Noname manuscript No.
(will be inserted by the editor)

Using the Butterfly Effect to Uncover How the Sun Generates Acoustic Noise

Subphotospheric Seismic Holography of p-Mode Emission in Numerical Simulations of Convection

Charles Lindsey · **Matthias Rempel**

Received: TBD, 2018 / Accepted: TBD, 2018

Abstract A major encumbrance to recognition of episodes of p-mode emission has been the accumulation over hours of p-modes long from long before recent local emission episodes. This is true in simulations just as it is in the solar environment itself. Local seismic signatures of acoustic radiation accumulated over hours drowns out the signature of newly emitted "acoustic pings" from any location. This problem could be alleviated in simulations by periodically damping the accumulated acoustic radiation—if this can be done benignly, i.e., in such a way the the onset transient of the damping does not emit its own acoustic noise. We introduce an introductory way of doing this based upon a character of the *butterfly effect* applied to compressible radiative MHD simulations of convection that excites p-modes. This gives us an encouraging preview of what further development of this utility offers for an understanding of the character of p-mode generation in convective atmospheres.

Keywords Sun · Convection · Helioseismology · p-Modes

1 Introduction

We are now approaching the sixtieth year since the discovery, by Leighton, Noise & Simon [12] and Evans, Michard & Servajean [6, 7], of the solar oscillations, the surface manifestations of waves in the solar interior with periods ranging from about a hundred seconds to more than an hour. These waves are called "p-modes," and have opened our first spatially discriminating view into

---

C. Lindsey
NorthWest Research Associates
Tel.: +303-415-9701 X238
Fax: +303-415-9702
E-mail: clindsey@nwra.com

M. Rempel
NCAR/HAO

the interiors of our own star, the Sun, and a growing variety of others. This has been the object of the far-reaching fields of solar and stellar seismology, the former more formally called helioseismology. Both have had a remarkably successful and diverse development in the succeeding six decades. Solar seismology has played a major role in the the development of the "Standard Solar Model" [4] mapping the thermal structure and mass distribution of the Sun from its core to its surface. This was crucial to the deep mystery of how the Sun's core produces the energy that has kept our planet alive for the billions of years needed for sentient human life to emerge on it. It helped us to penetrate the terrifically stubborn solar neutrino problem, that tortued some of our most incisive thinkers throughout the later 20th and early 21st centuries, leading to the spectacular discovery of neutrino oscillations [**?**]. Today, solar seismology is being used to monitor large active regions in the Sun's far hemisphere [15,**?**,2] for applications in space-weather forecasting. And, its stellar namesake is being used to probe the interiors of distant stars, some of which could possibly host extraterrestrial life yet unknown, a scientific discovery that would arguably be the greatest in world history. What can this tell us about the great confluence of apparent accidents, on a planetary scale, that led to the emergence of life forms on Earth?

Because the Sun is a high-quality resonant cavity, it has been possible to infer a great deal about its internal thermal structure from p-modes based only on the Sun's resonant frequencies, knowing relatively little about how they are excited. However, there remain significant aspects of solar seismology that could be greatly improved by a better understanding of how the Sun produces its acoustic noise. These are more what we call the "local" aspects of solar seismology, how to discriminate the properties of local anomalies from the generality of the mean global structure. Given certain conditions, global properties of a subject, e.g., the length and radius of an organ pipe and the speed of sound within it, can be determined by its resonant frequencies, and global helioseismology has done an excellent job of this with the solar interior. However, to determine the location of a local anomaly somewhere within the medium, we need something more like bat technology. The bat acoustically illuminates a mosquito with a *ping* and listens for the immediate echo. The temporal relationship between the ping and its echo is now crucial. So, we want the ablity to identify the sources and character of the individual pings in the solar acoustic-noise engine from which they emanate. How are these pings distributed over the Sun's surface? What is their surface manifestation? To what degree are they episodic?

