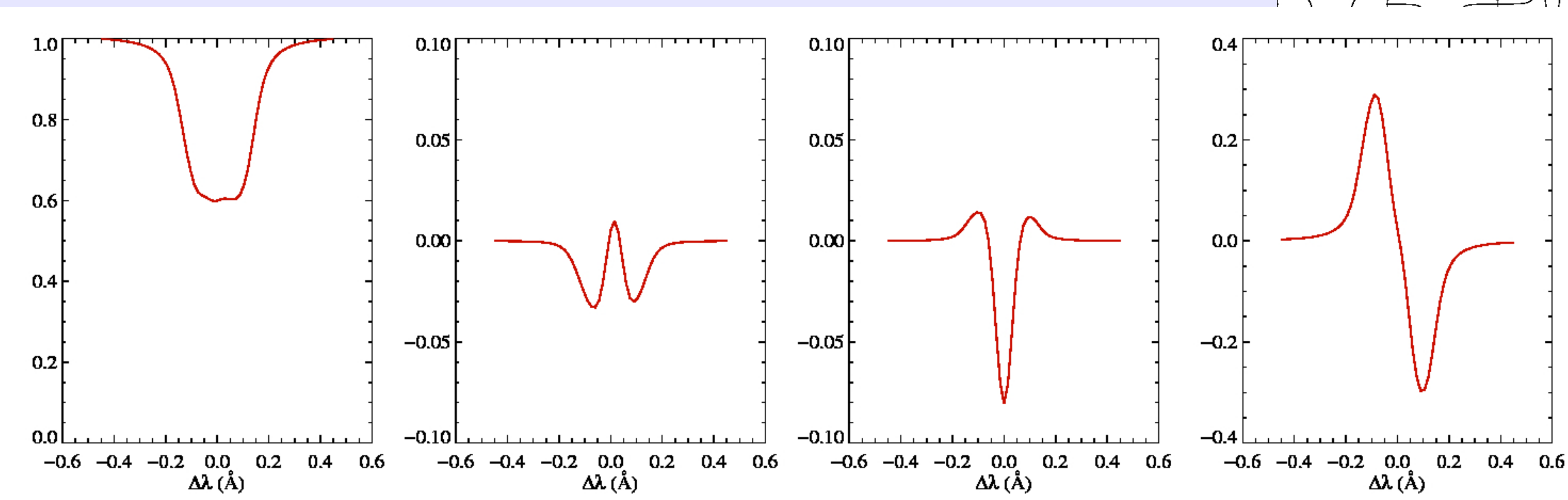


Ok, let's look at a sample:

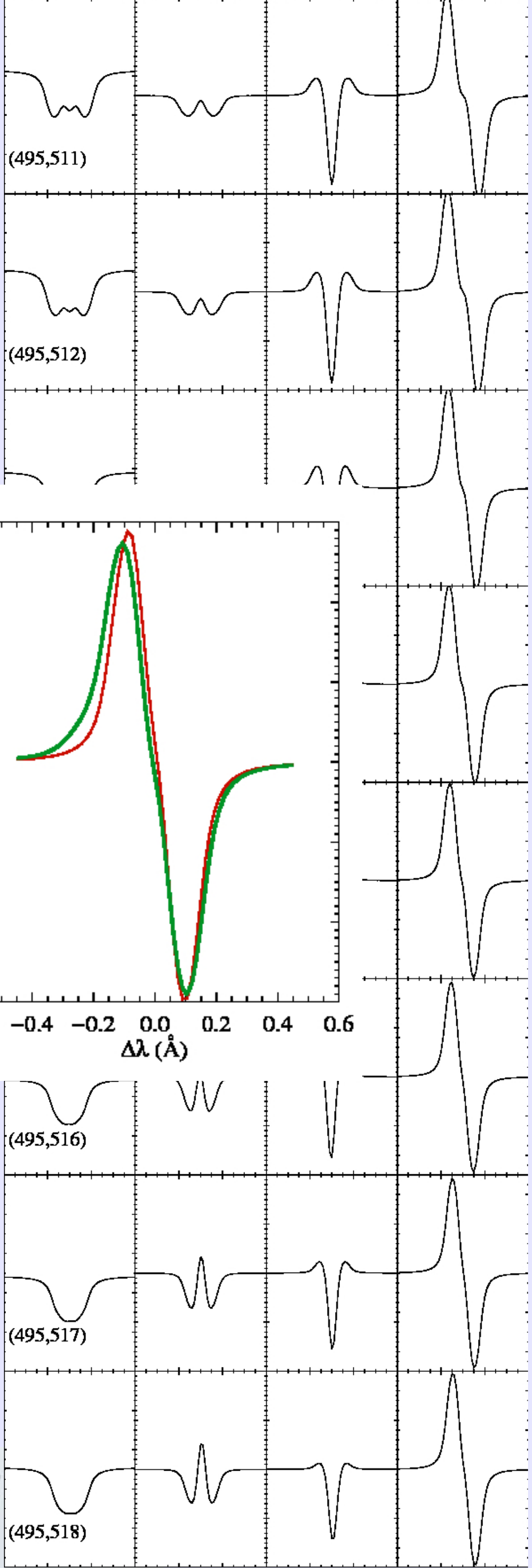


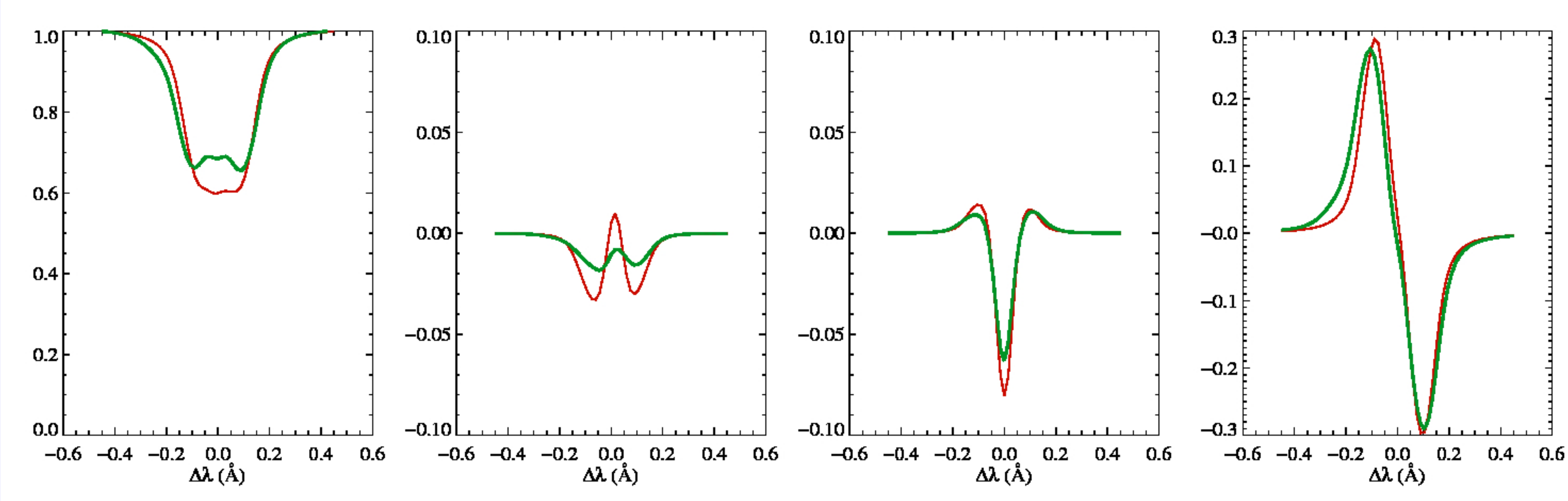
<I> + <P>:


(495,511)
(495,512)
(495,516)
(495,517)
(495,518)


1.0
0.8
0.6
0.4
0.2
0.0
0.10
0.05
0.00
−0.05
−0.10
0.4
0.2
−0.2
−0.4
−0.6 −0.4 −0.2 0.0 0.2 0.4 0.6
Δλ (Å)

demodulated(<I+/-P>) ≠ <I> + <P>

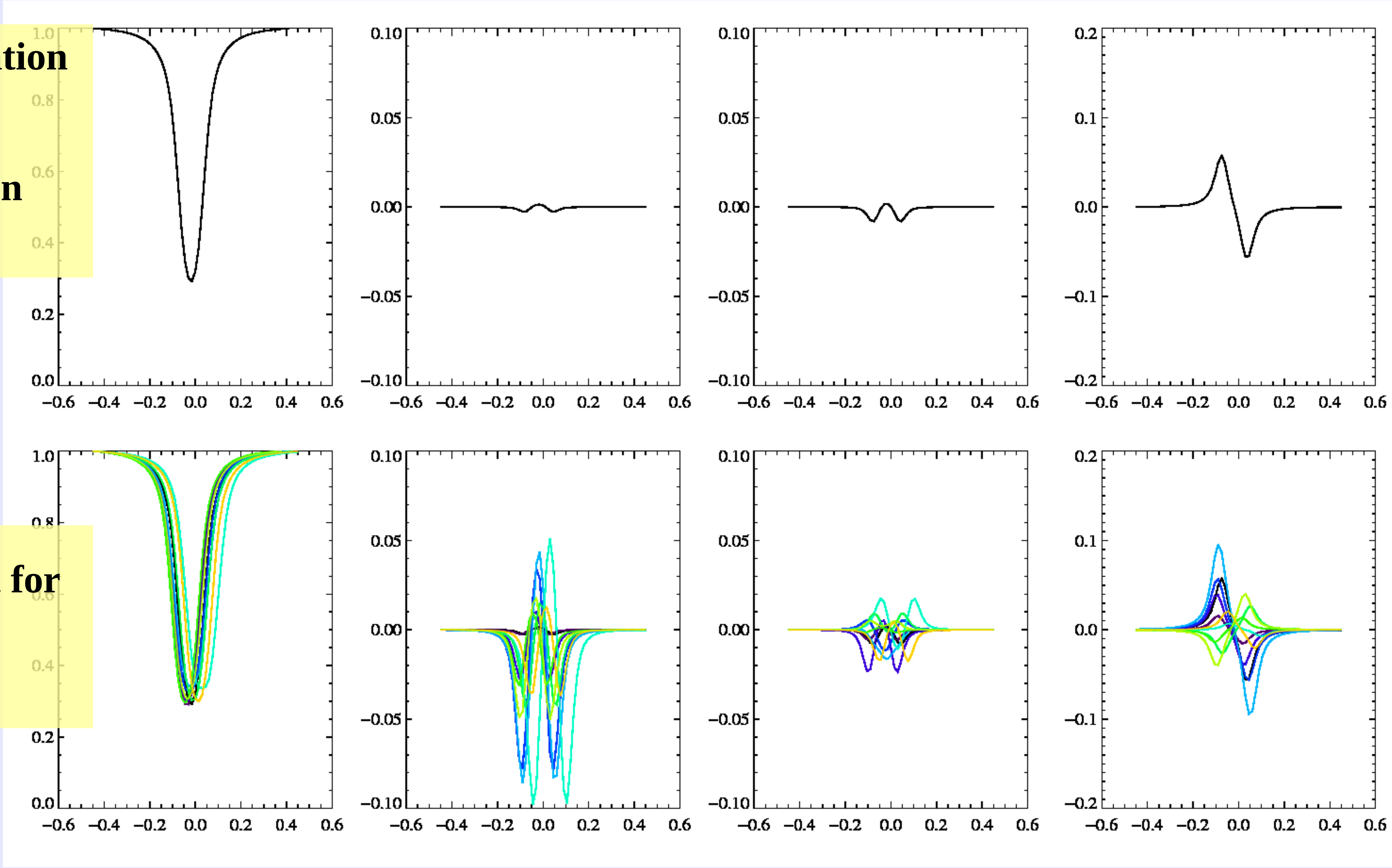




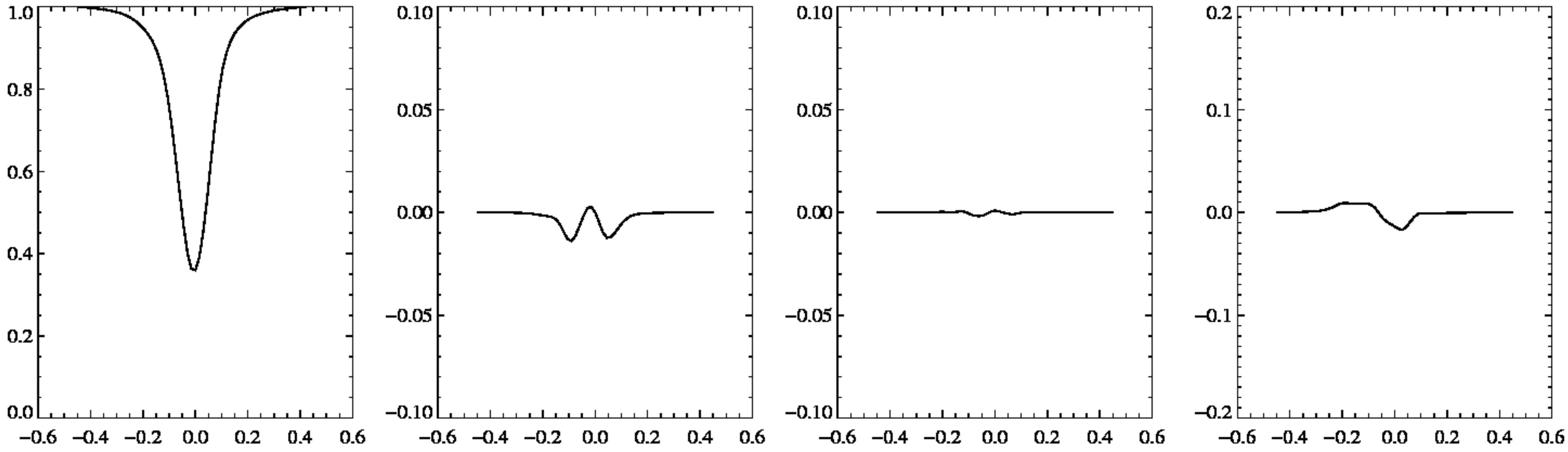
Temporal Resolution:

One location in full-spatial-resolution data.

