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#### **Key Points:**

- A spectrum of gravity waves is excited from an ocean wave packet
- The gravity waves have phase speeds greater, equal, and less than the ocean wave
- These gravity waves create significant perturbations in the thermosphere

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# Excitation of gravity waves by ocean surface wave packets: Upward propagation and reconstruction of the thermospheric gravity wave field

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**Abstract** In this paper, we derive the atmospheric gravity waves (GWs) and acoustic waves excited by an ocean surface wave packet with frequency  $\omega_F$  and duration  $\chi$  in an f plane, isothermal, windless, and inviscid atmosphere. This packet is modeled as a localized vertical body force with Gaussian depth  $\sigma_z$ . The excited GW spectrum has discrete intrinsic frequencies ( $\omega_{lr}$ ) at  $\omega_F$  and  $\omega_F \pm 2\pi/\chi$  ("sum" and "difference") and has a "continuum" of frequencies for  $\omega_{lr} < \omega_F + 2\pi/\chi$ . The momentum flux spectrum peaks at  $\omega_{lr} \sim \omega_F$  and decreases rapidly as  $\omega_{lr}$  decreases. To simulate the effect these GWs have on the thermosphere, we present a new scheme whereby we sprinkle  $\mathcal{N}$  GW spectra in the ocean wave packet region, ray trace the GWs, and reconstruct the GW field. We model the GWs excited by ocean wave packets with horizontal wavelengths of  $\lambda_H = 190$  km, periods of  $\tau_F = 2\pi/\omega_F = 14 - 20$  min and  $\chi = 30 - 50$  min. The excited GWs begin to arrive at z = 250 km at  $t \sim 75 - 80$  min. Those with the largest temperature perturbations T' have large  $\omega_{lr}$  and arrive at  $t \sim 90 - 130$  min. If  $|\alpha| = \omega_F + 2\pi/\chi$  is a solution of the GW dispersion relation and  $|\alpha|$  is less than the buoyancy frequency at z = 250 km, the sum and highest-frequency continuum GWs have much larger phase speeds and arrive 50–60 min earlier with larger T' than the GWs with frequency  $\omega_F$ . For a packet with  $\lambda_H = 190$  km,  $\tau_F = 14$  min,  $\chi = 30$  min, and height  $h_0 = 1.3$  m, the maximum T' at z = 250 km is  $\sim 9$ , 22, and 40 K for  $\sigma_z = 1$ , 2, and 4 m, respectively.

#### 1. Introduction

Atmospheric gravity waves (GWs) with large phase speeds, frequencies, and vertical wavelengths can transfer significant amounts of momentum and energy from the lower atmosphere to the midthermosphere because (1) GW amplitudes increase exponentially with altitude [*Hines*, 1960] and (2) the thermosphere is a viscous fluid, thereby causing every GW to dissipate there [*Pitteway and Hines*, 1963; *Hickey and Cole*, 1988; *Vadas and Fritts*, 2005; *Vadas*, 2007; *Heale et al.*, 2014]. When a GW dissipates, it heats/cools the surrounding fluid [*Walterscheid*, 1981; *Liu*, 2000; *Medvedev and Klaassen*, 2003; *Becker*, 2004; *Yiğit and Medvedev*, 2009; *Vadas*, 2013] and accelerates the fluid in the direction that it was propagating [*Fritts and Alexander*, 2003; *Vadas and Fritts*, 2006; *Miyoshi and Fujiwara*, 2008; *Vadas and Liu*, 2009; *Yiğit et al.*, 2009; *Vadas*, 2013]. Thus, energy and momentum are transferred from the GW to the thermosphere upon dissipation. Such processes have been studied extensively for deep convection via modeling [*Vadas and Liu*, 2009, 2013; *Vadas*, 2013; *Vadas et al.*, 2014].

GWs from deep convection have been observed in the ionosphere for decades [e.g., *Bauer*, 1958; *Röttger*, 1977; *Hung and Kuo*, 1978; *Kelley*, 1997; *Hocke and Tsuda*, 2001; *Bishop et al.*, 2006]. GWs excited by mountain wave breaking have also been observed in the ionosphere [*Vadas and Nicolls*, 2009; *Smith et al.*, 2013]. Recently, interest has turned toward understanding the role that GWs excited by ocean surface waves have on the thermosphere and ionosphere. *Djuth et al.* [2010] postulated an ocean wave source for many of the GWs observed by the incoherent scatter radar at the Arecibo Observatory (AO). *Nicolls et al.* [2014] determined the direction of propagation for the GWs observed at the AO during a 3 day campaign. Although deep convection was postulated as a likely source, some of the GWs could have been excited by ocean surface waves. In a different study at Wallops Island, *Vadas and Crowley* [2010] found that most of the observed GWs were likely secondary GWs from Tropical Storm Noel. While this is still the most likely source for those GWs

©2015. American Geophysical Union. All Rights Reserved. having phase speeds > 300 m/s, it is possible that some of the subsound speed GWs could have been excited by ocean surface waves.

Tsunamis are a series of ocean waves created by earthquakes beneath the ocean surface, volcanos, landslides, large meteor impacts, etc. It has been theorized that GWs launched by tsunamis can propagate well into the thermosphere and can couple into the ionosphere [*Hines*, 1960; *Najita et al.*, 1974; *Peltier and Hines*, 1976]. Observations made during several tsunamis in the Indian and Pacific Oceans over the past decade have confirmed this coupling process. *Artru et al.* [2005] used the dense GPS Earth Observation Network of dual-frequency receivers located in Japan to image the ionospheric response created by the tsunami resulting from the 2001 Peruvian earthquake. Additional observations of tsunami-generated GWs in the ionosphere made using radio techniques were reported for the 2004 Sumatra tsunami [e.g., *Liu et al.*, 2006; *Lognonné et al.*, 2006; *Occhipinti et al.*, 2006], as well as essentially every subsequent tsunami in the Pacific Ocean [e.g., *Rolland et al.*, 2010; *Galvan et al.*, 2011; *Makela et al.*, 2011; *Occhipinti et al.*, 2013; *Garcia et al.*, 2014].

The excitation and propagation of GWs from tsunamis have been studied previously using various models [Peltier and Hines, 1976; Hickey et al., 2009, 2010; Occhipinti et al., 2006, 2011; Matsumura et al., 2011; Makela et al., 2011; Galvan et al., 2012; Kherani et al., 2012; Broutman et al., 2014]. Most of these studies found that some of the excited GWs can propagate into the midthermosphere (i.e.,  $z \sim 200 - 300$  km). However, simplifications were made in many of these models which limit their application and future predictive capabilities. Occhipinti et al. [2006, 2011] did not include thermospheric dissipation and assumed that the Boussinesq approximation was appropriate. However, the thermosphere is characterized by exponentially increasing viscosity. Therefore, neglecting viscosity yields GW amplitudes that are far too large, even for GWs with large phase speeds of  $c_{H} \sim 200 \text{m/s}$  [Vadas, 2007]. Thus, GW excitation amplitudes at the ocean surface had to be adjusted by hand to yield reasonable ionospheric amplitudes that matched observations. Additionally, the Boussinesq approximation is only valid if a GW's vertical wavelength,  $|\lambda_z|$ , is small compared to  $2\pi H$ , where  $\mathcal{H}$  is the neutral density scale height. This is a poor assumption for the largest-amplitude GWs that are able to propagate to z ~250 km, as we show in this paper. Hickey et al. [2009] used a sum of steady state GW solutions to calculate the tsunami-induced GW solutions in the thermosphere. However, such a sum likely cannot represent the general, time-dependent atmospheric solutions to the fluid equations, since each steady state solution for each wave component peaks too high in altitude. This occurs because the GWs excited by a steady state system continuously pump energy and momentum into the system in order to maintain it, thereby pushing the GW effects up in altitude by several density scale heights [Vadas and Nicolls, 2012]. Additionally, a tsunami is highly localized temporally and spatially and is therefore poorly approximated as a steady state system because the excited GWs cannot supply a continuous stream of energy and momentum to the same (x, y, z) location in the thermosphere. Finally, Peltier and Hines [1976], Occhipinti et al. [2008], and Hickey et al. [2009] approximated a tsunami as a steady state, plane ocean surface wave, rather than as a localized ocean surface wave packet. (Here a "wave packet" refers to ocean surface waves having similar periods and wave vectors that are approximately localized in space and time.) This approximation led to an oversimplified assumption concerning the phase speeds and frequencies of the excited GWs. The steady state assumption treated the tsunami as a "mountain" in the presence of a constant wind, thus simplifying the solution to a stationary mountain wave solution with vertically propagating lee waves. This assumption, combined with the plane wave assumption, led to the assumption that the GWs excited by a tsunami have the same phase speed and frequency,  $\omega_F = c_H/\lambda_H$ , as the fundamental wave (with horizontal wavelength  $\lambda_H$ ) within the tsunami (as determined from the Fourier transform of the ocean surface displacement). As we shall see in this paper, an ocean surface wave packet excites a spectrum of GWs containing distinct frequencies and a continuum of frequencies, thereby resulting in GWs having phase speeds that are larger, smaller, and equal to that of the ocean surface wave. This occurs primarily because the ocean wave packet is localized in time and space.

As described previously, there have been many ionospheric observations of tsunami-generated GWs. This points to the distinct possibility of using ionospheric observations to monitor for tsunamis. However, significant work still needs to be done to quantitatively relate the amplitudes, phase speeds, and timing of these ionospheric perturbations to the amplitudes and phases of the ocean-level perturbations. Indeed, such quantitative linkages likely require significant improvements in the modeling of tsunami-generated GWs as they are excited from the ocean and then propagate and dissipate in the thermosphere.

Real and modeled ocean surface displacements in tsunami events involve complicated interactions of shallow water wave models with coastline, bathymetry, existing surface waves, and a variety of surface forcings in addition to the triggering event. The atmospheric response to the full complexity of a real tsunami forcing is too large an undertaking given the development of GW and acoustic wave (AW) modeling to date. Instead, we show below that the tsunami forcing is abstracted to be represented by a body force proportional to the vertical component of the surface displacement amplitude by a wave packet that is consistent with observed and modeled tsunami scales, perhaps most appropriately associated with the leading edge of a tsunami event. Qualitative support for this abstraction is taken from sources like *Salmon* [2014] and *Gill* [1982]. Note that diffusive effects caused by the boundary layer (Ekman layer) at the ocean-atmosphere interface or from diffusion in the planetary boundary layer are neglected here.