We are already apprised that the sources of p-modes lie in a thin layer beneath the Solar surface a few hundred km thick [11,9,10], i.e., about the outer third of a thousandth of the Sun's visible radius. And, we have good acoustic diagnostics for imaging acoustic sources we know to be local [14,15,5,1], very much the way our eyes do this in electromagnetic radiation. The stubborn problem has been that the very qualities that make the Sun an excellent resonant cavity also make it highly obscurant of individual acoustic sources, episodic or otherwise. The primary reason is that its surface is an efficient multiple reflector of acoustic waves. The Sun is an echo chamber, its deepest penetrating waves reverberating between opposing hemispheres repeatedly before they eventually tire out and fade. Because of this, acoustic energy in the solar interior accumulates for hours or days in sharply resonant modes, constantly pervading its entire interior volume. Any episode of newly produced acoustic emission from a local source becomes a minor party to a terrific din of accumulated acoustic noise constantly passing through the source region, multiply reflected noise that was produced somewhere else, far away and long ago.[1]

2 Simulations

One of the most significant developments in our understanding of chaotic turbulence over the past half century has been the ability to simulate it to ever higher degrees of realism on powerful computing machines [21–23]. A crucial part of the development of solar seismology has been the comparative application of our diagnostics to simulations of fully compressible radiative MHD simulations of convection that reproduce the production of p-modes by the solar granulation, the layer from which we are now convinced nearly all of the p-modes are excited. This is a tremendously powerful facility, not only because it shows us the interior region into which the p-mode waves are emitted but, further, because we can make changes in the computations to see what difference various components of the simulation make in the consequent behavior, differences that may be prohibitively obscure to inference from first principles. This leads to two questions: (1) What is the meaning of what the simulations show us? and (2) Are the simulations realistic? The first has an analogy in how observations of the Sun itself confront us. We can pursue this question in the simulation domain on similar terms, with tools that are much more powerful, penetrating and closer at hand. The second may never be fully resolved, but, we can—and do—pursue it by comparing what we see of the surface of the simulation to what the surface of the Sun itself shows us. This seems to lead to better simulations, and it often brings our attention to features in the solar observations we had yet to notice in the reality.

So, then, this study reports the relative beginning of an ongoing process. As such, it will capitalize upon a particular diagnostic utility that simulations open to the effort to understand p-mode excitation that to our knowledge has yet to be explored.

---

[1] The problem that confronts local recognition of recent acoustic sources enjoys an interesting analogy in the electromagnetic spectrum to *Olbers' Paradox*. Olbers (1758–1840) proposed that if the universe were infinite then the entire sky, both night and day, should be as bright as the Sun—which would make our own Sun invisible for lack of contrast against the daytime sky. This mercifully fails in our universe, due to the red shift caused by its rapid expansion. Somewhat ironically, Olbers' principle works only too well in (1) crowded restaurants with efficiently reflecting walls and bodies, and (2) the solar acoustic environment, in the latter case, at least, of which there is no significant such expansion.

3 The Butterfly Effect

Most of us are familiar with a broad understanding that in a chaotically turbulent medium the effect of an infinitesimal component of the causal influences, e.g., the momentary waft from a single butterfly fluttering over Salto Angel in South America, on an atmospheric system of planetary dimensions can grow with time to such an extent that, at length, it makes a major difference in macroscopic outcomes, e.g., whether a subsequent hurricane makes landfall along the Gulf Coast to flood New Orleans or veers and misses the Gulf entirely to wreak its havoc along the eastern seaboard (Lorenz 1969)[17]. We distantly envision that if we could unravel the intricate causal morass that separates these two alternative outcomes by more than a thousand km, then the mere momentary fluttering of the single butterfly in Venuzeula months before could be held criminally accountable for the difference.

The practicality of such an accounting is far beyond our present technical horizon, the needed control requiring the equivalent of free time travel months or years into the future and back. However, a distantly realistic facsimile of such a time travel is actually quite practical for computer simulations of a growing variety of chaotic phenomena. This benefits from the ability of modern parallel processors with thousands of cross-linked CPUs to repeat a computation entailing trillions of floating-point operations on each CPU to arrive at precisely the same result as the original for lack of a single error in any component of the computing hardware down to the last decimal place. The convective turbulence that excites the p-modes turns out to be such an application. Convective turbulence in the solar environment, while terrifically complex, is far simpler than turbulence in Earth's atmosphere, since it is free of adverse complications to its input, such as mountains, skyscrapers, giraffs and diverse other anomalies sticking into the atmosphere from below; the influence of oceans, seas and lakes of varying salinity, diurnally heated in radically different ways by a distant source; etc., to further multiply the complexity of the task. Numerical simulations of turbulence now reproduce observed qualities of the solar granulation and the field of acoustic noise attached to it with remarkable realism and clarity. So, going on a half century after Lorenz (1969), we are at the doorstep of a tremendous opportunity.