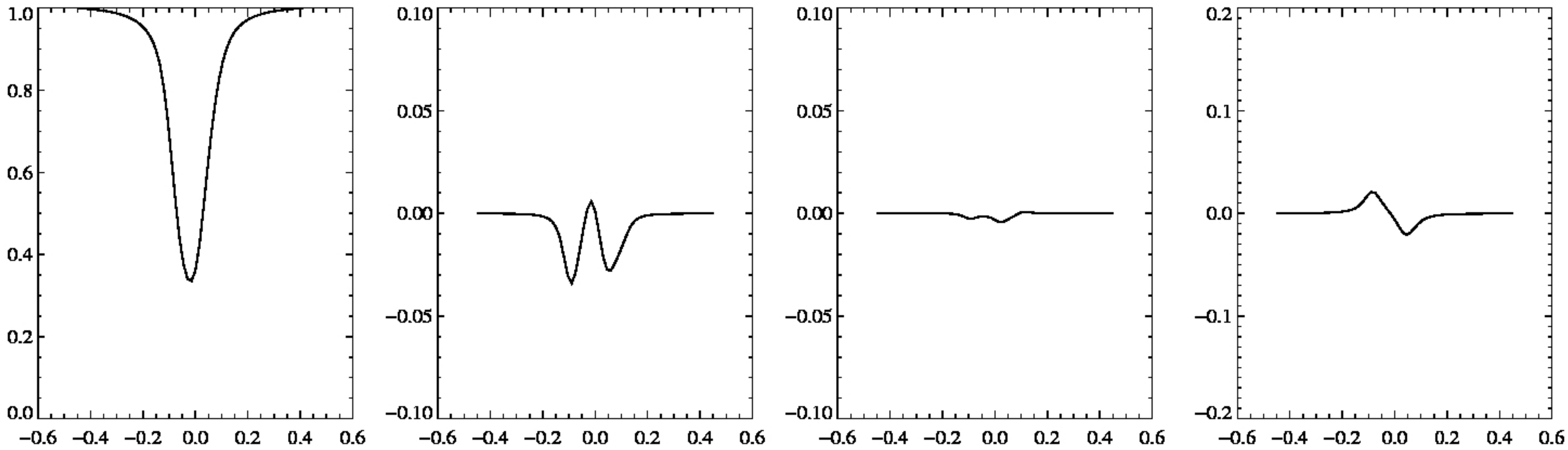
Watch it for about 10 minutes:



<I> + <P>:



demodulated(<I+/-P>) ≠ <I> + <P>



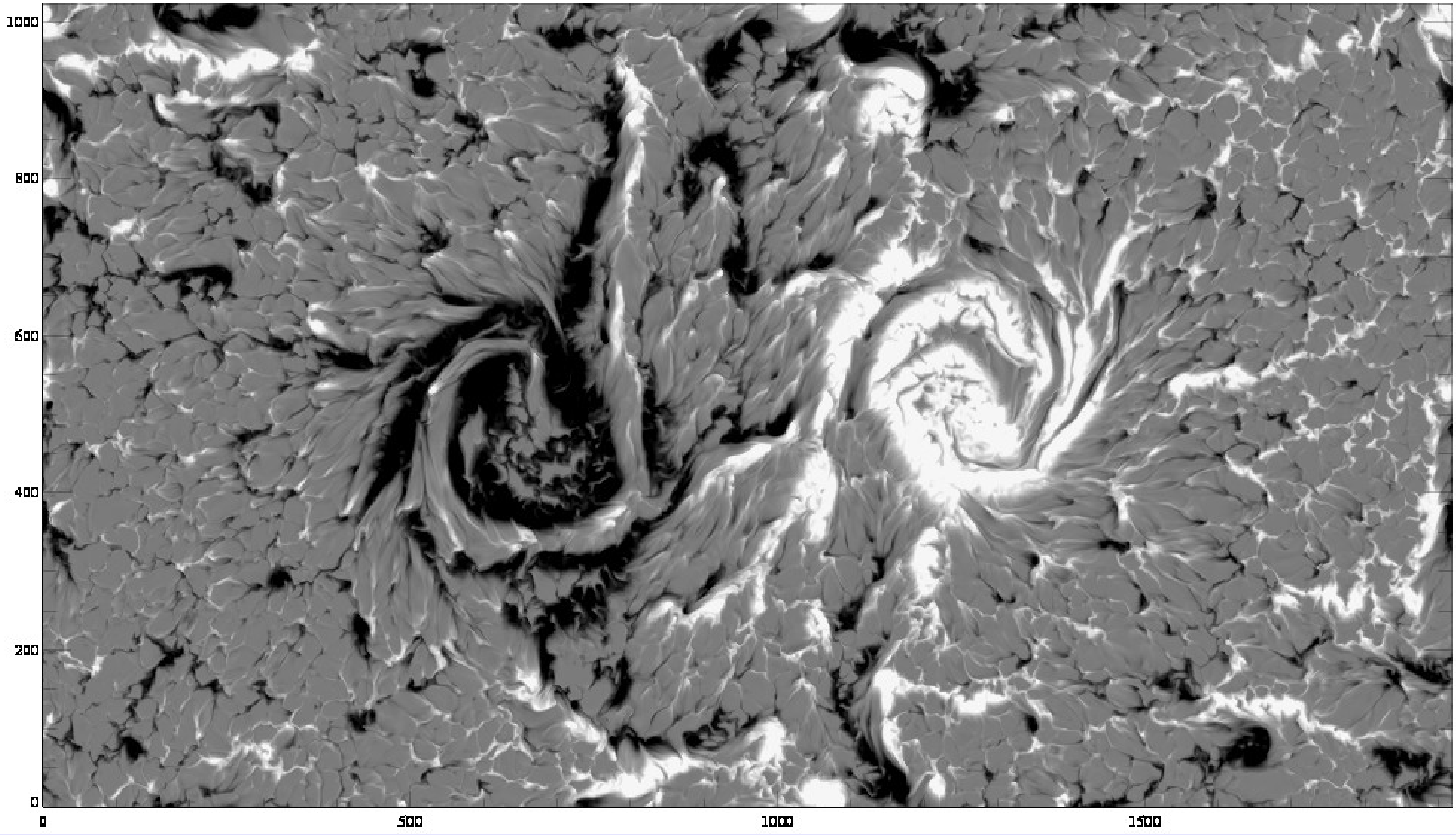
(demodulated (<I+/-P>) is what is truly observed)

Why is this difference a problem?