In this paper, we introduce a new model which determines the GW field in the thermosphere excited by an ocean surface wave *packet* (i.e., localized in time and space). This model consists of two parts. The first part consists of a model which calculates the spectral solutions for the GWs (and AWs) excited by an ocean surface wave packet. These solutions are compressible, not Boussinesq, and are therefore not constrained by the value of  $|\lambda_z|$ . Additionally, the GW amplitudes are obtained directly from this model and are therefore not ad hoc. The second part consists of "sprinkling" hundreds or thousands of GW spectra (obtained from the first part) throughout the ocean wave packet region, ray tracing the GWs (with phases) into the thermosphere, and reconstructing the GW field in the thermosphere. Because we ray trace the GWs, the thermosphere can include realistic viscosity, thereby allowing for a significant improvement over inviscid models. This sprinkling scheme is novel and is tailored to result in solutions that very closely resemble the exact Fourier-Laplace solutions for the response to an ocean surface wave packet in an isothermal, windless, and inviscid atmosphere.

We review the method used to derive the compressible *f* plane solutions to general body forces and heatings in section 2. In section 3, we argue that the acceleration of air above an ocean surface wave packet can be modeled as a vertical body force. We then derive the solutions to an ocean surface wave packet. We describe our new GW sprinkling/ray trace scheme and show how to reconstruct the GW field in section 4. In section 5, we ray trace the GWs excited by medium-scale ocean wave packets into the thermosphere and reconstruct the GW fields. We also determine the relative importance of the GWs with the fundamental, sum, difference, and continuum frequencies at the ocean surface and in the thermosphere. Our conclusions are provided in section 6. The appendices contain special case wave packet solutions and the compressible *f* plane solutions to a steady state ocean surface wave.

#### 2. Excitation of GWs and AWs From Body Forces and Heatings

In this section, we review the equations and solution methods used to determine the fluid response to general 3-D body forces and heatings having separable spatial and temporal dependencies in an isothermal, inviscid, and windless atmosphere.

#### 2.1. Compressible, f Plane Fluid Equations

The compressible momentum, mass, and energy equations for general body forces and heatings in a 3-D inviscid fluid satisfying the f plane approximation are (equations (1), (2), and (4) in *Vadas* [2013]):

$$\frac{\partial \mathbf{v}}{\partial t} + \frac{1}{\rho} \nabla \rho - \mathbf{g} + 2\Omega \times \mathbf{v} = \mathbf{F}(\mathbf{x}) \mathcal{F}(t), \tag{1}$$

$$\frac{\mathsf{D}\rho}{\mathsf{D}t} + \rho \nabla \cdot \mathbf{v} = 0, \tag{2}$$

$$\frac{\mathsf{D}p}{\mathsf{D}t} + \gamma p \nabla \cdot \mathbf{v} = \frac{p}{T} J(\mathbf{x}) \mathcal{F}(t), \tag{3}$$

where  $\mathbf{x} = (x, y, z)$ ,  $D/Dt = \partial/\partial t + (\mathbf{v}.\nabla)$ ,  $\mathbf{v} = (u, v, w)$  is velocity vector, u, v, and w are zonal, meridional, and vertical velocities, respectively, T is temperature,  $\rho$  is density, p is pressure,  $\Omega$  is Earth's rotation vector, and  $\mathbf{g} = -g\hat{\mathbf{k}}$  is the gravitational force. We use the ideal gas law,  $p = r\rho T$ , where  $r = (8308/X_{MW}) \text{ m}^2 \text{s}^{-2} \text{K}^{-1}$ ,  $X_{MW}$  is the mean molecular weight of a particle in the gas (in g/mol),  $\gamma - 1 = r/C_v$ , and  $C_v$  is the mean specific heat at constant volume. The body force and heating have time dependence  $\mathcal{F}(t)$ . The spatial portion of the 3-D body force is  $\mathbf{F}(\mathbf{x}) = (F_x, F_y, F_z)$  and that of the heat/cooling is  $J(\mathbf{x})$ . The functions  $\mathbf{F}$  and J can be any arbitrary (but continuous and derivable) functions of  $\mathbf{x}$ .

The mean molecular weight decreases from  $X_{MW} = 28.9$  in the lower atmosphere to  $X_{MW} = 16$  in the upper thermosphere as the molecular composition changes from primarily diatomic N<sub>2</sub> and O<sub>2</sub> to monatomic O [*Roble and Ridley*, 1994]. At the same time,  $\gamma$  increases from  $\gamma = 1.4$  to  $\gamma = 1.67$  because of this change. The transition from diatomic to monatomic species occurs at  $z \sim 150$  to 300 km. In order to solve equations (1)–(3) analytically, we assume that the fluid is composed of a single species, and that  $X_{MW}$  and  $\gamma$  are locally constant. We also assume that the fluid is isothermal (constant temperature in **x**), inviscid, and unsheared. (Without these assumptions, it is not possible to derive analytic solutions, which are necessary for ray tracing.) Variations in the background temperature, wind, dissipation,  $X_{MW}$  and  $\gamma$  are included via ray tracing away from the source region (see section 5) [e.g., *Vadas*, 2007; *Fritts and Vadas*, 2008].

#### 2.2. Solution Method

We now review the method we use to derive the linear solutions to equations (1)–(3). We follow the formalism developed in *Vadas* [2013]. Note that we leave the temporal dependence of the force/heatings,  $\mathcal{F}(t)$ , unspecified here.

We expand the variables as mean (overlines) plus perturbations (primes) in local Cartesian coordinates:

$$u = U + u', \quad v = V + v', \quad w = w',$$
  

$$\rho = \overline{\rho} + \rho', \quad T = \overline{T} + T', \quad p = \overline{p} + p'. \tag{4}$$

We neglect the Earth's curvature and assume that the fluid obeys the *f* plane approximation. For medium and high-frequency waves with periods less than a few hours, f = 0 is an excellent approximation at any latitude, because the Coriolis force is ineffective at these time scales. This approximation is satisfied for an ocean wave packet if its fundamental period is less than a few hours. Note that typical tsunami periods are ~ 10 to ~ 60 min. The hydrostatic equation (derived from equation (1) for  $\mathbf{v} = \mathbf{F} = J = 0$ ) is  $d\overline{p}/dz = -g\overline{p}$ , yielding (Hines, 1960)

$$\overline{\rho} = \overline{\rho}_0 e^{-z/\mathcal{H}}, \quad \overline{p} = \overline{\rho}_0 e^{-z/\mathcal{H}}, \tag{5}$$

where  $\mathcal{H} = -\overline{\rho}(d\overline{\rho}/dz)^{-1} = r\overline{T}_0/g$  is the neutral density scale height, and  $\overline{\rho}_0$ ,  $\overline{T}_0$ , and  $\overline{p}_0$  are the mean density, temperature, and pressure at z = 0, respectively. (Note that our isothermal assumption implies that  $\overline{T} = \overline{T}_0$ .) We assume that the force/heat/cooling amplitudes are small enough that wave-mean flow and wave-wave interactions can be neglected. The resulting linearized equations are given by equations (10)–(12) in *Vadas* [2013]. We define the following variables:

$$\begin{aligned} \xi &= e^{-z/2\mathcal{H}}u', \quad \sigma = e^{-z/2\mathcal{H}}v', \quad \eta = e^{-z/2\mathcal{H}}w', \\ \phi &= e^{z/2\mathcal{H}}\rho'/\overline{\rho}_0 = e^{-z/2\mathcal{H}}\rho'/\overline{\rho}, \quad \psi = e^{z/2\mathcal{H}}p'/\overline{\rho}_0 = e^{-z/2\mathcal{H}}p'/\overline{\rho}, \\ \zeta &= e^{-z/2\mathcal{H}}T'/\overline{T}_0. \end{aligned}$$
(6)

We also define the "scaled" force/heat/coolings (denoted by the subscript "s") as

$$F_{xs} = e^{-z/2H} F_x, \quad F_{ys} = e^{-z/2H} F_y,$$
  

$$F_{zs} = e^{-z/2H} F_z, \quad J_s = e^{-z/2H} rJ.$$
(7)

We expand  $\xi$ ,  $\sigma$ ,  $\eta$ ,  $\phi$ ,  $\psi$ ,  $\zeta$ ,  $F_{xs}$ ,  $F_{ys}$ ,  $F_{zs}$ , and  $J_s$  as Fourier series. For example,

$$\xi(x, y, z, t) = \frac{1}{(2\pi)^3} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-i(kx+ly+mz)} \widetilde{\xi}(k, l, m, t) dk dl dm,$$
(8)

where "~" denotes the Fourier transform,  $\mathbf{k} = (k, l, m)$  is the wave vector, and k, l, and m are the zonal, meridional, and vertical wave numbers, respectively. Additionally, the horizontal wave number is  $k_H = \sqrt{k^2 + l^2}$ . Equations (1)–(3) become (equations (16)–(20) in *Vadas* [2013]):

$$\frac{\partial \widetilde{\xi}}{\partial t''} - ik\widetilde{\psi} - f\widetilde{\sigma} = \widetilde{F_{xs}}\mathcal{F}(t), \tag{9}$$

$$\frac{\partial \widetilde{\sigma}}{\partial t''} - il\widetilde{\psi} + f\widetilde{\xi} = \widetilde{F_{ys}}\mathcal{F}(t), \tag{10}$$

10.1002/2015JA021430

$$\frac{\partial \widetilde{\eta}}{\partial t''} - im_s \widetilde{\psi} + g \widetilde{\phi} = \widetilde{F_{zs}} \mathcal{F}(t), \tag{11}$$

$$\frac{\partial \widetilde{\phi}}{\partial t''} - i\left(k\widetilde{\xi} + l\widetilde{\sigma} + m_s\widetilde{\eta}\right) = 0, \tag{12}$$

$$\frac{\partial \widetilde{\psi}}{\partial t''} + \delta \widetilde{\eta} - ic_s^2 (k\widetilde{\xi} + l\widetilde{\sigma}) = \widetilde{J}_s \mathcal{F}(t), \tag{13}$$

where  $\partial/\partial t'' = \partial/\partial t - i(k\overline{U} + I\overline{V})$ , the sound speed is  $c_s$ , and

$$c_s^2 = \gamma g \mathcal{H} = \gamma r \overline{T}_0, \tag{14}$$

$$\delta = g(\gamma - 1) - ic_s^2 m_s, \tag{15}$$

$$m_{\rm s} = m - i/2\mathcal{H}.\tag{16}$$

Since  $p = r\rho T$ , then  $p'/\overline{p} = \rho'/\overline{\rho} + T'/\overline{T}$ . Using this and equation (6), the scaled temperature perturbation is then  $\widetilde{\zeta}$ 

$$=\frac{\gamma}{c_s^2}\widetilde{\psi}-\widetilde{\phi}.$$
(17)