Figure 1 (top frame) shows results of a computer simulation of convective turbulence in the outer 20 Mm of the solar convection zone. Represented in color is the local vertical velocity of the medium, which we designate v_z^{ref}, as this will serve as a reference for later comparison. This is rendered in a vertical plane as a function of horizontal location and depth, in which the full domain is a 3-dimensional rectilinear volume. The approximate photospheric surface is marked by the upper extreme of a jagged yellow fringe slightly beneath the top of the frame. What predominates the volume beneath this surface is downwardly plummeting plumes of gas that have been cooled by convective exposure to interplanetary space within the previous several hours, and are sinking back toward the bottom of the convection zone. The speed of these plumes is transonic just below the solar surface, which, to our understanding, makes it especially conducive to the excitation of acoustic noise [8,13]. At great depth, the velocity amplitude of the acoustic noise produced by these plumes is very small compared to that of the speed of the plumes themselves. There are practical analytical methods for discriminating the acoustic motion from that of the plumes, weak though it is. However, we propose change course here, because this is where a diagnostic that draws upon the butterfly effect becomes of great use, not only in discriminating the acoustic waves from the plumes but in discriminating *recently emitted waves from the great preponderance of obscurant acoustic noise* (from far away, excited long ago).

4 Lepidoptery of the Solar Granulation

To introduce this concept[2] into the disucssion, we consider, as a reference, the simulation whose vertical velocity, $v_z^{\rm ref}$, is rendered in the top panel of Figure 1. We go back to some time midway in the term of this simulation and artifically introduce into the field v_z a weak, random "butterfly perturbation" at that moment, spread over a layer within which convective motion is quite strong, i.e., into the near-surface domain representing the solar granulation. We then independently rerun the simulation from that moment forward, denoting by $v_z^{\rm lep}$ the vertical velocity in the rerun. At length, we turn our attention to the "butterfly difference,"

$$\Delta^{\rm lep} v_z \;\equiv\; v_z^{\rm lep} \;-\; v_z^{\rm ref} \tag{1}$$

between the two simulations and how it grows with time. Initially, $\Delta^{\rm lep} v_z$ is practically imperceptible to the eye, only the momentary, extremely small butterfly perturbation itself, and a proportionately weak acoustic transient that propagates quickly downward from it and disappears. Among the features that are enjoyably absent in $\Delta^{\rm lep} v_z$ is the *accumulated acoustic noise* in the medium that equally had pervaded both of the indiviual simulations. It soon becomes evident, though, that $\Delta^{\rm lep} v_z$ is subject to an instability very similar to that which drives convection in the original simulation, and that the butterfly perturbation has excited the instability. Indeed, $\Delta^{\rm lep} v_z$ grows quasi-exponentially, at a rate that is roughly the reciprocal of the convective turnover time. So, almost no matter how weak the initial perturbation, in only a relatively few convective turnover times it becomes significant. As it grows in amplitude, $\Delta^{\rm lep} v_z$ developes plumes, which penetrate to ever increasing depths. At length, the difference in the reference and butterfly MHD fields develop difference acoustic sources, and these begin to emit freshly minted acoustic noise.

With appropriate qualifications, the butterfly difference, $\Delta^{\rm lep} v_z$ can be treated as a turbulent field that shares certain of the statistical attributes of the reference simulation. This is rendered in color the lower panel of Figure 1, ∼14 min after the butterfly perturbation has been introduced. The noise it emits travels downward, at the speed of sound, hence ahead of the newly