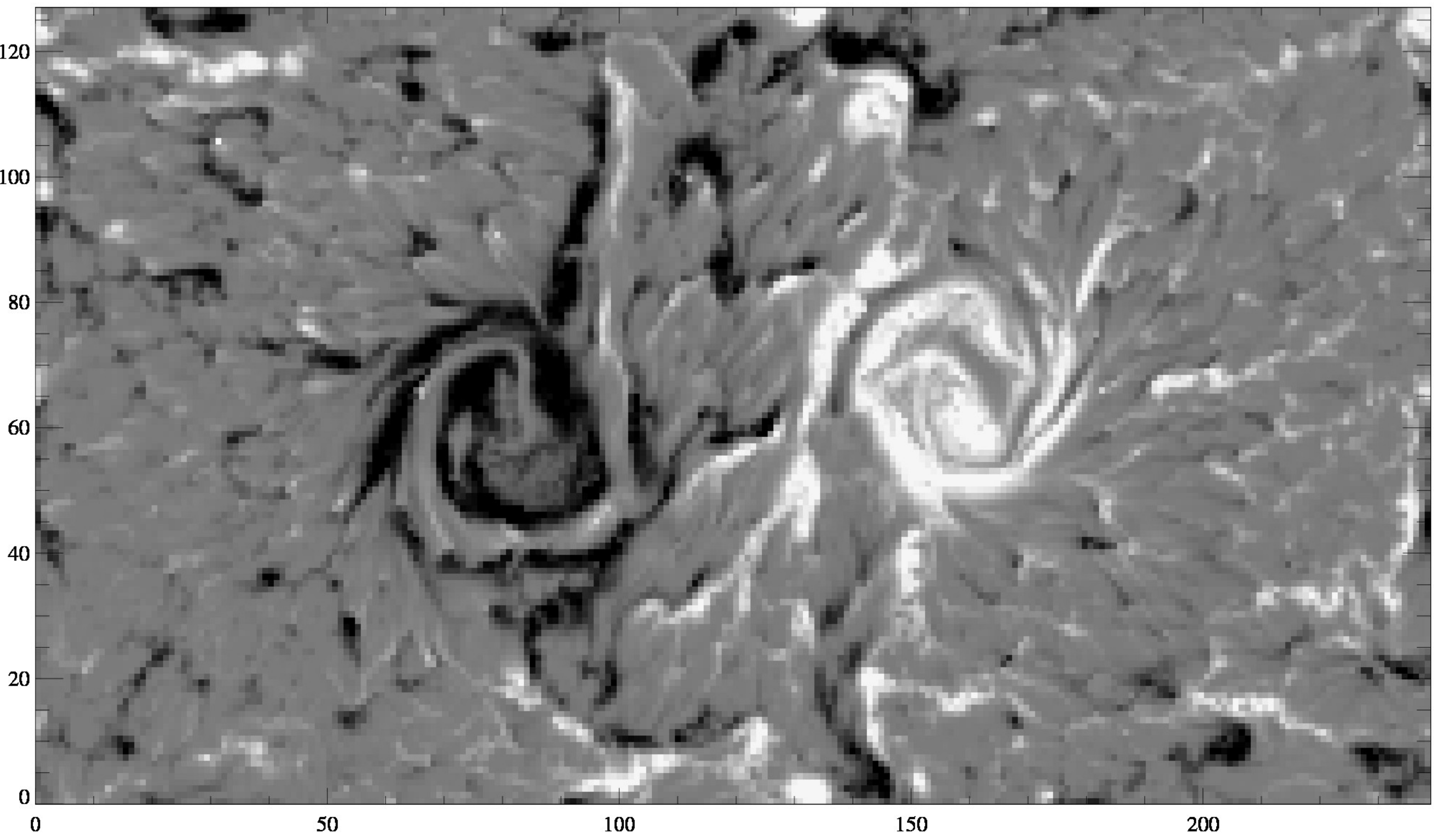
- A linear combination of Stokes Parameters can be solved for with a consistent magnetic models
- An intensity-weighted average with unknown weighting of Stokes parameters can not (necessarily).
- Resulting Stokes Spectra may (or may not) be consistent with a single magnetic model.
- May implicate velocity gradients, and/or multiple-component atmospheres, when the cause is limited resolution.

Combining both effects:

Original boundary

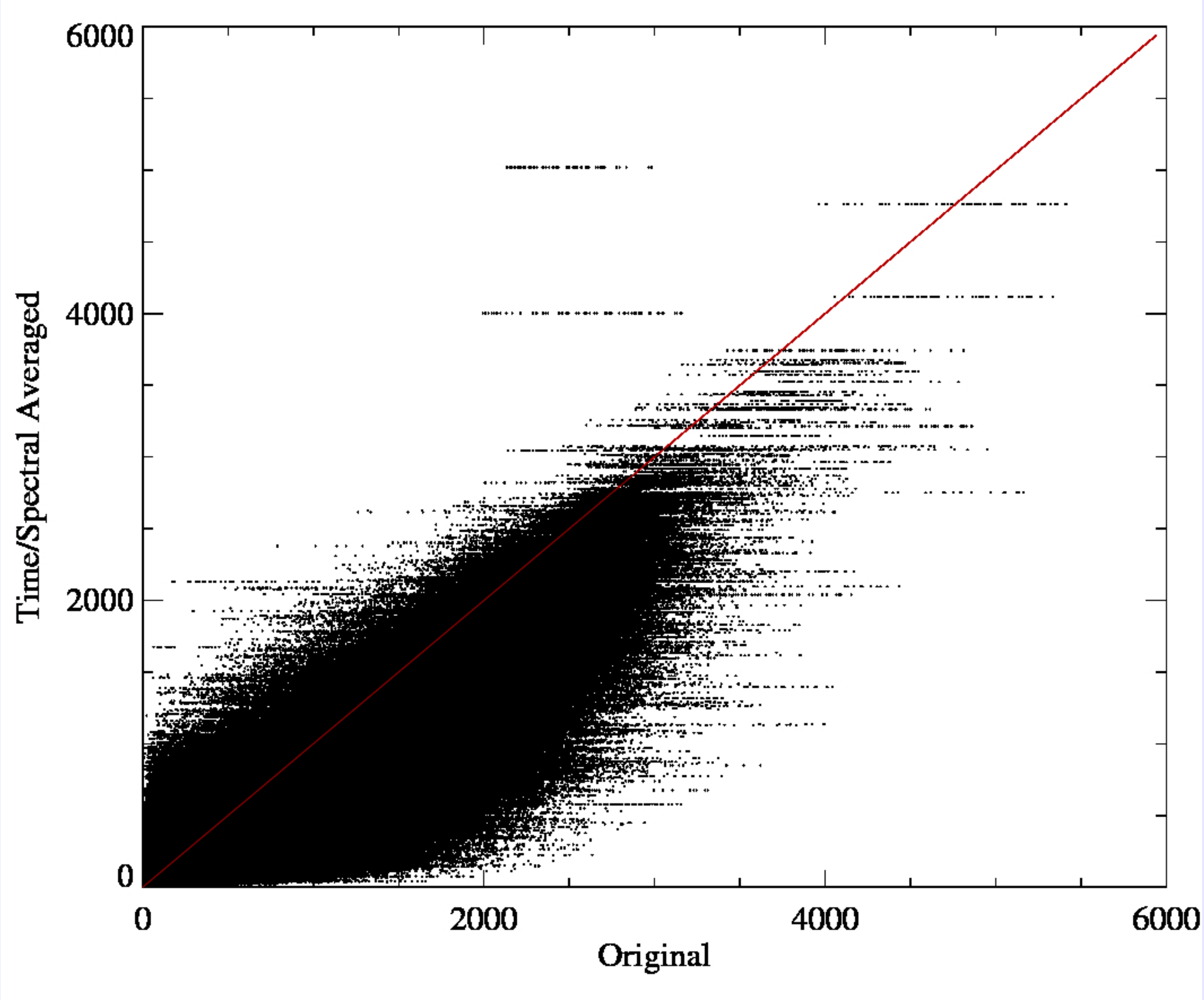


Inverted temporal/spectral averaged data



Limited spatial and temporal resolution:

Comparison of original (x-axis) and 10-min, 1'' average |fB|, f=1.0 (y-axis)



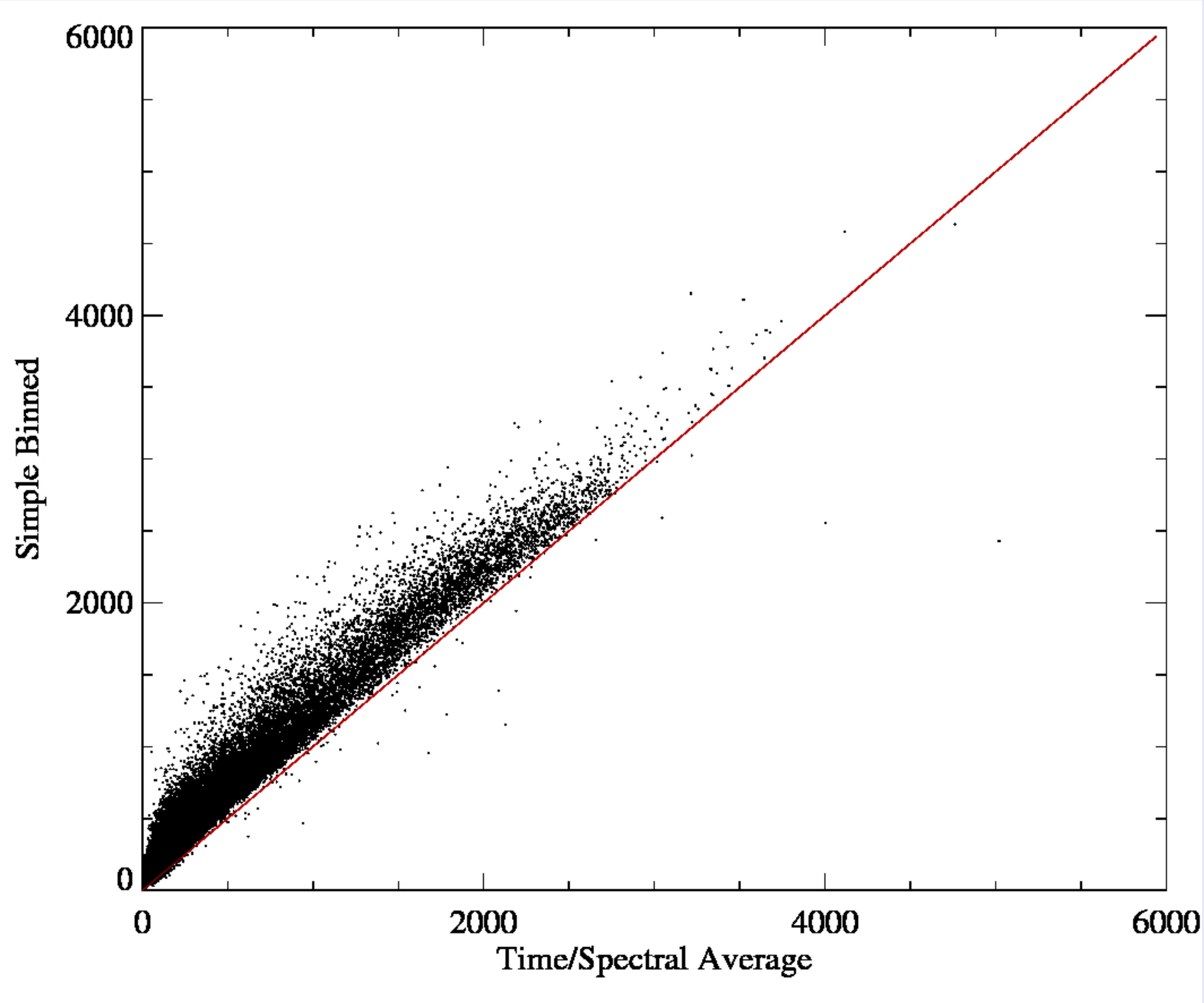
Overall decrease in detected field,
but not always.

"Hey, KD, isn't it just the same (and a lot easier) to 'rebin' the magnetogram?"

Comparison of 10-min, 1" average |fB|, f=1.0 (x-axis), and a simple "rebin" average of the original data:

No, it's not the same. **It's a systematic difference.**

Simple rebinning (spatially) generally preserves the "flux", whereas real observations at different spatial resolutions, as modeled and shown here, *do not.*



First Summary:

• Temporal, Spatial resolution limitations have similar behavior, in terms of resulting Zeeman Stokes Polarization spectra.

• Resulting limited-resolution spectra are not the same as averaged Stokes spectra: strongly weighted by intensity.

• Mixed (as observed) signals may increase, or decrease, true asymmetries.

• Re-inverting demonstrates that spatial/temporal averaging as observations do, is not the same as spatially/temporally averaging the magnetograms.

Objects of Desire **from Vector Field Data:**

- magnetic field strength and direction
- gradients of the magnetic fields
- current density ***J***
- magnetic "twist"
- magnetic shear angle (difference from potential)
- magnetic free energy
- plasma velocity ***v***
- magnetic "helicity"
- forces & torques
- coronal field from force-free extrapolation

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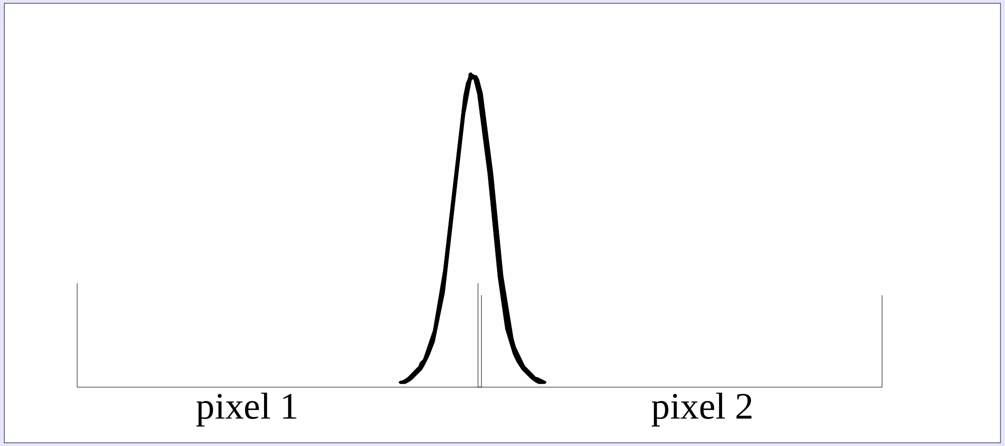
Each of these depends at some level, on the <u>spatial</u> and/or <u>temporal gradient</u> of the inferred field.

Question: How does one analyze unresolved field, e.g. when $f < 1.0$?