We take the Laplace transform of equations (9)–(13). For example, the Laplace transform of  $\tilde{\psi}$  is [Abramowitz and Stegun, 1972]:

$$\mathcal{L}_{\widetilde{\psi}} = \mathcal{L}(\widetilde{\psi}) = \int_0^\infty e^{-s_r t} \widetilde{\psi}(t) dt, \qquad (18)$$

where

$$s = s_r - i(k\overline{U} + I\overline{V}), \tag{19}$$

and the intrinsic and ground-based wave frequencies are  $\omega_{ir} = -is$  and  $\omega_r = -is_r$ , respectively. (Here a Laplace transform in time is utilized (rather than a Fourier transform in time) in order to preserve the explicit time dependence of the solution caused by the envelope force. Such time-dependent terms can often be linear in t [for example, Vadas and Fritts, 2001]. The resulting equations are then functions of  $\mathcal{L}_{\tilde{\xi}}, \mathcal{L}_{\tilde{\sigma}}, \mathcal{L}_{\tilde{\eta}}, \mathcal{L}_{\tilde{\phi}}$ , and  $\mathcal{L}_{\tilde{\psi}}$ . We solve these five linear equations for  $\mathcal{L}_{\widetilde{w}}$  and obtain

$$\mathcal{L}_{\widetilde{\psi}} = \frac{1}{s(s^{2} - s_{1}^{2})(s^{2} - s_{2}^{2})} \left\{ \left[ (s^{2} + im_{s}g)(s^{2} + f^{2})\widetilde{\psi}(0) - \delta(s^{2} + f^{2})(s\widetilde{\eta}(0) - g\widetilde{\phi}(0)) + ic_{s}^{2} \left[ \left\{ k\widetilde{\xi}(0) + l\widetilde{\sigma}(0) \right\} s + \left\{ k\widetilde{\sigma}(0) - l\widetilde{\xi}(0) \right\} f \right] (s^{2} + N_{B}^{2}) \right] + \left[ ic_{s}^{2}(sA_{F} + fB_{F}) \left( s^{2} + N_{B}^{2} \right) - \delta s(s^{2} + f^{2})\widetilde{F_{zs}} + (s^{2} + im_{s}g)(s^{2} + f^{2})\widetilde{J_{s}} \right] \mathcal{L}_{F} \right\}.$$
(20)

Here the Laplace transform of  $\mathcal{F}(t)$  is  $\mathcal{L}_{\mathcal{F}}$ , the buoyancy frequency is  $N_{\beta}$ ,

$$V_{B}^{2} = (\gamma - 1)g^{2}/c_{s}^{2}$$
(21)

$$A_{\rm F} = k\widetilde{F_{\rm xS}} + l\widetilde{F_{\rm yS}} \tag{22}$$

$$B_{\rm F} = k\widetilde{F_{\rm vs}} - l\widetilde{F_{\rm xs}}.$$
(23)

(Note that equation (20) was not shown in Vadas [2013].) The corresponding compressible f plane dispersion relation (applicable to both AWs and GWs) is

$$s^{4} + \left[f^{2} + c_{s}^{2}(\mathbf{k}^{2} + 1/(4\mathcal{H}^{2}))\right]s^{2} + c_{s}^{2}\left[k_{H}^{2}N_{B}^{2} + f^{2}(m^{2} + 1/(4\mathcal{H}^{2}))\right] = 0.$$
(24)

The two roots of equation (24) are

$$s_1^2 = -\omega_1^2 = -\frac{a}{2} \left[ 1 - \sqrt{1 - 4b/a^2} \right],$$
(25)

#### GRAVITY WAVES FROM OCEAN WAVE PACKET

**AGU** Journal of Geophysical Research: Space Physics

$$s_2^2 = -\omega_2^2 = -\frac{a}{2} \left[ 1 + \sqrt{1 - 4b/a^2} \right],$$
(26)

and correspond to the GWs and AWs, respectively. Here

$$a = -\left(s_1^2 + s_2^2\right) = \left[f^2 + c_s^2(\mathbf{k}^2 + 1/(4\mathcal{H}^2))\right],$$
(27)

$$b = s_1^2 s_2^2 = c_s^2 \left[ k_H^2 N_B^2 + f^2 (m^2 + 1/(4\mathcal{H}^2)) \right].$$
<sup>(28)</sup>

The intrinsic GW frequency is  $\omega_{GW} = \omega_1 = -is_1$ , while the intrinsic AW frequency is  $\omega_{AW} = \omega_2 = -is_2$ .

Given a specified analytic function for the temporal dependence of the force/heating  $\mathcal{F}(t)$ , we compute  $\mathcal{L}_{\mathcal{F}}$  analytically, plug it into equation (20), take the inverse Laplace transform of equation (20), and then solve for  $\tilde{\psi}(\mathbf{k}, t)$ . We also differentiate and rearrange equations (9)–(13) to obtain

$$\frac{\partial^2 \widetilde{\xi}}{\partial t^2} + f^2 \widetilde{\xi} = ik \frac{\partial \widetilde{\psi}}{\partial t} + ilf \widetilde{\psi} + f \widetilde{F_{ys}} \mathcal{F} + \widetilde{F_{xs}} \frac{\mathrm{d}\mathcal{F}}{\mathrm{d}t},$$
(29)

$$\frac{\partial^2 \widetilde{\sigma}}{\partial t^2} + f^2 \widetilde{\sigma} = i l \frac{\partial \widetilde{\psi}}{\partial t} - i k f \widetilde{\psi} - f \widetilde{F_{xs}} \mathcal{F} + \widetilde{F_{ys}} \frac{d\mathcal{F}}{dt}, \tag{30}$$

$$\frac{\partial^2 \widetilde{\eta}}{\partial t^2} + N_B^2 \widetilde{\eta} = \frac{g}{c_s^2} \left( i\gamma \mathcal{H} m_s - 1 \right) \frac{\partial \widetilde{\psi}}{\partial t} + \frac{g}{c_s^2} \left( \gamma \mathcal{H} \widetilde{F_{zs}} \frac{\mathrm{d}\mathcal{F}}{\mathrm{d}t} + \widetilde{J_s} \mathcal{F} \right), \tag{31}$$

$$\frac{\partial^2 \widetilde{\phi}}{\partial t^2} + N_B^2 \widetilde{\phi} = \frac{1}{c_s^2} \frac{\partial^2 \widetilde{\psi}}{\partial t^2} + im_s \frac{N_B^2}{g} \widetilde{\psi} + \frac{g(\gamma - 1)}{c_s^2} \widetilde{F}_{zs} \widetilde{F} - \frac{1}{c_s^2} \widetilde{J}_s \frac{\mathrm{d}F}{\mathrm{d}t}.$$
(32)

Here we have substituted  $t'' \to t$  for ease of notation. Then,  $\tilde{\xi}(\mathbf{k}, t)$ ,  $\tilde{\sigma}(\mathbf{k}, t)$ ,  $\tilde{\eta}(\mathbf{k}, t)$ , and  $\tilde{\phi}(\mathbf{k}, t)$  are obtained from  $\tilde{\psi}(\mathbf{k}, t)$  by analytically integrating equations (29)–(32).  $\tilde{\zeta}$  is obtained from equation (17). Note that  $\tilde{\psi}, \tilde{\xi}, \tilde{\sigma}, \tilde{\eta}, \tilde{\phi}$ , and  $\tilde{\zeta}$  are functions of the wave vector **k** and time *t*.

#### 3. Excitation of GWs and AWs From an Ocean Surface Wave Packet

Consider a linear ocean surface wave packet far from shore. We first understand how this wave affects the air directly above it and then solve the atmospheric equations of motion.

#### 3.1. Acceleration of Air Above an Ocean Surface Wave

We first consider the simplest case of a parcel of air located just above the surface of the ocean which is perturbed by a steady state plane ocean surface wave. As the surface wave travels under the air parcel, the air parcel is primarily displaced upward and downward in time, like a buoy, with little net horizontal motion. (Note that the parcels of water within the ocean surface wave form ellipses) [*Gill*, 1982, page 103]. Therefore, we assume that the ocean wave only accelerates the air parcels in the vertical direction. (However, note that both vertical and horizontal accelerations excite GWs) [*Vadas and Fritts*, 2001; *Vadas*, 2013]. If the displacement of the ocean depth about the mean depth is given by

$$h'(x, y, t) = h_0 e^{i(\omega_F t - kx - ly + q)},$$
(33)

then the vertical acceleration of the ocean surface is

$$\frac{\partial^2 h'}{\partial t^2} = -h_0 \omega_F^2 e^{i(\omega_F t - kx - ly + q)} = -\omega_F^2 h'.$$
(34)

Here  $h_0$  is the amplitude of the ocean surface wave,  $\omega_F$  is the frequency, q is the phase, and k and l are the zonal and meridional wave numbers, respectively. Because the ocean is ~1000 times denser than air, we assume that the ocean wave surface is not deformed by the back reaction of the excited atmospheric perturbations on the ocean. Then equation (34) equals the vertical acceleration of the air just above the ocean surface.

Although equation (34) describes the vertical acceleration caused by a steady state plane ocean wave, a similar argument holds for a localized ocean surface wave packet (i.e., that the air above the ocean wave packet is primarily accelerated vertically). We therefore describe the effect that an ocean surface wave packet has on the air just above it by inserting a localized vertical acceleration into the atmospheric equations of motion at the ocean surface.

#### 3.2. Atmospheric Perturbations Created by an Ocean Surface Wave Packet

We model the effect that an ocean surface wave packet has on the atmosphere as a vertical body force (i.e., a vertical acceleration). The spatial portion of this force is

$$\mathbf{F}(\mathbf{x}) = F_z(\mathbf{x})\mathbf{\hat{k}},\tag{35}$$

since our assumption is that  $F_x = F_y = 0$ . If we wanted to model the atmospheric response to a steady state plane ocean wave, we could choose  $\mathcal{F}(t) = \sin(\omega_F t)$  for  $t = [-\infty, \infty]$ . However, we are interested in the response to an ocean surface wave *packet*, since that best describes the leading edge of a tsunami [pg. 111 of *Salmon*, 2014]. Therefore, we choose  $\mathcal{F}$  to oscillate at the fundamental frequency  $\omega_F$  over an envelope of duration  $\chi$ :

$$F(t) = \frac{1}{\chi} \begin{cases} (1 - \cos \hat{a}t) \sin(\omega_F t) & \text{for } 0 \le t \le \chi \\ 0 & \text{for } t \le 0 \text{ and } t \ge \chi. \end{cases}$$
(36)

This function begins at t = 0 and ends at  $t = \chi$ . The envelope frequency is

$$\hat{a} \equiv 2\pi n/\chi, \tag{37}$$

where *n* is a positive integer. Although we derive the solutions for any *n*, we only employ n = 1 for the ocean wave excitation mechanism. Note that the fundamental period,

$$\tau_F \equiv 2\pi/\omega_F,\tag{38}$$

is unrelated to  $\chi$  in general. Figure 1 shows  $\mathcal{F}(t)$  with  $\tau_F = 15$  min for  $\chi = 1$  and 2 h. The  $\omega_F$  oscillation is modulated by a smooth envelope which turns on at t = 0, peaks at  $\chi/2$ , and ends at  $t = \chi$ . The envelope functions are also shown in Figure 1.