---

[2] Butterfly: Phylum: Anthropoda; Class: Insecta; Order: Lepidoptera

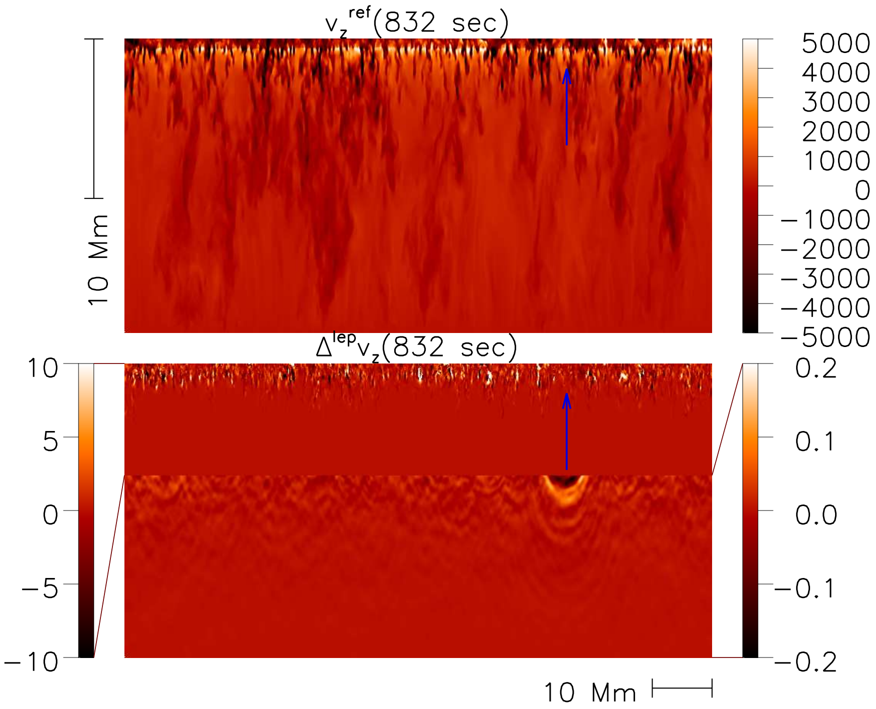

Fig. 1 Vertical-cut snapshots of compressible radiative-convective simulations of the solar granulation to which a weak butterfly perturbation is applied beneath the photosphere. Velocities are expressed in m/s. Note stretching of the vertical scale by a factor of 8/3 with respect to the horizontal. Top frame shows the vertical velocity, v_z^{ref}, in m/s, of a reference simulation 832 s (∼14 min) after a random perturbation is applied to its "butterfly counterpart," v_z^{lep}. Bottom frame shows the growing difference $\varDelta^{\mathrm{lep}} v_z \equiv v_z^{\mathrm{lep}} - v_z^{\mathrm{ref}}$ between the butterfly counterpart and the reference at this moment. Contrast of the lower 60% of the bottom frame is enhanced by a factor of 50—shows acoustic waves arriving 13 min thence.

developing, downwardly growing plumes, the underlying layers thus for a while devoid of any other disturbance. So, there is a significant period during which *the newly emitted acoustic noise is the only significant disturbance to inhabit the layers from a fraction of a Mm beneath the photosphere to the bottom of the computational domain.* We will call this the "purely acoustic region," a domain that shrinks as plumes growing downward from the overlying turbulence invade it from above.

As in the individual simulations, the acoustic noise in the butterfly difference, $\varDelta^{\mathrm{lep}} v_z$ in the purely acoustic region is much smaller in amplitude than the overlying turbulent motion that produces it. However, it becomes clearly visible, to be compared with its overlying convective source field, when the amplitude of $\varDelta^{\mathrm{lep}} v_z$ in this region is appropriately amplified, in this case by a factor of 50, which it is in the bottom 60% of the lower frame of Figure 1.

At least in the early evolution of the butterfly difference, the disturbances arriving into the purely acoustic region show evidence of an overlying source distribution that is perceptibly *episodic*. Some acoustic pings stand out clearly from the rest (vertical blue arrow). One suspects that this single example is a result of the pre-selected vertical plane in which the simulation has been sampled passing close to a conspicuous source at the surface.

The lack of a conspicuous overlying anomaly in $\Delta^{\rm lep} v_z$ at the location to which the blue arrow points transpires to be a result of the vertical sampling plane simply missing the source location, which would supposedly be more compact that the outwardly spread acoustic ping it emitted. This is clarified by Figure 2, by horizontally cospatial sampling over horizontal planes in the purely acoustic region and at the overlying base of the photosphere. This offers an overhead view of horizontal planes at the surface and 4.1 Mm beneath it. The upper-left frame shows the intensity, I_c, emanating from the surface of the reference simulation 698 s ($\sim$11.7 min) after the butterfly perturbation has been applied. The upper right frame shows the butterfly difference, $\Delta^{\rm lep} v_z$, at the same moment. The lower-left frame shows the butterfly difference, $\Delta^{\rm lep} v_z$, $\sim$11.6 min thence, over the horizontal plane 4.1 Mm beneath the upper simulation of the surface . The acoustic pings now appear as outwardly expanding ring-like fringe patterns.