Consider two pixels, and a magnetic field element with a field-strength distribution and location as shown.
One would infer:
$(fB)_{\text{pixel 1}} = (fB)_{\text{pixel 2}}$
$\Delta(fb)/\Delta x = 0$
(but, obviously, it's not true everywhere)



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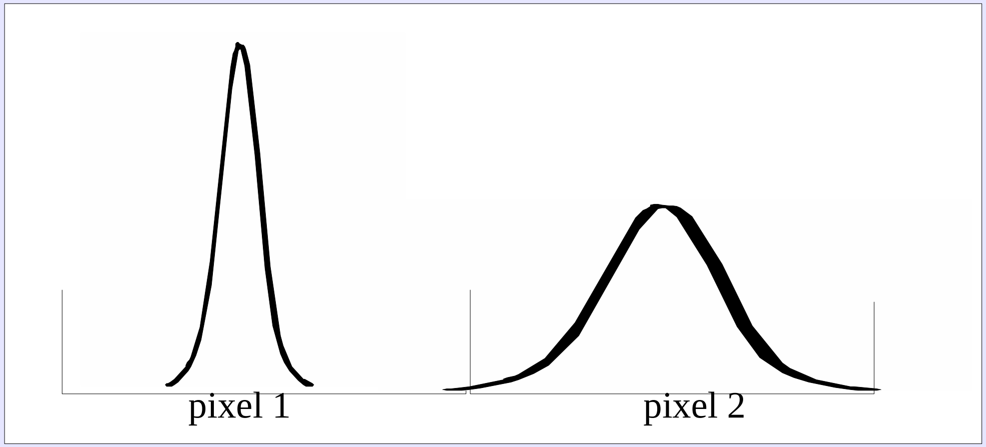
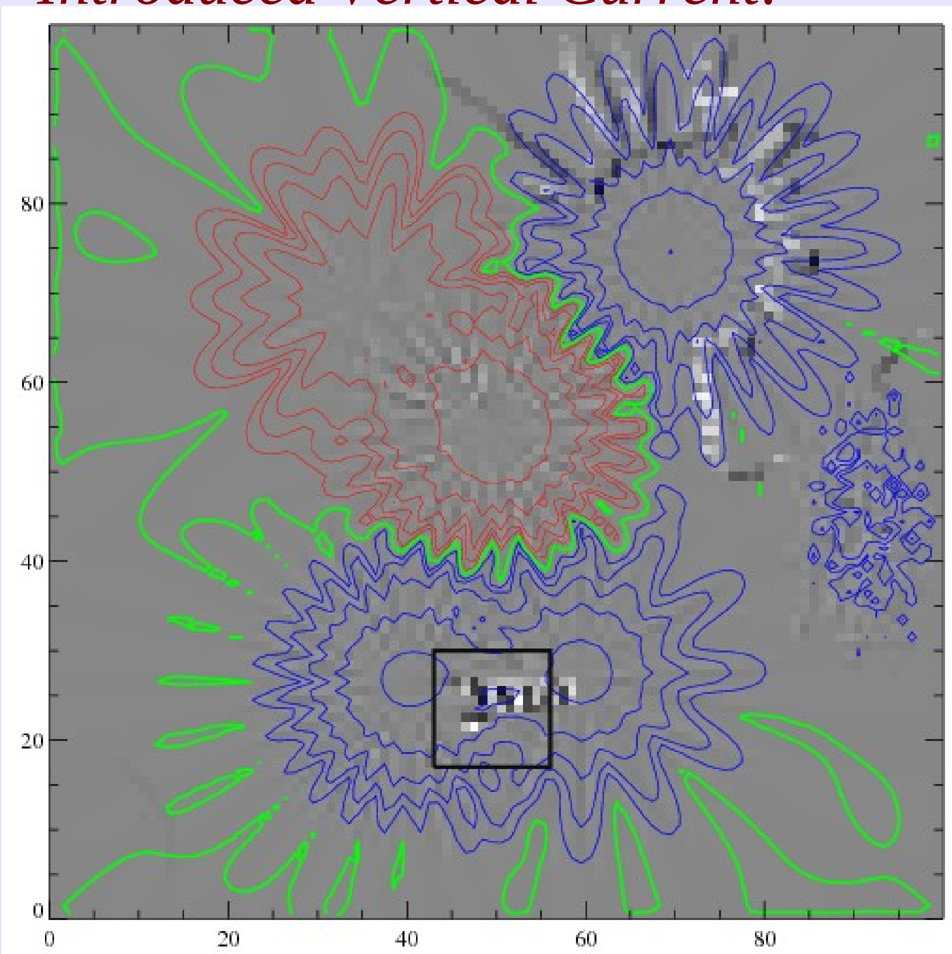


These two physically different scenarios are observationally indistinguishable.

*Arturo Lopéz Ariste reminds me that we debated this point (whether one can take derivatives of unresolved fields) 7 years ago and that I'm finally agreeing with him.

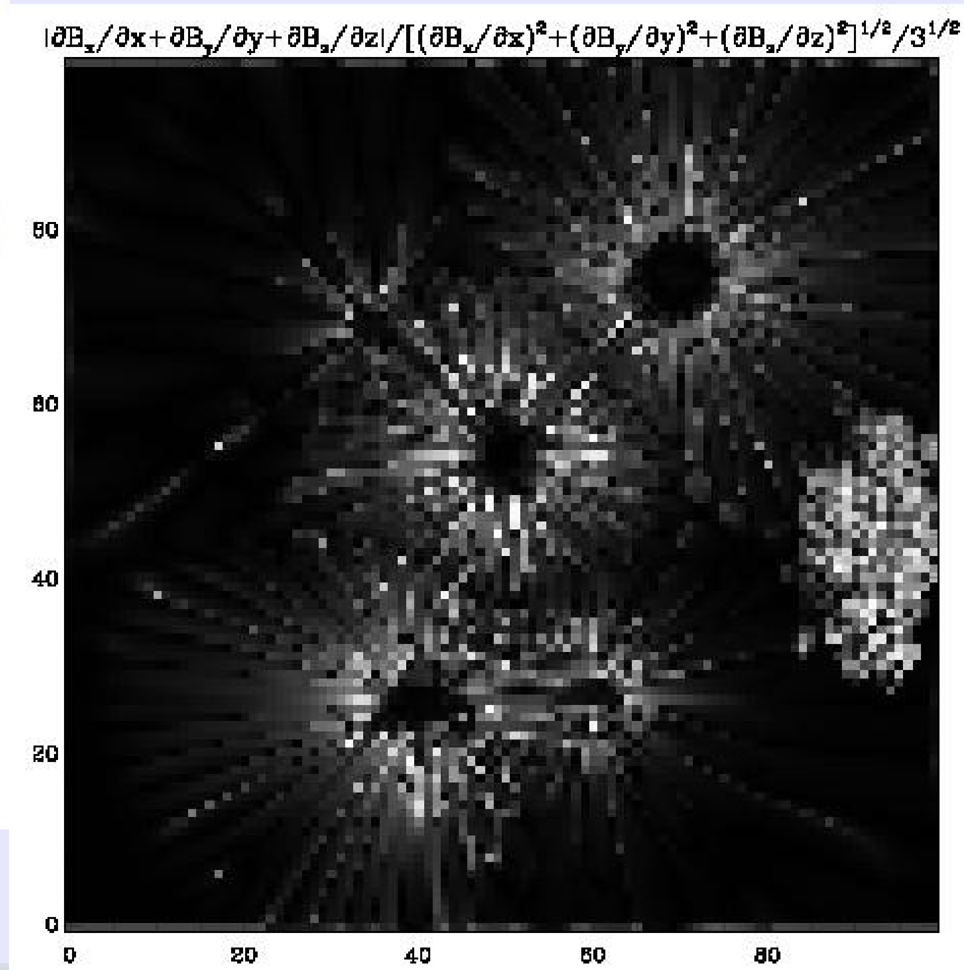
Approximations used for derivatives (finite-difference approaches) are susceptible to discontinuities introduced by unresolved structure. This can lead to:

Introduced Vertical Current:



Courtesy M. Georgoulis

$\nabla \bullet B \propto \partial Bx/\partial x + \partial By/\partial y \neq 0$

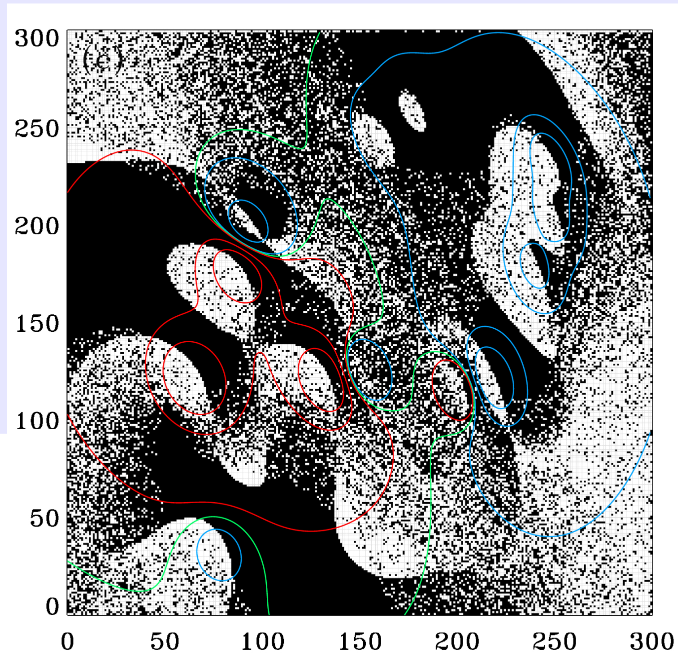


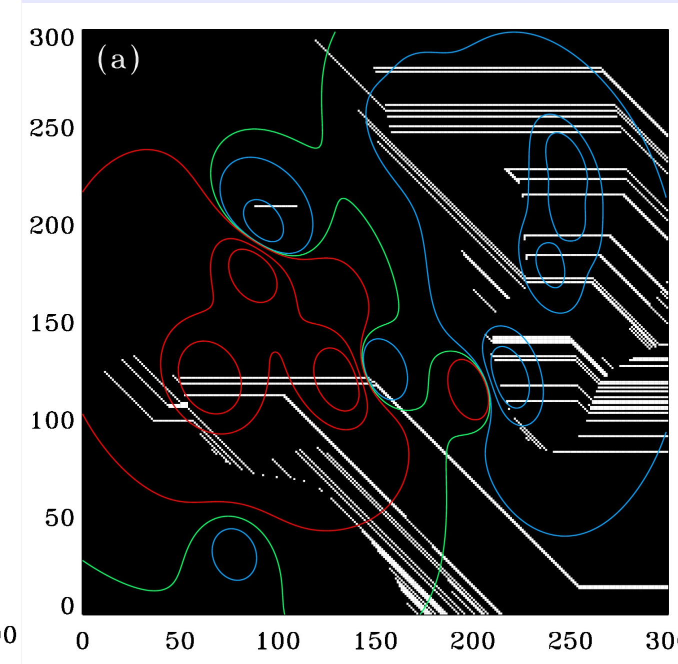
Original model was divergence-free and current-free.

What to do?

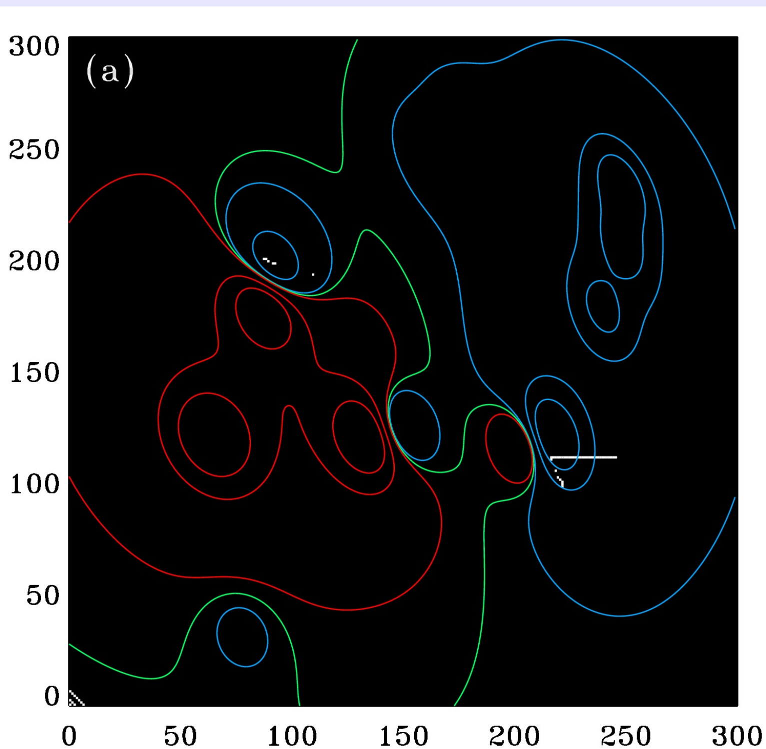
Relying upon local characterization of derivatives is risky with unresolved data:

Ambiguity-resolution algorithms which rely upon locally evaluating and minimizing $|\nabla\bullet B|$ generally fail (black: good, white: bad, red/blue are contours of model field):