We are only interested in the response of the atmosphere to the ocean surface wave packet. Therefore, we set the initial conditions to zero:  $\tilde{\xi}(0) = \tilde{\sigma}(0) = \tilde{\eta}(0) = \tilde{\phi}(0) = \tilde{\psi}(0) = \tilde{\zeta}(0) = 0$ . We also set  $F_x = F_y = 0$  and J = 0. Equation (20) becomes

$$\mathcal{L}_{\widetilde{\psi}} = \frac{-\delta(s^2 + f^2)\widetilde{F}_{zs}}{(s^2 - s_1^2)(s^2 - s_2^2)}\mathcal{L}_F.$$
(39)

Equation (39) is the Laplace transform of the scaled pressure perturbation caused by the general vertical body force,  $F_z \mathcal{F} \hat{\mathbf{k}}$ . Note that  $F_z$  can be any function of  $\mathbf{x}$  as long as it and its first derivatives are continuous.

We now determine the atmospheric solution for our chosen temporal dependence. For simplicity, we set the background mean winds to zero:  $\overline{U} = \overline{V} = 0$ . (This is equivalent to calculating the solution in the intrinsic frame of reference moving with the air at the ocean surface.) The Laplace transform of equation (36) is then

$$\mathcal{L}_{F} = \frac{1}{\chi} \left\{ \frac{\omega_{F} - e^{-s\chi}(s\sin\omega_{F}\chi + \omega_{F}\cos\omega_{F}\chi)}{(s^{2} + \omega_{F}^{2})} + \frac{\alpha - e^{-s\chi}(s\sin\alpha\chi + \alpha\cos\alpha\chi)}{2(s^{2} + \alpha^{2})} + \frac{\beta - e^{-s\chi}(s\sin\beta\chi + \beta\cos\beta\chi)}{2(s^{2} + \beta^{2})} \right\},$$
(40)

where

$$\alpha = -\omega_F - \hat{a},\tag{41}$$

$$\beta = -\omega_F + \hat{a}. \tag{42}$$

Here " $-\alpha$ " and " $-\beta$ " are the sum and difference frequencies of the fundamental ocean wave frequency ( $\omega_F$ ) with the envelope frequency ( $\hat{a}$ ), respectively.

Plugging equation (40) into equation (39), we obtain

$$\mathcal{L}_{\widetilde{\psi}} = -\frac{\delta F_{zs}}{\chi} \left\{ \mathcal{L}_{\widetilde{Q}(\omega_F)} + \frac{1}{2} \mathcal{L}_{\widetilde{Q}(\alpha)} + \frac{1}{2} \mathcal{L}_{\widetilde{Q}(\beta)} \right\},\tag{43}$$

where

$$\mathcal{L}_{\tilde{Q}(\Upsilon)} = \frac{(s^2 + f^2)[\Upsilon - e^{-s_{\chi}}(s\sin\Upsilon\chi + \omega_F\cos\Upsilon\chi)]}{(s^2 + \Upsilon^2)(s^2 - s_1^2)(s^2 - s_2^2)}$$
(44)



**Figure 1.**  $\mathcal{F}(t) \times 1000$  versus time for  $\tau_F = 15$  min (solid lines). We show (a)  $\chi = 1$  h and (b)  $\chi = 2$  h. The envelope function  $(1 - \cos at)$  (with n = 1) is shown as dotted lines (values are shown on the right-hand y axis).

and " $\Upsilon$ " represents the frequency  $\omega_F$ ,  $\alpha$ , or  $\beta$ . We rewrite equation (43) in terms of simple partial fractions (such as  $1/(s^2-s_1^2)$ ) [Abramowitz and Stegun, 1972]. We then take the inverse Laplace transform of equation (43) in a straightforward but tedious way and determine the pressure perturbation  $\tilde{\psi}$  as a function of  $\mathbf{k}$  and t. (Note that  $\tilde{\psi}$  is therefore not a function of the Laplace variable s). We then calculate the velocity and density perturbations  $\tilde{\xi}$ ,  $\tilde{\sigma}$ ,  $\tilde{\eta}$ , and  $\tilde{\phi}$  by analytically integrating equations (29)–(32). These perturbations are also functions of  $\mathbf{k}$  and t. The final forced solutions for  $t \ge 0$  are

$$\widetilde{\xi}_{F}(t) = -\frac{i\delta\widetilde{F}_{zs}}{\chi} \left\{ \frac{A}{(f^{2} - \omega_{F}^{2})} (k\omega_{F}\cos\omega_{F}t + lf\sin\omega_{F}t) + \frac{B}{(f^{2} - \alpha^{2})} (k\alpha\cos\alpha t + lf\sin\alpha t) + \frac{C}{(f^{2} - \beta^{2})} (k\beta\cos\beta t + lf\sin\beta t) + \frac{1}{(f^{2} - \omega_{1}^{2})} \left[ (k\omega_{1}D + lfE)\cos\omega_{1}t + (-k\omega_{1}E + lfD)\sin\omega_{1}t \right] + \frac{1}{(f^{2} - \omega_{2}^{2})} \left[ (k\omega_{2}F + lfG)\cos\omega_{2}t + (-k\omega_{2}G + lfF)\sin\omega_{2}t \right] \right\},$$

$$(45)$$

$$\widetilde{\sigma}_{F}(t) = -\frac{i\delta\widetilde{F}_{zs}}{\chi} \left\{ \frac{A}{(f^{2} - \omega_{F}^{2})} (l\omega_{F}\cos\omega_{F}t - kf\sin\omega_{F}t) + \frac{B}{(f^{2} - \alpha^{2})} (l\alpha\cos\alpha t - kf\sin\alpha t) \right. \\ \left. + \frac{C}{(f^{2} - \beta^{2})} (l\beta\cos\beta t - kf\sin\beta t) + \frac{1}{(f^{2} - \omega_{1}^{2})} \left[ (l\omega_{1}D - kfE)\cos\omega_{1}t - (l\omega_{1}E + kfD)\sin\omega_{1}t \right] \right. \\ \left. + \frac{1}{(f^{2} - \omega_{1}^{2})} \left[ (l\omega_{2}F - kfG)\cos\omega_{2}t - (l\omega_{2}G + kfF)\sin\omega_{2}t \right] \right\},$$

$$(46)$$

$$\widetilde{\eta}_{F}(t) = \frac{\left[N_{B}^{2} - c_{s}^{2}(m^{2} + \frac{1}{4H^{2}})\right]\widetilde{F}_{zs}}{\chi} \left\{ \frac{A\omega_{F}}{N_{B}^{2} - \omega_{F}^{2}} \cos \omega_{F}t + \frac{B\alpha}{N_{B}^{2} - \alpha^{2}} \cos \alpha t + \frac{C\beta}{N_{B}^{2} - \beta^{2}} \cos \beta t + \frac{\omega_{1}}{N_{B}^{2} - \omega_{1}^{2}} (D \cos \omega_{1} t - E \sin \omega_{1} t) + \frac{\omega_{2}}{N_{B}^{2} - \omega_{2}^{2}} (F \cos \omega_{2} t - G \sin \omega_{2} t)\right\} + \frac{(1 - \mathcal{Z})\widetilde{F}_{zs}}{\chi} \left\{ \frac{\omega_{F} \cos \omega_{F} t}{N_{B}^{2} - \omega_{F}^{2}} + \frac{\alpha \cos \alpha t}{2\left(N_{B}^{2} - \alpha^{2}\right)} + \frac{\beta \cos \beta t}{2\left(N_{B}^{2} - \beta^{2}\right)}\right\},$$

$$(47)$$

$$\widetilde{\psi}_{F}(t) = -\frac{\delta F_{zs}}{\chi} \left\{ A \sin \omega_{F} t + B \sin \alpha t + C \sin \beta t + D \sin \omega_{1} t + E \cos \omega_{1} t + F \sin \omega_{2} t + G \cos \omega_{2} t \right\},$$
(48)

$$\begin{split} \widetilde{\phi}_{F}(t) &= \frac{\delta \widetilde{F}_{zs}}{\chi c_{s}^{2}} \left\{ \frac{A[\omega_{F}^{2} - im_{s}(\gamma - 1)g]}{N_{B}^{2} - \omega_{F}^{2}} \sin \omega_{F} t + \frac{B[\alpha^{2} - im_{s}(\gamma - 1)g]}{N_{B}^{2} - \alpha^{2}} \sin \alpha t + \frac{C[\beta^{2} - im_{s}(\gamma - 1)g]}{N_{B}^{2} - \beta^{2}} \sin \beta t \\ &+ \frac{[\omega_{1}^{2} - im_{s}(\gamma - 1)g]}{N_{B}^{2} - \omega_{1}^{2}} (D \sin \omega_{1} t + E \cos \omega_{1} t) + \frac{[\omega_{2}^{2} - im_{s}(\gamma - 1)g]}{N_{B}^{2} - \omega_{2}^{2}} (F \sin \omega_{2} t + G \cos \omega_{2} t) \right\} \\ &+ \frac{(1 - \mathcal{Z})(\gamma - 1)g\widetilde{F}_{zs}}{\chi c_{s}^{2}} \left\{ \frac{\sin \omega_{F} t}{N_{B}^{2} - \omega_{F}^{2}} + \frac{\sin \alpha t}{2(N_{B}^{2} - \alpha^{2})} + \frac{\sin \beta t}{2(N_{B}^{2} - \beta^{2})} \right\}, \end{split}$$
(49)

where the subscript "F" denotes the forced solution, and  $\mathcal Z$  is the Heaviside step function:

$$\mathcal{Z} = \begin{cases} 0 \text{ for } 0 \le t < \chi \\ 1 \text{ for } t \ge \chi. \end{cases}$$
(50)