At this point, we do not yet recognize any conspicuous anomaly in the continuum intensity, I_c, emanating from the surface of the reference simulation (upper-left frame) corresponding to the expectation source locations of the underlying pings, i.e., directly overlying centers of their ring-like fringe patterns (see red crosses). However, we see *conspicuous anomalies* in the *butterfly difference*, $\Delta^{\rm lep} v_z$, of the vertical velocity at these surface locations (upper right). These features tend to appear fairly compact to the eye. The horizontal location of the vertical plane in which $v_z^{\rm lep}$ and $\Delta^{\rm lep} v_z$ are rendered in Figure 1, is represented in the lower-left frame of Figure 2 by a blue line. It misses the source of the ping apparent toward the "north-east"[3] corner of that frame, passing 2.2 Mm to the "north" of it. But, the butterfly vertical-velocity difference, $\Delta^{\rm lep} v_z$ directly overlying the conspicuous ping, 2.2 Mm south of the blue line, shows a distinctive, compact horizontally cospatial signature (see red crosshairs in the north-east corner of the upper-right frame).

The lower-right frame applies an acoustic imaging algorithm, *computational seismic holography* [14,15], to the acoustic field in the lower left frame that coherently projects its source-power distribution to the overlying surface. This shows stigmatically the horizontal match between the overlying source of the pings and the respective compact anomalies in the butterfly-difference to which they are evidently attached.

Figure 3 shows a microcosmic view of the granulation patterns (left column) and the butterfly-vertical-velocity differences (right column) associated with the three most intense acoustic sources (middle column, $|H_+|^2$) that appear in

---

[3] The simulations do not associate geodetic directionality, neither terrestrial nor solar, with the computational grid. We arbitrarity designate the grid boundary toward the top of the page "north" and the rightward "east," reserving "upward" and "downward" for reference with respect to gravity.

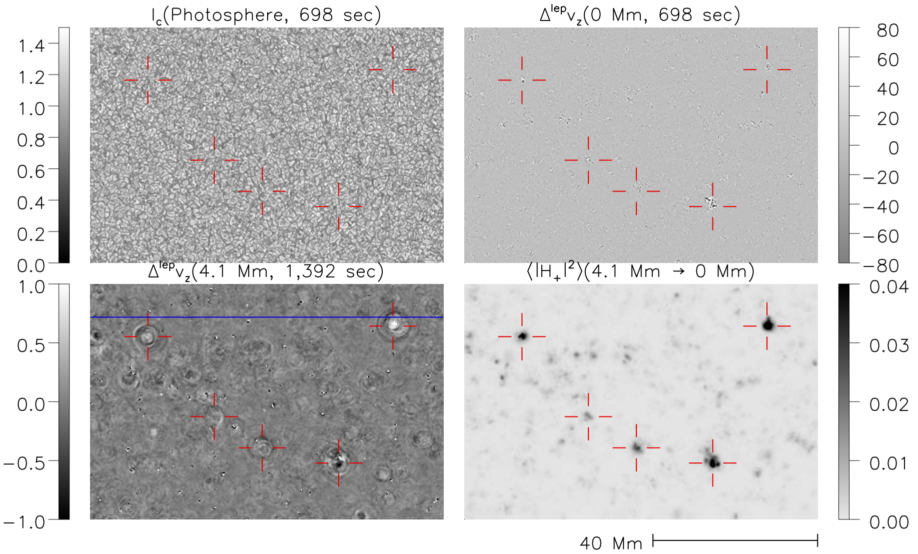

Fig. 2 Comparative samplings of compressible radiative-convective simulations of the solar granulation in the photosphere (top row) and 4.1 Mm beneath it (bottom left). Top left shows continuum intensity, I_c, emanating directly upward from the reference photosphere, normalized to the mean, 698 s (11.6 min) after the butterfly perturbation has been applied. Top-right shows the butterfly difference, $\Delta^{\rm lep} v_z$, in the vertical velocity, in m/s, at the base of the photosphere at the same moment. Bottom-left shows $\Delta^{\rm lep} v_z$ at depth 4.1 Mm 1,392 s (23.2 min) after the butterfly perturbation, at which point it represents acoutic noise generated by the overlying butterfly turbulence 694 s (11.6 min) before. Bottom-right shows source power distribution, $|H_+|^2$, of the acoustic field in lower-left frame holographically extrapolated back to the overlying surface (and technically some 700 s back in time). It is integrated over the 1,500-s interval beginning at the application of the butterfly perturbation. Horizontal locations of conspicuous episodes of seismic emission in the butterfly difference are indicated in all frames by red crosshairs. The blue horizontal line near the top of the lower-left frame indicates the location of the plane over which v_z and $\Delta^{\rm lep} v_z$ were rendered in Figure 2.