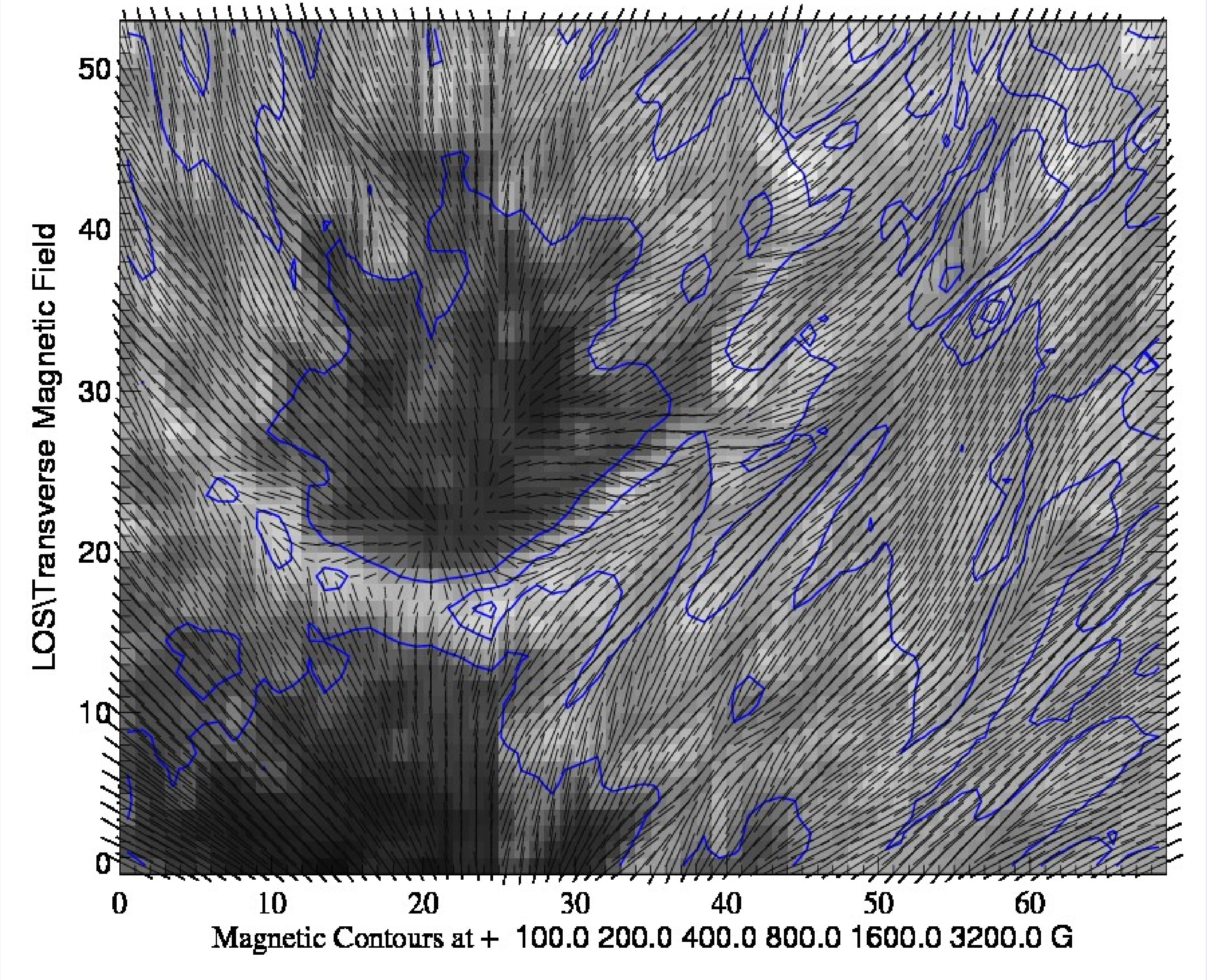
Algorithms which minimize $|\nabla\bullet B|$ *globally*, however, *suceed:*



Does photospheric $J_z \propto \nabla \times B_h$ actually exist? (c.f., Parker 1996)

On the one hand... any individual pixel is suspicous.

Yet as we keep increasing the resolution, there *appears* to be non-potential field...



And statistically, vertical current density and other finite-difference-dependent quantities correlate to solar activity, implying stored magnetic energy...

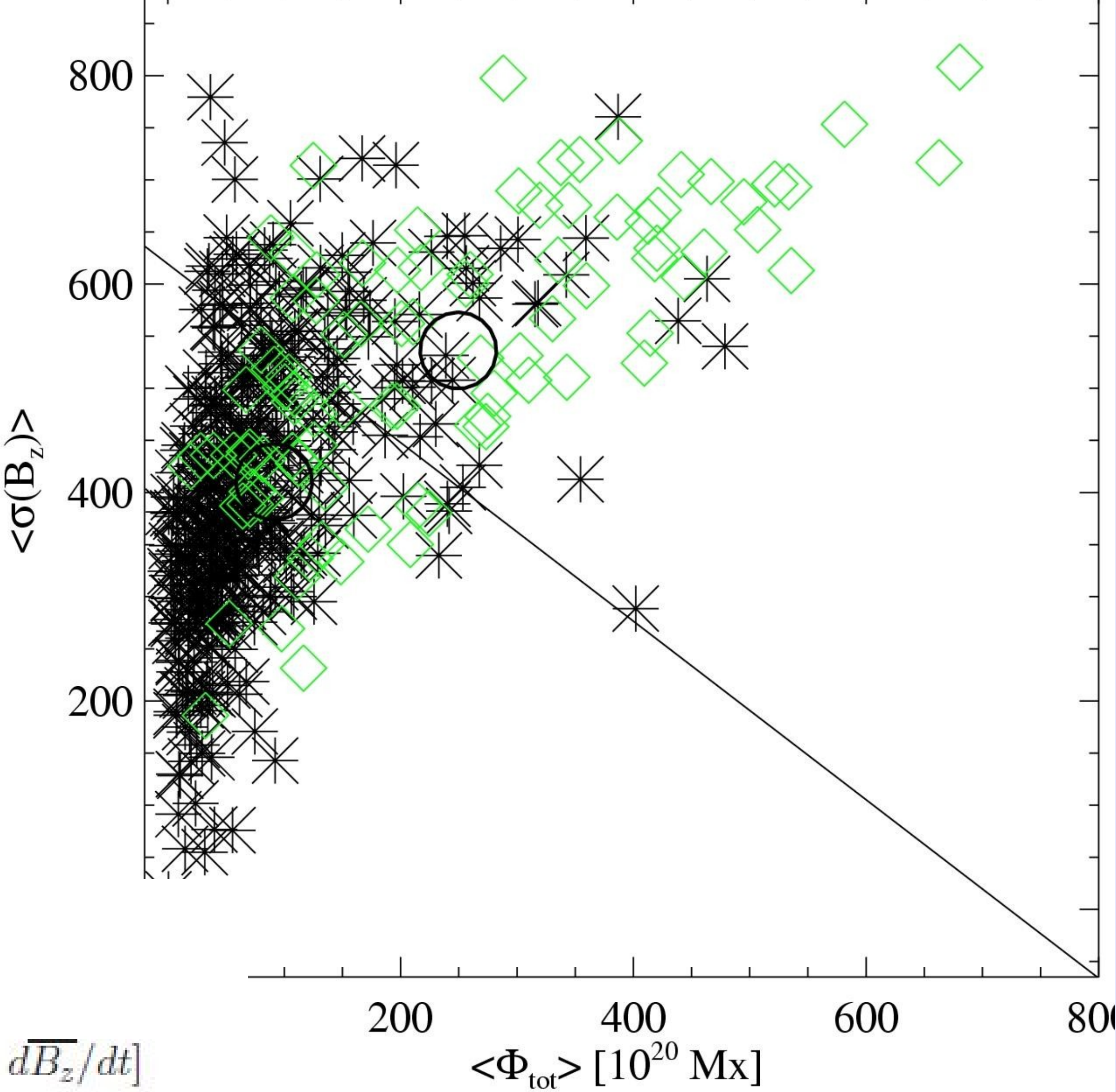


Table 8. Classification table for $[\langle\sigma(B_h)\rangle,\ \langle\varsigma(J_z^h)\rangle,\ \langle\kappa(J_z^h)\rangle,\ \langle A(\psi > 80^\circ)\rangle,\ d|\alpha_{\rm ff}|/dt,\ d\overline{B_z}/dt]$

		Predicted	
		flare	no flare
Observed	flare	10	0
	no flare	0	14

From Leka & Barnes 2003, 2007

Second (Final) Summary:

- Lack of resolution does not completely invalidate results
  - Objects of desire can be calculated and used

However

- The most (only) reliable use of unresolved data are *statistical* or *global* in nature.
  - Analysis and algorithms which rely on finite-differences and inferred magnetic evolution must be aware of the effects of limited resolution.