The time-independent coefficients are

$$A = \frac{(\mathcal{Z} - 1)(\omega_F^2 - f^2)}{(-\omega_1^2 + \omega_F^2)(-\omega_2^2 + \omega_F^2)},$$
(51)

$$B = \frac{(\mathcal{Z} - 1)(\alpha^2 - f^2)}{2(-\omega_1^2 + \alpha^2)(-\omega_2^2 + \alpha^2)},$$
(52)

$$C = \frac{(\mathcal{Z} - 1)(\beta^2 - f^2)}{2\left(-\omega_1^2 + \beta^2\right)\left(-\omega_2^2 + \beta^2\right)},$$
(53)

$$D = \frac{\left(-\omega_1^2 + f^2\right)}{\omega_1\left(-\omega_1^2 + \omega_2^2\right)} \left(\frac{\left(\omega_F - \mathcal{Z}p_1(\omega_F)\right)}{\left(\omega_F^2 - \omega_1^2\right)} + \frac{\left(\alpha - \mathcal{Z}p_1(\alpha)\right)}{2\left(\alpha^2 - \omega_1^2\right)} + \frac{\left(\beta - \mathcal{Z}p_1(\beta)\right)}{2\left(\beta^2 - \omega_1^2\right)}\right),\tag{54}$$

$$E = -\frac{\left(-\omega_1^2 + f^2\right)\mathcal{Z}}{\omega_1\left(-\omega_1^2 + \omega_2^2\right)}\left(\frac{q_1(\omega_F)}{\left(\omega_F^2 - \omega_1^2\right)} + \frac{q_1(\alpha)}{2\left(\alpha^2 - \omega_1^2\right)} + \frac{q_1(\beta)}{2\left(\beta^2 - \omega_1^2\right)}\right),\tag{55}$$

$$F = -\frac{\left(-\omega_{2}^{2} + f^{2}\right)}{\omega_{2}\left(-\omega_{1}^{2} + \omega_{2}^{2}\right)} \left(\frac{\left(\omega_{F} - \mathcal{Z}p_{2}(\omega_{F})\right)}{\left(\omega_{F}^{2} - \omega_{2}^{2}\right)} + \frac{\left(\alpha - \mathcal{Z}p_{2}(\alpha)\right)}{2\left(\alpha^{2} - \omega_{2}^{2}\right)} + \frac{\left(\beta - \mathcal{Z}p_{2}(\beta)\right)}{2\left(\beta^{2} - \omega_{2}^{2}\right)}\right),$$
(56)

$$G = \frac{\left(-\omega_{2}^{2} + f^{2}\right)\mathcal{Z}}{\omega_{2}\left(-\omega_{1}^{2} + \omega_{2}^{2}\right)}\left(\frac{q_{2}(\omega_{F})}{\left(\omega_{F}^{2} - \omega_{2}^{2}\right)} + \frac{q_{2}(\alpha)}{2\left(\alpha^{2} - \omega_{2}^{2}\right)} + \frac{q_{2}(\beta)}{2\left(\beta^{2} - \omega_{2}^{2}\right)}\right).$$
(57)

Additionally, the variables p and q are functions of  $\Upsilon$ , and are defined as:

$$p_{j}(\Upsilon) = \Upsilon \cos(\Upsilon \chi) \cos(\omega_{j} \chi) + \omega_{j} \sin(\Upsilon \chi) \sin(\omega_{j} \chi), \qquad (58)$$

$$q_{i}(\Upsilon) = -\Upsilon \cos(\Upsilon \chi) \sin(\omega_{i} \chi) + \omega_{i} \sin(\Upsilon \chi) \cos(\omega_{i} \chi),$$
(59)

where " $\Upsilon$ " is the frequency  $\omega_F$ ,  $\alpha$ , or  $\beta$ , and the subscript "j" is either "1" (GW) or "2" (AW). For t < 0 (prior to the wave packet),  $\tilde{\xi}_F = \tilde{\sigma}_F = \tilde{\eta}_F = \tilde{\varphi}_F = 0$ .

We dub equations (45)-(49) the exact "Fourier-Laplace" (FL) wave packet solutions because they were derived using Fourier and Laplace transform techniques. Note that these solutions are functions of the wave vector **k** and time *t*. It can be shown that for t = 0,  $\tilde{\xi}_F(0) = \tilde{\sigma}_F(0) = \tilde{\eta}_F(0) = \tilde{\psi}_F(0) = \tilde{\phi}_F(0) = 0$ . Special case solutions are given in Appendix A. When  $\omega_1$  or  $\omega_2$  equals  $\omega_F$ ,  $-\alpha$ , or  $-\beta$ , some of the coefficients in equations (51)–(57) (e.g., *A*, *B*) "blow up", which one might think would cause the solutions to "blow up." They do not, however, because of cancelations with factors in the numerators of the FL solutions. We have derived the solutions for  $\omega_1 = \omega_F + \epsilon$  as  $\epsilon \to 0$ ,  $\omega_1 = -\alpha + \epsilon$  as  $\epsilon \to 0$ , etc. to prevent this from occurring in the Fortran 90 model.

For completeness, we also derived the steady state ocean wave solutions for  $\mathcal{F}(t) = \sin(\omega_F t)$ . These solutions are contained in Appendices B and C.

In equations (45)–(49), all terms proportional to  $\cos \omega_1 t$  and  $\sin \omega_1 t$  represent freely propagating GWs, while all terms proportional to  $\cos \omega_2 t$  and  $\sin \omega_2 t$  represent freely propagating AWs. These waves obey the dispersion relation, equation (24). Those terms proportional to  $\cos \omega_F t$ ,  $\cos \alpha t$ ,  $\cos \beta t$ ,  $\sin \omega_F t$ ,  $\sin \alpha t$ , and  $\sin \beta t$  represent forced oscillations created by the vertical forcing. These forced oscillations are only nonzero while the force is occurring (i.e., for  $0 \le t \le \chi$  or  $\mathcal{Z} = 0$ ) and may or may not represent propagating waves

(i.e., GWs or AWs). Only for wave vectors (k, l, m) which satisfy equation (24) for  $\omega_1 = \omega_F$ ,  $\omega_1 = -\alpha$ ,  $\omega_1 = -\beta$ ,  $\omega_2 = \omega_F$ ,  $\omega_2 = -\alpha$ , or  $\omega_2 = -\beta$  do the forced oscillations propagate in the atmosphere away from the ocean surface as GWs or AWs.

Equations (45)–(49) show that the atmospheric waves (GWs and AWs) excited by an ocean surface wave packet (as modeled by a vertical body force oscillating at the fundamental frequency  $\omega_F$  for the duration  $\chi$ , as given by equation (36)) have the following frequencies: (i) the fundamental ocean wave frequency  $\omega_F$  (GWs if  $\omega_F < N_B$  and AWs if  $\omega_F > N_B$ ); (ii) the sum frequency  $|\alpha|$  (GWs if  $|\alpha| < N_B$  and AWs if  $|\alpha| > N_B$ ); (iii) the difference frequency  $|\beta|$  (GWs if  $|\beta| < N_B$  and AWs if  $|\beta| > N_B$ ); (iii) the spatial and temporal scales of the wave packet when the GW dispersion relation is satisfied.

Only the first set of GWs (with frequency  $\omega_F$ ) was included in recent "steady state" tsunami models [*Hickey et al.*, 2009; *Occhipinti et al.*, 2006, 2011], even though GWs having continuum frequencies are also excited by steady state ocean surface waves (see Appendix B). As we shall see in section 5, GWs having sum and continuum frequencies are very important for understanding the thermospheric response to an ocean surface wave packet. This includes the occurrence of GWs with phase speeds faster than that of the ocean wave. Certainly, a tsunami is a highly localized ocean wave packet, not a steady state ocean wave, and is therefore most accurately modeled as an ocean surface wave packet. Thus, the packet envelope provides complexity to the free and forced GW spectra that would not otherwise be present.

#### 3.3. Horizontal Phase Speeds of the Ocean Wave and Excited GWs

The horizontal phase speed of an ocean surface wave with horizontal wavelength  $\lambda_H$  and period  $\tau$  is [Gill, 1982]

$$c = \frac{\lambda_H}{\tau}.$$
 (60)

A typical value for the ocean depth is  $D \sim 5$  km [*Gill*, 1982; *Salmon*, 2014]. For the shallow water approximation (i.e., if  $\lambda_H \gg D$ ),  $c \propto \sqrt{D}$ . A tsunami obeys the shallow water approximation, because  $\lambda_H \sim$  several hundred kilometers. For this case, the phase velocity equals the group velocity [*Salmon*, 2014, page 27].

The horizontal phase speed of a GW is  $c_H = \omega_r/k_H$ , where  $k_H = 2\pi/\lambda_H$  [e.g., *Fritts and Alexander*, 2003]. This is the same as the phase speed for the ocean surface wave (see equation (60)). An ocean wave packet excites GWs having the fundamental, sum, difference, and continuum frequencies. The speed of those GWs having the fundamental frequency  $\omega_F$  and horizontal wavelength  $\lambda_H$  is

$$C_H(\omega_F) = \frac{\omega_F}{k_H} = \frac{\lambda_H}{\tau_F}.$$
(61)

Thus, detectors in the ocean and upper atmosphere will note the arrival of the ocean wave and these "fundamental" GWs nearly simultaneously. On the other hand, the horizontal phase speed of the "sum" GWs is

$$c_H(|\alpha|) = \frac{|\alpha|}{k_H} = \frac{\lambda_H}{\tau_F} + \frac{\lambda_H}{\chi} = c_H(\omega_F) + \frac{\lambda_H}{\chi}.$$
(62)

Therefore, the horizontal phase speed of the sum GWs is larger than that of the ocean wave by  $\lambda_H/\chi$ . For example, if  $\lambda_H = 200$  km and  $\chi = 40$  min, this "addition" to the horizontal phase speed is quite large:  $\lambda_H/\chi = 83$  m/s. Note that these sum GWs could have played a role in the fast GWs observed by *Makela et al.* [2011]. In that work, GWs likely excited by the tsunami were observed in the ionosphere ~1 h before the tsunami reached Hawaii (see section 6).

#### 3.4. GWs Excited by a Localized, Vertically Oscillating Ocean Surface Wave

Consider an ocean surface wave with horizontal wavelength  $\lambda_{H}$  that oscillates vertically at the fundamental frequency  $\omega_{F}$  over the duration  $\chi$ . The angle perpendicular to the phase lines is  $\theta$  measured counterclockwise from east. As we will see, this oscillation excites GWs which propagate approximately parallel to and antiparallel to  $\theta$ .

We model the effect the oscillating ocean surface has on the atmosphere as a vertical body force with a center at  $(x_0, y_0, z_0)$ . We first translate our coordinate system into one that is centered on the force:

$$x' = x - x_0,$$
  
 $y' = y - y_0,$   
 $z' = z - z_0.$  (63)

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**Figure 2.** GW temperature perturbations, T', at z = 10 km and at t = 31, 41, 51, and 61 min for the exact FL solutions for the vertical body force given by equation (65) with  $\lambda_H = 10$  km,  $\chi = 25$  min,  $\tau_F = 10$  min,  $\theta = 0$ ,  $w_{ocean} = 0.01$  m/s,  $\sigma_z = 1.0$  m,  $x_L = \lambda_H$ , and  $y_L = 2\lambda_H$ . Minimum and maximum values are shown in the title of each image.