Figure 2. As already mentioned, the granulation pattern, rendered in continuum intensity, $I_c^{\rm ref}$, emanating directly upward from the reference simulation (left column), does not show any features nearly as distinctive to our eyes as those clearly visible in the butterfly-vertical-velocity difference, $\Delta^{\rm lep} v_z$, (right column). However, it does show from *where* in the granulation pattern the individual acoustic sources emanate to within the diffraction limit of the acoustic diagnostic. In our present perception, the acoustic sources gravitate toward the edges of the granules. This technically conforms to the major statistical class of seismic emmiters identified by Lindsey & Donea (2013, see Region I in Figures 3 and 4 in their study)[16].

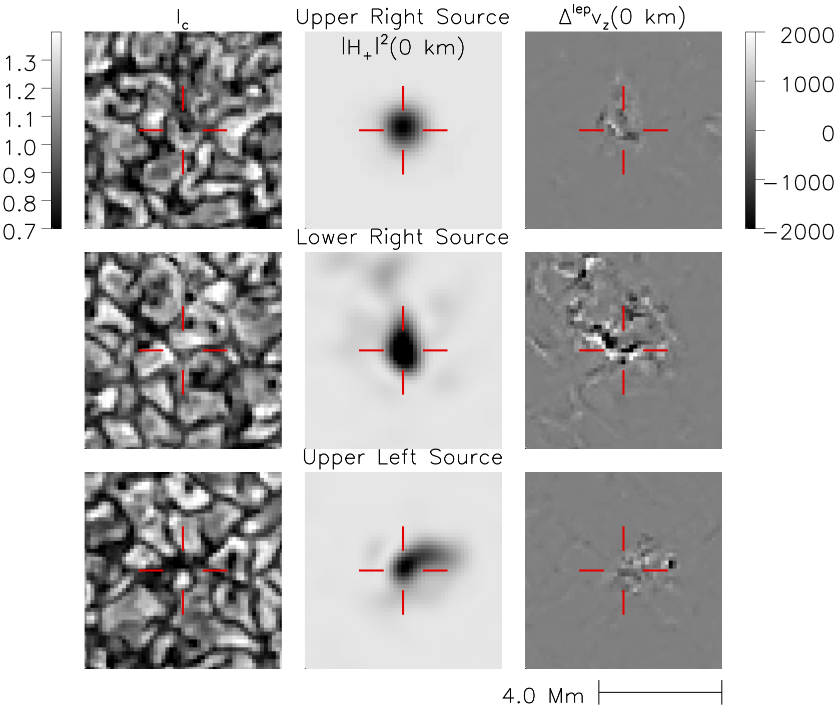

Fig. 3 The three most intense helioseismic sources, as rendered in $|H_+|^2$ in the bottom right frame of Figure 2, are shown in the middle column of this Figure next to respective concurrent cospatial granulation patterns as rendered in vertical continuum visible intensity (left column) and next to concurrent cospatial photospheric butterfly vertical velocity differences, $\Delta^{\rm lep}v_z$ (right column). In this Figure, each of the helioseismic signatures (middle frame), rather than integrated over a 1,500-s period as they are in Figure 2, is rendered instantaneously, at the moment at which it is individually most compact, with I_c (left frame) and $\Delta^{\rm lep}v_z$ (right frame) are rendered at that same respective moment.

5 Discussion

Goldreich & Kumar (1990) [8] reckon the efficiency with which turbulence characteristic of the outer convection zone emits noise to be in proportion to a high power (17/2) of the Mach number of its motion. This leads to an expectation that the source distribution is confined to a very thin layer, ~300 km thick, in the extreme outer convection zone, a thesis consistent with heliosismic observations [11,9,10]. It also suggests that acoustic sources should be both compact, so as to occupy a small filling factor spatially [3]. As small spatial filling factor translates ergotically to a small filling factor in space-time, hence small temporally as well as spatially, i.e., *episodic*, at least to our expectation. This appears to be the case for the pings the butterfly difference releases—at least in the early stages of its growth. While this may can be regarded as a likely indication of some significant character of the emission, it would be dangerous to conclude at this point that what we see in the butterfly difference is fully characteristic of emission in the reference simulation itself. Since there is a significant interval before which any significant seismic emission has yet to arrive to a depth of 4.1 Mm in the butterfly simulation, it might should be rather expected there will be a period during which only a few pings have arrived. So, the important issue of episodicity begs for more careful consideration.

The samples shown in Figure 3 support the statistical results of Lindsey & Donea (2013). This evidence is not only very thin at this point, even if it is borne out by further statistics, it may represent only a limited class of emitters. While the butterfly signatures are highly suggestive of the locations of real acoustic sources, they could be highly selective as to which sources they show us at various times since the application of the butterfly perturbation. Where we see a distinctive butterfly signature characteristic of acoustic emission, we are justified in believing that some amount of emission did indeed emanate. However, the *absence* of a butterfly signature *cannot* generally secure an absence of strong acoustic emission in either the reference simulation or its butterfly counterpart. The conspicuous butterfly signatures are selective of regions in which the the butterfly *difference* in emission *has grown most rapidly* since the butterfly perturbation, not those from which the absolute acoustic emission is most intense—at least to our present understanding. Moreover, even where an episodic butterfly signature is present, we cannot expect it to be realistically representative of the *acoustic flux* carried by it, neither in the reference simulation nor in its butterfly counterpart.

The major theme of this study, then, is rather to identify to what kinds of features butterfly-like experiments can guide us that any amount of statistics over of just surface observations, whether of simulations of the Sun or of the reality, cannot. We see, for example, that the butterfly-difference signature designated "Lower Right" (middle row) in Figure 3 is highly sheared in the acoutic source region, and that the preponderance of it somewhat conforms to the intergranular lane separating granules surrounding the center of the acoustic source distribution (middle frame). That is eye catching and likely significant, a feature that cannot be expected to emerge statistics compiled, such as by Lindsey & Donea (2013) [16], from millions of granules.

The results emerging from this study beg for a more extensive view of emitting regions that we have brought to bear as yet. The basic need is less in the realm of improved acoustic diagnostics as the ability, once an episode of acoustic emission is recognized, and its source location and time and accurately secured, to look directly at it in immaculate detail. The various dynamical components can be isolated by embedding them into otherwise undisturbed acoustic environments to observe the transition from turbulent convective motion into propagating acoustic disturbances to whatever degree of detail is needed. The algorithm that ran the simulations shown in Figures 1–3 sequestered the hydromagnetic conditions of the medium only in the single plane, that in which $\varDelta v_z^{\mathrm{lep}}$ was rendered in Figures 2 and 3. A great deal of analytical power can certainly be brought to bear by extending this domain, both upward into the photosphere and several hundred km beneath it. Even once its spatial and temporal locality is identified, a realistic understanding of the dynamics of p-mode excitation begs for analysis thereof over three full dimensions, not just a single surface.

Perhaps the most fruitful development to emerge from the butterfly experiments will be the motivation to devise a more direct alternative means of disposing of the accumulated acoustic radiation that drowns out new acoustic sources. We think this could be accomplished by an *acoustic mop* i.e., an algorithm that can be applied at the analysist's discretion to the sourceless acoustic environment underlying the solar granulation (in a simulation thereof) to cleanse it of old accumulated acoustic noise by appropriate damping mechanisms. After each application of this acoustic mop, there follows a period during which the only significant disturbance in the sourceless region is newly emitted acoustic radiation propagating downward from the overlying source region. A crucial difference between this acoustic radiation and that which emanates in the butterfly experiments is that the former will carry the full power of the acoustic sources emitting it.

Such a utility promises the facility to address some pointed questions: How do the acoustic sources move? Do they plummet alongside of the transonic plume that drives them? Are acoustic sources associated with downward-propagating transonic plumes? To what depths does the emission persist? Can our diagnostics discriminate acoustic emission due to implosive cooling [23, 21, 22, 19, 20]? (The results of Lindsey & Donea (2013) [16] suggest they can.) If so, how episodic is this, bearing in mind that the driver is now rapid cooling rather a high power of a turbulent velocity? What observational signatures distinguish underlying acoustic emitters? Can an understanding of this be used to improve local helioseismic diagnostics? We think standard seismic diagnostics applied to butterfly experiments can credibly address these question and a host of others emerging alongside of them.

6 Summary

1. Because the Sun's surface is an efficient reflector of p-modes in the 2.5–4.5 mHz range, acoustic noise emitted by the solar granulation accumulates over hours, even days.
2. This accumulated noise encumbers the recognition of newly emitted acoustic radiation, both in simulations of the Sun and in the Sun itself.
3. The butterfly effect offers a means of provisionally eliminating old accumulated noise in simulations.
4. This reveals a distinctive class of newly emitted acoustic radiation and its source regions. It offers a tantalizing observational glimpse into the dynamics of acoustic emission in simulations of this in the Sun's outer convective zone.
5. A highly desirable facility for the study of acoustic emission in the Sun's outer convective zone is an *acoustic mop*, i.e., an algorithm that can benignly (without exciting acoustic emission of its own) damp old accumu-

lated acoustic radiation in the outer convection zone that obscures newly emitted acoustic radiation. Use of the butterfly effect for this purpose can be regarded as a preliminary such facility, showing us the diagnostic potential of a working acoustic mop for compressible, radiative simulations of convection.