Next, we rotate the coordinate system by  $\theta$ 

$$x'' = x' \cos \theta + y' \sin \theta,$$
  

$$y'' = -x' \sin \theta + y' \cos \theta.$$
(64)

Here x'' and y'' are the coordinates perpendicular and parallel to the ocean surface phase lines, respectively. We define the spatial portion of the force to be

$$F_z = F_0 \sin(k_H x'') \exp\left[-\left(\frac{x''}{x_L}\right)^2 - \left(\frac{y''}{y_L}\right)^2\right] \exp\left[-\frac{(z')^2}{2\sigma_z^2}\right],\tag{65}$$

where  $F_0$  is amplitude and  $x_L$  and  $y_L$  are x'' and y'' length scales of the force, respectively. Here we chose a Gaussian function to model the vertical acceleration of air above the ocean surface. For this function, the air above the surface is significantly accelerated over the vertical distance  $\sim 2\sigma_z$ . It is likely that  $\sigma_z$  cannot be much smaller than  $\frac{1}{2}h_0$ , and that  $\sigma_z$  cannot be larger than  $\sim 10h_0$  because air parcels above that height are unlikely to directly "feel" this oscillation. Although studies have examined the ocean/atmosphere coupling for small-scale ocean waves [e.g., *Hristov et al.*, 2003], similar studies for medium-scale ocean waves (i.e.,  $\lambda_H \sim$ hundreds of kilometers) do not seem to exist. Therefore, we treat  $\sigma_z$  as a free parameter in this paper. We discuss how this parameter might be determined in section 6.

In order to show that the reconstructed ray trace solutions give the same solutions as the exact FL solutions, we consider a small-scale force given by equation (65) with  $\lambda_H = 10$  km,  $\chi = 25$  min,  $\tau_F = 10$  min,  $\theta = 0$ ,  $\sigma_z = 1.0$  m,  $x_L = \lambda_H$ ,  $y_L = 2\lambda_H$ ,  $x_0 = y_0 = 0$ , and  $z_0 = 4.0$  m. The background atmosphere is isothermal, inviscid, and windless, with  $\overline{T}_0 = 239$  K,  $N_B = 0.02$ rad/s, f = 0,  $\gamma = 1.4$ ,  $X_{MW} = 28.9$ , g = 9.8m/s<sup>2</sup>, and sound speed  $c_s = 310$ m/s. The number of grid points in the x, y, and z directions are  $n_x = n_y = 64$  and  $n_z = 8192$ .

The grid spans x = [-57.1, 57.1] km, y = [-57.1, 57.1] km, and z = [-10.7, 16.7] km, with grid sizes of  $\Delta x = \Delta y = 1.8$  km and  $\Delta z = 3.3$  m. Most of the grid points are in the vertical direction in order to resolve both the force and the excited GWs. We set the maximum vertical velocity of the atmosphere just above the ocean surface to be  $w_{\text{ocean}} = 0.01$  m/s. This equals the maximum vertical velocity of the ocean surface wave packet. This requires setting  $F_0 = 119.9$ m/s (via solving equations (47) and (6)). Writing the maximum vertical velocity of the ocean surface wave at a fixed location as  $w' = w_{\text{ocean}} \cos(\omega_F t)$ , the maximum vertical displacement of the ocean surface due to the wave is then

$$h_0 = \int_0^{\tau_F/4} w' \mathrm{d}t = \frac{W_{\text{ocean}}}{\omega_F} \sin(\omega_F t) \big|_0^{\tau_F/4} = \frac{W_{\text{ocean}} \tau_F}{2\pi}.$$
 (66)

This ocean wave packet has a relatively large height of  $h_0 = 0.95$  m. However, because this height perturbation is spread out over the large horizontal distance  $\sim \lambda_H/4$ , this wave would not cause a significant tilt to the ocean surface in the open ocean. Figure 2 shows the exact FL GW temperature perturbations, T', at z = 10 km. Here T' is obtained by combining equations (17), (48), and (49), taking the inverse Fourier transform of  $\tilde{\zeta}$ , then utilizing equation (6). The excited GWs have linear phase lines with  $\lambda_H = 10$  km. The GWs at x > 0 propagate eastward from the force region, while those at x < 0 propagate westward. The excited GWs appear to propagate horizontally within wave packets, with the center of the eastward (westward) packet moving eastward (westward) in time. However, this is a false appearance because GWs move vertically and horizontally simultaneously. In reality, the T' maxima moves away from the force center because the slower GWs have larger periods and therefore propagate with larger angles from the vertical in a windless, isothermal atmosphere [e.g., *Vadas et al.*, 2009]:

a

$$p_{lr} \sim N_B \cos \Psi,$$
 (67)

where  $\Psi$  is the propagation angle of the GW from the vertical.

#### 4. Ray Tracing and Reconstruction of the GW Field

In this section, we describe a new scheme developed to ray trace the GWs excited by an ocean surface wave packet into the atmosphere. This scheme is quite different from that used previously for a convective plume; there, we inserted a single GW spectrum at the central location and time of the plume [*Vadas and Fritts*, 2009; *Vadas and Liu*, 2009, 2013; *Vadas et al.*, 2014]. That approach will not work here, however, because the ocean surface wave packet is spread out horizontally and evolves in time. We also determine the normalization factor needed to convert the GW spectral amplitudes to the real-space amplitudes via comparison with the exact FL solutions for the small-scale vertical body force considered in the previous section.

#### 4.1. Sprinkling of GW Spectra and Setup for Ray Tracing

We wish to model the GWs excited by the body force given by equation (65) with  $\theta = 0$ . Because of the nonspatial localization of the Fourier transform, we cannot relate the reduced-amplitude GWs near the edges of the ocean wave packet with their excitation location. Instead, we follow a different approach. We calculate the GW spectrum excited by a nonspatially localized vertical body force having periodic variations in the *x* direction:

$$F_z = F_0 \sin(kx)Q(z), \tag{68}$$

where the function Q contains the vertical dependence of this force. This spectrum is computed in Fourier space from equations (45)–(49) when the body force is finished (i.e., at  $t = \chi$ ), and so is a function of the wave vector (k(0), l(0), m(0)). Here the argument "(0)" refers to  $\theta = 0$ . The average GW momentum flux amplitude for each wave vector is then computed by averaging each GW's momentum flux from  $t = \chi$  to  $t = \chi + 2\pi/\omega$  numerically. Here the frequency  $\omega$  is computed from the anelastic GW dispersion relation for the wave vector (k(0), l(0), m(0)). Because there is no y variation in equation (68), we only need two grid points in the y direction to calculate the excited GW spectrum. (Although this force only varies in x, it will be generalized to any angle  $\theta$  via a rotation transformation, as described below.) The zonal and meridional momentum fluxes for each GW in the spectrum are  $\tilde{\xi}\tilde{\eta}\Delta k\Delta/\Delta m$  and  $\tilde{\sigma}\tilde{\eta}\Delta k\Delta/\Delta m$ , respectively. Here  $\Delta k$ ,  $\Delta l$ , and  $\Delta m$  are the k, l, and m spectral grid sizes, respectively, in the Fourier transform used to calculate the excited GW spectrum [*Vadas and Fritts*, 2009, equation (16)]:

$$\Delta k = \frac{2\pi}{n_x \Delta x}, \quad \Delta l = \frac{2\pi}{n_y \Delta y}, \quad \Delta m = \frac{2\pi}{n_z \Delta z}, \tag{69}$$

where  $n_x \Delta x$ ,  $n_y \Delta y$ , and  $n_z \Delta z$  are the x, y, and z domain lengths, respectively.

We now determine the GW spectrum for a body force with horizontal variation in the direction determined by the angle  $\theta$  (counterclockwise from east):

$$F_{z} = F_{0} \sin(k_{H} x'') Q(z).$$
(70)

This is accomplished by applying a rotation transformation to the horizontal wave vector and momentum flux vector:

$$k(\theta) = k(0)\cos\theta - l(0)\sin\theta, \tag{71}$$

$$l(\theta) = k(0)\sin\theta + l(0)\cos\theta, \tag{72}$$

$$\overline{\widetilde{\xi}\widetilde{\eta}}(\theta) = \overline{\widetilde{\xi}\widetilde{\eta}}(0)\cos\theta - \overline{\widetilde{\sigma}\widetilde{\eta}}(0)\sin\theta,$$
(73)

$$\overline{\widetilde{\sigma}\widetilde{\eta}}(\theta) = \overline{\widetilde{\xi}\widetilde{\eta}}(0)\sin\theta + \overline{\widetilde{\sigma}\widetilde{\eta}}(0)\cos\theta.$$
(74)

We then incorporate the spectral localization of the force given by equation (65) by "sprinkling" N of these GW spectra throughout the approximate region for the ocean wave packet via randomly selecting x'' and y'' between

$$x'' = [-2x_L, 2x_L]$$
 and  $y'' = [-2y_L, 2y_L],$  (75)

then determining x and y from equations (63)–(64). We also randomly select the launch time t between

t

$$= [0, \chi], \tag{76}$$

and set z to be a fixed value.

We now incorporate the amplitude dependence of the force,  $\exp[-(x''/x_L)^2 - (y''/y_L)^2]$ , into our ray trace model. From equations (45)–(49), a GW's amplitude is proportional to the force amplitude. Thus, the amplitude of a GW at  $x'' = -x_L$  should be smaller by  $\sim e^{(-1)} = 0.37$  than if the GW was located at x'' = 0. Therefore, we weight each GW's amplitude by  $\exp[-(x''/x_L)^2 - (y''/y_L)^2]$ . Additionally, the amplitude of a GW launched at  $t = \chi/10$  should be smaller than if it was launched at  $t = \chi/2$ . Thus, we weight each GW's amplitude by  $\sin^2(\pi t/\chi)$  from equation (36), since  $1 - \cos at = 2 \sin^2(\pi t/\chi)$  for n = 1. Finally, to simplify our computations, we calculate the GW spectrum for the fixed (arbitrary) forcing amplitude of  $F_0 = 100$ m/s. Then, we compute the atmospheric response just above the ocean surface wave for this value of  $F_0$  to determine the maximum vertical velocity of the ocean surface wave, which we call  $w_{max}$ . If we wish to model an ocean wave with a vertical velocity of  $w_{ocean}$  instead of  $w_{max}$ , we would then weight each GW's amplitude by  $w_{ocean}/w_{max}$ . Combining all of these factors, for a given launch location (x, y, z) and launch time t, we weight each GW's amplitude by

$$\exp[-(x''/x_L)^2 - (y''/y_L)^2]\}\sin^2(\pi t/\chi)(w_{\text{ocean}}/w_{\text{max}}).$$
(77)

Then the spectral momentum flux amplitude for each GW,  $\widetilde{u_H'}\widetilde{w'}$ , equals the amplitude of the FL solution,  $(\widetilde{u_H'}\widetilde{w'})_F$ , times the square of the factors in equation (77):

$$\overline{\widetilde{u_H'w'}} = (\overline{\widetilde{u_H'w'}})_F \frac{\left\{ \exp[-(x''/x_L)^2 - (y''/y_L)^2] \right\}^2 \sin^4(\pi t/\chi)}{W} \left( \frac{w_{\text{ocean}}}{w_{\text{max}}} \right)^2.$$
(78)

Here W is the sum of the weights over all  $\mathcal N$  launch locations and times,

$$W = \sum_{i=1}^{N} \left\{ \exp[-(x''/x_L)^2 - (y''/y_L)^2] \right\}^2 \sin^4(\pi t/\chi),$$
(79)

 $u_{H}'$  is the GW horizontal velocity perturbation, and  $(\overline{u_{H}'\widetilde{w'}})_{F} = \sqrt{(\widetilde{\xi}\widetilde{\eta})^{2} + (\widetilde{\sigma}\widetilde{\eta})^{2}}$ . If a different force is modeled instead, the function inside the curly brackets in equations (77)–(79) will differ. We discuss how to convert the spectral to real-space GW momentum flux in section 4.2.

Finally, we set the initial phase for each GW. The spatial portion is

$$\Phi_{xy} = \begin{cases} (kx + ly) & \text{for } k_{\text{dom}} > 0, \\ -(kx + ly) & \text{for } k_{\text{dom}} < 0, \end{cases}$$
(80)

where  $k_{dom} = k$  if the GW is mainly propagating in the *x* direction (i.e., |k| > |I|), and  $k_{dom} = I$  if the GW is mainly propagating in the *y* direction (i.e., |I| > |k|). The initial phase for each GW is then

Φ

$$=\Phi_{xy}-\omega_r t.$$
(81)

#### 4.2. Excitation, Ray Tracing, and Reconstruction of the GW Field in the Troposphere

We calculate the GWs excited by the same small- $\lambda_H$  body force as used for Figure 2. We utilize equation (70) with

$$Q(z) = \exp\left[-\frac{(z')^2}{2\sigma_z^2}\right]$$
(82)

and set  $\lambda_H = 10$  km,  $\chi = 25$  min,  $\tau_F = 10$  min,  $\theta = 0$ ,  $\sigma_z = 1.0$  m,  $w_{\text{ocean}} = 0.01$  m/s,  $x_L = \lambda_H$ ,  $y_L = 2\lambda_H$ ,  $x_0 = y_0 = 0$ , and  $z_0 = 4$  m. (Note that for this body force with  $F_0 = 100$ m/s,  $w_{\text{max}} = 0.01$  m/s.) The atmosphere we utilize for generating the GW spectrum is the same as used in Figure 2. We employ a grid with  $n_x = 16$ ,  $n_y = 2$ , and  $n_z = 8192$  points in the x, y, and z directions, respectively. The grid lengths are x = [-20, 20] km, y = [-2.5, 2.5] km, and z = [-5, 13] km, and the grid sizes are  $\Delta x = \Delta y = 2.5$  km and  $\Delta z = 2.3$  m. Here  $|\lambda_x|$  ranges from 5 to 40 km and  $|\lambda_z|$  ranges from 4 to 18 km. Each spectrum contains  $n_x n_y n_z/2 \sim 130,000$  upward propagating GWs. Additionally, the z range extends to negative values in order to prevent artificial truncation (in  $\lambda_z$ ) of the GW spectrum. Because the GW momentum fluxes from the ray trace model must equal the GW momentum fluxes from the FL model, we divide the binned, spectral zonal and meridional GW momentum fluxes,  $\xi \tilde{\chi} \Delta k \Delta l \Delta m$  (which have units of momentum flux times a volume), respectively by the volume factor

$$\Xi = \Delta x \Delta y \Delta z \Delta t / \mathcal{G}.$$
(83)

Here  $\Delta x$ ,  $\Delta y$ ,  $\Delta z$ , and  $\Delta t$  are the cell sizes for the grid in the ray trace model, and *G* is the normalization factor with units of time. The zonal and meridional GW momentum fluxes (per unit mass) in real space at the cell located at (*x*, *y*, *z*, *t*) are then

$$\overline{u'w'}(x,y,z,t) = \Sigma(\overline{\xi}\widetilde{\eta}\Delta k\Delta l\Delta m/\Xi), \quad \overline{v'w'}(x,y,z,t) = \Sigma(\overline{\widetilde{\sigma}\eta}\Delta k\Delta l\Delta m/\Xi),$$
(84)

respectively, where the sums are over all of the GWs that enter this xyzt cell.

By comparing the reconstructed ray trace solutions with the exact FL solutions (see section 4.3), we find that

$$G = 150 \times (2/n_v) \,\mathrm{s}$$
 (85)

best reproduces the exact FL solutions. This is similar to the factor currently employed in this ray trace model for convective plumes (i.e., G = 60 s) [*Vadas et al.*, 2012, 2014].

Figure 3 shows the zonal component of the GW spectrum (normalized to an arbitrary number):

$$km\tilde{\xi}\tilde{\eta}\Delta k\Delta l\Delta m/\Xi.$$
 (86)

Here we multiply by *km* because a GW on any contour in the spectrum has the same momentum flux (per unit mass) in real space at the altitude *z* in an isothermal, windless atmosphere [*Vadas and Fritts*, 2009]. Although only the GWs with  $|\lambda_x| = 10$  km have relatively large momentum fluxes, the spectrum contains a range of  $\lambda_z$  and intrinsic periods  $\tau_{lr} = 2\pi/\omega_{lr}$ . The sum, fundamental, and difference periods are 7, 10, and 17 min, respectively. The sum GWs are seen as a distinct peak at  $|\lambda_z| \simeq 8$  km and  $\tau_{lr} \simeq 7$  min. There is a gap, followed by the fundamental GWs at  $|\lambda_z| \simeq 5$  km and  $\tau_{lr} \simeq 10$  min. For smaller  $|\lambda_z|$  and larger  $\tau_{lr}$ , the momentum flux decreases rapidly, with a faster decline initially followed by a slower decline for  $\tau_{lr} \ge 20$  min. The GWs with  $\tau_{lr} > 10$  min contain both continuum and difference frequencies, although there is no obvious peak at the difference frequency.

Dispersion causes GWs with different wavelengths and periods to travel at different speeds, with estimated vertical group velocities of [*Fritts and Alexander*, 2003]

$$c_{gz} \sim \frac{\omega_{lr}}{|m|} \sim \frac{k_H N_B}{m^2} \sim \frac{\lambda_z^2}{\lambda_H \tau_B}.$$
(87)

The time it takes to reach the altitude *z* is then  $t = \int dz/c_{gz} \simeq z/c_{gz}$ . From Figure 3, we estimate the propagation time to reach z = 10 km to be t = 8 min (for the sum waves), 20 min (for the fundamental waves), ~1 h (for the difference waves), and ~3.7 h for the GWs with  $|\lambda_z| \sim 1.5$  km. Thus, GW activity is expected to last for nearly 4 h at this altitude because of wave dispersion and the complexity of the GW spectrum.

We sprinkle  $\mathcal{N} = 200$  of these (identical) GW spectra throughout the ocean wave packet region as given by equations (75)–(76), set z = 0.1 km, and weight each GW's amplitude by equation (77). Here we choose a large enough value for  $\mathcal{N}$  so that the reconstructed GW field in the troposphere converges, is smooth, and



**Figure 3.** Normalized momentum flux spectra (using an arbitrary normalization factor) for the GWs excited by an ocean surface wave packet with  $\lambda_H = 10$  km,  $\chi = 25$  min and  $\tau_F = 10$  min. (a) Momentum flux spectrum summed over  $\lambda_z$  for each  $\lambda_H$  bin. (b) Momentum flux spectrum summed over  $\lambda_H$  for each  $|\lambda_z|$  bin. (c) Momentum flux spectrum at  $\lambda_H = 10$  km as a function of wave period.

looks similar to the exact solution (see section 4.3). Figure 4a shows the random locations and times of the GW spectra, and Figure 4b shows the corresponding weight factors. Note that  $k_{dom} = k$  because the excited GWs propagate mainly zonally for this force.

We then ray trace every upward propagating GW from all  $\mathcal{N}$  spectra from these locations and times and calculate the average GW momentum fluxes, wave vectors, and phases in each cell. These cells span a four-dimensional (4-D) "grid" (in *x*, *y*, *z*, and *t*), and are required in order to reconstruct the GW field via inserting the average GW momentum fluxes, wave vectors, and phases into the GW dissipative polarization relations [*Vadas and Fritts*, 2005, 2009]. Because we need the average wave vectors, we ray trace those GWs having k > 0 (eastward) and k < 0 (westward) separately so that the average  $\overline{k}$  and  $\overline{l}$  values calculated in each *xyzt* "cell" are accurate; they otherwise might incorrectly average to zero if GWs traveling in opposite directions propagate through the same cell. (Note that GWs which are evanescent, reach critical levels, or leave the 4-D grid are automatically eliminated by the ray trace model.)

We ray trace the GWs from the initial height up to z = 12 km. The 4-D grid that we employ for reconstructing the GW field spans x = [-60, 60] km, y = [-60, 60] km, z = [4, 12] km, and t = [0, 90] min. The number of cells in x, y, z, and t are  $N_x = N_y = 240$ ,  $N_z = 10$ , and  $N_t = 45$ , yielding cell sizes of  $\Delta_x = \Delta_y = 0.5$  km,  $\Delta_z = 0.8$  km,



**Figure 4.** (a) Locations of the  $\mathcal{N} = 200$  GW spectra (diamonds). The times are shown for each spectrum using colors from t = 0 (blue) to t = 25 min (red). (b) Same as Figure 4a but the colors show the weight factors  $\exp[-(x''/x_L)^2 - (y''/y_L)^2]\sin^2(\pi t/\chi)$  from 0 (blue) to 1 (red).



**Figure 5.** T' for the (a) eastward and (b) westward GWs at z = 10 km and t = 31 min created by an ocean wave packet with  $\lambda_H = 10$  km,  $\chi = 25$  min,  $\tau_F = 10$  min,  $\theta = 0$ ,  $w_{ocean} = 0.01$  m/s,  $\sigma_z = 1.0$  m,  $x_L = \lambda_H$ , and  $y_L = 2\lambda_H$ . (c) The sum of Figures 5a and 5b. (d–f) Same as Figures 5a–5c but for t = 41 min. (g–i) Same as Figures 5a–5c but for t = 51 min. (j–l) Same as Figures 5a–5c but for t = 51 min. (j–l) Same as Figures 5a–5c but for t = 61 min. Minimum and maximum values are shown in the title of each image.