References

1. Alvarado Gómez, J. D., Buitrago Casas, J. C., Martínez Oliveros, J. C., Lindsey, C. & Hudson, H., Magneto-Acoustic Energetics Study of the Seismically Actie Flare of 15 February 2011, Solar Phys., 280, 335 (2012)
2. Braun, D. C. & Lindsey, C., Seismic Imaging of the Far Hemisphere of the Sun, Ap. J., 560, 189 (2001)
3. Brown, T. M., Bogdan, T. J., Lites, B. W. & Thomas, J. H., Localized Sources of Propagating Acoustic Waves in the Solar Photosphere, Ap. J. Lett., 394, L65 (1992) AAAA
4. Christensen-Dalsgaard, J, Proffitt, C. R. & Thompson, M. J., Effects of Diffusion on Solar Models and their Oscillation Frequencies, Ap. J., 403, 75 (1993)
5. Donea, A.-C., Braun, D. C. & Lindsey, C., Seismic Images of a Solar Flare, Ap. J., 513, 143 (1999)
6. Evans J. W., Main, P., Michard R. & Servajean, R., Correlations in the Time Variations of Macroscopic Inhomogeneities in the Solar Atmosphere, Ap. J., 136, 682 (1962)
7. Evans J. W., Michard R., Servajean, R., Observational Study of Macroscopic Inhomogeneities in the Solar Atmosphere. V. Statistical Study of Variations of Solar Inhomogeneities, An. Ap. , 26, 368 (1963)
8. Goldreich, P. & Kumar, P., Wave Generation by Turbulent Convection, Ap. J., 363, 694 (1990)
9. Kumar, P., Solar Oscillations with Frequencies above the Acoustic Cutoff Frequency, PASP, 42, 15 (1993)
10. Kumar, P., Properties of Acoustic Sources in the Sun, Ap. J., 428, 827 (1994)
11. Kumar, P. & Lu, E., The Location of the Source of High-Frequency Solar Acoustic Oscillations, Ap. J. Lett., 375, L35 (1991) AAAA
12. Leighton R. B., Noyes R. W., Simon G. W., Velocity Fields in the Solar Atmosphere. I. Preliminary Report, Ap. J., 135, 474 (1962)
13. Lighthill, M. J., On Sound Generated Aerodynamically. I. General Theory, Proc. Roy. Soc. London, A211, 564 (1952)
14. Lindsey, C. & Braun, D. C., Helioseismic Holography, Ap. J., 485, 895 (1997)
15. Lindsey, C. & Braun, D. C. Seismic Images of the Far Side of the Sun, Science , 287, 1799 (2000)
16. Lindsey, C. & Donea, A.-C., Statistics of Local Seismic Emission from the Solar Granulation, J. Phys. Conf. Series, 440a, 2044L (2013)
17. Lorenz, E. N., Atmospheric Predictability as Revealed by Naturally occurring Analogues, Journal of the Atmospheric Sciences, 26, 636 (1969)
18. Noyes R. W., Leighton R. B., Velocity Fields in the Solar Atmosphere. II. The Oscillatory Field, Ap. J., 138, 631 (1963)
19. Rast, M. P., Photospheric Downflows: How Deep, How Coherent, How Important?, in SCOREe'96: Solar Convection and Oscillations and their Relationship, ed. F. P. Pijpers, J. Christensen-Dalsgaard, & C. S. Rosenthal (Dordrecht: Kluwer), p. 135 (1997)
20. Rast, M. P. 1999 The Thermal Starting Plume as an Acoustic Source, Ap. J. Lett., 524, 462 (1999)
21. Stein, R. F. & Nordlund, Å, Simulations of Solar Granulation. I. General Properties, Ap. J., 499, 914 (1998a)
22. Stein, R. F. & Nordlund, Å, Convection and p-Modes, in Structure and Dynamics of the Interior of the Sun and Sun-Like Stars, Proc. SOHO 6/GONG 98 Workshop, ed. S. G. Korzennik & A. Wilson (ESASP-418), p. 693 (1998b)
23. Stein, R. F. & Nordlund, Å. Solar Oscillations and Convection. II. Excitation of Radial Oscillations, Ap. J., 546, 585 (2001)