**Figure 6.** Test case comparison of T' for the FL solutions and reconstructed ray trace solutions at y = 0 and z = 10 km at (a) t = 31, (b) 41, (c) 51, and (d) 61 min. The FL solutions are shown as solid lines, while the reconstructed ray trace solutions are shown as triangles.

and  $\Delta_t = 2$  min. It may seem that the *z* cell size is too large, since  $\sigma_z$  is very small in comparison. However, the GWs excited by this force have  $|\lambda_z| \sim 1.5 - 9$  km from Figure 3. Thus, our vertical cell size is adequate to resolve the excited GWs. This brings up an important point— that the GW vertical wavelengths are not related to the ocean wave height. Instead, the GW amplitudes are proportional to the ocean wave height. A GW's vertical wavelength is determined solely from the GW dispersion relation, given  $\lambda_H$  and  $c_H$ .

The atmosphere we utilize for ray tracing is the same as in Figure 2. Figure 5 shows T' at z = 10 km for the same times as in Figure 3. Figures 5a, 5d, 5g, and 5j show that the eastward GWs appear to move eastward as a distinct wave packet with a center at  $x \sim 10$ , 20, and 30 km for t = 31, 41, and 51 min, respectively. A similar phenomenon occurs for the westward GWs. As before, this apparent horizontal motion occurs because slower GWs with larger periods reach this altitude (at later times) further from the force center. Figures 5c, 5f, 5i, and 5l show the total T' for the eastward plus westward GWs. Eastward and westward GW packets appear to propagate away from the vertical body force at this altitude, as expected, yielding results similar to Figure 2.

#### 4.3. Comparison of the Reconstructed Ray Trace and Exact FL Solutions

Figure 6 shows a comparison of T' for the reconstructed ray trace solutions and the exact FL solutions. In general, the ray trace solutions are quite similar to the FL solutions, although there tends to be a phase mismatch at earlier times (t < 51 min). Additionally, some differences occur for  $|x| \ge 20$  km at t = 61 min. This may occur because the periodic boundary conditions in the FL solutions at  $x = \pm 57$  km cause GWs to reflect back toward x = 0, thereby creating larger T' at large |x| and t. However, there are no periodic boundary conditions in the ray trace model; if a GW goes outside the grid, it is eliminated. Note that the value of G in equation (85) was chosen so that the reconstructed ray trace and FL solutions would agree well in Figure 6.



**Figure 7.** Normalized momentum flux spectra (using an arbitrary normalization factor) for the GWs excited by an ocean surface wave packet with  $\lambda_H = 190$  km,  $\tau_F = 14$  min and  $\chi = 30$  min. (a) Momentum flux spectrum summed over  $\lambda_z$  for each  $\lambda_H$  bin. (b) Momentum flux spectrum summed over  $\lambda_H$  for each  $|\lambda_z|$  bin. (c) Momentum flux spectrum at  $\lambda_H = 190$  km as a function of the intrinsic wave period. (d) Momentum flux spectrum at  $\lambda_H = 190$  km as a function of the intrinsic horizontal phase speed.

#### 5. Excitation, Ray Tracing, and Reconstruction of the GW Field in the Thermosphere

While the exact FL solutions in real space (i.e., in x, y, z, and t) are important, they cannot be used to calculate the GW field in the thermosphere because they cannot incorporate horizontal wind, temperature, and viscosity variations. The ray trace model, however, can incorporate these effects. We therefore utilize the FL spectral solutions to calculate the excited GW spectrum from an ocean wave packet in spectral space (i.e., in k, l, m, and t). We then sprinkle these spectra into the ocean wave packet region, ray trace the GWs into the thermosphere through realistic winds, temperatures, and viscosity, and reconstruct the GW field there.

#### 5.1. GW Field in the Thermosphere From a Medium-Scale Ocean Wave Packet

We study the excitation and propagation of GWs from a southward moving, medium-scale ocean wave packet with  $\lambda_{H} = 190$  km,  $\tau_{F} = 14$  min,  $\chi = 30$  min, and phase speed  $c = \omega_{F}/\lambda_{H} = 226$  m/s. Such a packet would excite GWs similar to the waves observed by *Makela et al.* [2011] from the Tohoku-Oki tsunami. We model this packet's effect on the atmosphere via the force given by equations (70) and (82), with  $\lambda_{H} = 190$  km,  $\tau_{F} = 14$  min,  $\chi = 30$  min,  $w_{ocean} = 0.01$  m/s,  $\theta = 90^{\circ}$ ,  $\sigma_{z} = 1.0$  m,  $x_{L} = \lambda_{H}$ ,  $y_{L} = 3\lambda_{H}$ ,  $x_{0} = y_{0} = 0$ , and  $z_{0} = 4.0$  m. Here we choose  $\sigma_{z} = 1.0$  m because we are interested in an ocean wave height that is a fraction of a meter to several meters. We calculate the GW spectrum using an arbitrary force amplitude of  $F_{0} = 100$  m/s, which results in the maximum upward velocity of  $w_{max} = 3.2 \times 10^{-3}$  m/s. From equation (66), the ocean wave height is  $h_{0} = 1.3$  m. Again, in the open ocean the tilt of the ocean surface caused by this wave is very small because  $\lambda_{H}/4 \gg h_{0}$ . As was seen in Figure 2, a single force creates GWs which propagate parallel and antiparallel to the direction given by the angle  $\theta$ . In Appendix D, we show how the solutions derived in section 3.2 can be used to model a propagating plane ocean wave. This involves adding a second force that is phase shifted by 1/4 of a wavelength and 1/4 of a period, which results in the cancelation of the antiparallel GWs and the amplitude doubling of the parallel GWs, for example. The waveform is otherwise unchanged. Therefore, to



**Figure 8.** Background atmosphere for ray tracing. (a) Temperature  $\overline{T}$ . (b) The mean molecular weight  $X_{MW}$  (in g/mol) (solid line, lower *x* axis) and the ratio of mean specific heats  $\gamma$  (dashed line, upper *x* axis) as a function of  $-\ln(\overline{\rho})$ , where  $\overline{\rho}$  has units of g/m<sup>3</sup> (left-hand y axis). The corresponding altitudes *z* (in km) are shown on the right-hand *y* axis. (c) Mean background density  $\overline{\rho}$ . (d) Density-scale height  $\mathcal{H}$ . (e) Sound speed  $c_s$ .

increase the speed of the ray trace calculations, we instead utilize a single force and ray trace only the parallel or antiparallel GWs in this paper.

For generating the GW spectrum, we use a grid with  $n_x = 4$ ,  $n_y = 2$ ,  $n_z = 131072$ , x = [-95, 95] km, y = [-47.5, 47.5] km, z = [-121, 141] km,  $\Delta x = \Delta y = 47.5$  km, and  $\Delta z = 2$  m. This grid is centered at z = 5 km and extends "below the Earth's surface" to z = -121 km in order that the upward and downward propagating, large- $\lambda_z$  GWs are not truncated in  $|\lambda_z|$  (i.e., are properly resolved). The largest  $\lambda_H$  we can excite here is  $\lambda_H = 190$  km. The atmosphere we utilize is isothermal, inviscid, and windless, with  $\overline{T}_0 = 239$  K,  $N_B = 0.02$  rad/s, f = 0,  $\gamma = 1.4$ , diatomic  $X_{MW} = 28.9$ , g = 9.8m/s<sup>2</sup>, and  $c_s = 310$ m/s.

Figure 7 shows the momentum flux spectrum of the excited GWs (with an arbitrary normalization factor) using equation (86). The spectrum consists of GWs with  $\lambda_{H} = 190$  km only and contains discrete and continuous values of  $\lambda_{z}$ ,  $\tau_{Ir}$ , and intrinsic phase speeds  $c_{IH}$ . The sum and difference periods are  $2\pi/|\alpha| = 9.5$  min and  $2\pi/|\beta| = 26$  min, respectively, from equation (42). The fundamental and difference GWs have  $|\lambda_{z}| = 133$  and 43 km, respectively. For  $\lambda_{H} = 190$  km and  $\mathcal{H} = 7$  km, the smallest period allowed by the GW dispersion relation that is excited by the ocean wave packet is ~12.3 min, which corresponds to  $|\lambda_{z}| = 263$  km. Thus, this ocean



**Figure 9.** Reconstructed T' at z = 250 km for the southward propagating GWs every 10 min from t = 68 to 178 min, as labeled. The ocean wave packet has  $\tau_F = 14$  min,  $\lambda_H = 190$  km, and  $\chi = 30$  min.



**Figure 10.** (a) Maximum T' at z = 250 km for the GWs excited by an ocean wave packet with  $\tau_F = 14$  min,  $\lambda_H = 190$  km, and  $\chi = 30$  min. The average (b)  $\tau_{Ir}$ , (c) meridional wavelength  $\lambda_y$ , (d) vertical wavelength  $\lambda_{zr}$  (e) meridional phase speed  $c_y = \omega_r / \lambda_y$ , and (f) vertical phase speed  $c_z = \omega_r / \lambda_z$  are shown at the location where T' is maximum in Figure 10a.

wave packet cannot excite a sum GW. Note that  $2\pi H \sim 44$  km. Therefore, the Boussinesq approximation (i.e.,  $|\lambda_z| << 2\pi H$ ) does not hold for those GWs with intrinsic periods  $\tau_{lr} < 25$  min, which includes the fundamental GWs. Using  $c_{lH} = \lambda_H / \tau_{lr}$ , we compute intrinsic horizontal phase speeds of  $c_{lH} \simeq 258$ , 226, and 122 m/s for the GWs with  $\tau_{lr} = 12.3$ , 14, and 26 min.

The  $\tau_{lr} = 12.3$  min GWs have momentum flux amplitudes that are ~ 25% that of the fundamental GWs in Figures 7c and 7d. These GWs are followed by the fundamental GWs. There is another gap in the spectrum, followed by continuum GWs having  $\tau_{lr} \sim 16-20$  min,  $|\lambda_z| \sim 60-90$  km, and  $c_{lH} \simeq 160-195$  m/s. This is followed by the rest of the continuum GWs having  $\tau_{lr} \sim 20-120$  min,  $|\lambda_z| \sim 5-60$  km, and  $c_{lH} \simeq 20-160$  m/s. Thus, the continuum GWs have  $\omega_{lr} < \omega_F$  here and have momentum flux amplitudes that decrease rapidly as  $\tau_{lr}$  increases. The difference GWs only appear as a small bump in the spectrum. The momentum flux is maximum for  $\tau_{lr} = 16.5$  min but is only slightly smaller for  $\tau_F = 14$  min. Note that if we set  $n_x = 16$  and increase the *x* grid length by a factor of 4 to allow for the excitation of larger- $\lambda_H$  GWs, we find that only GWs with  $\lambda_H = 190$  km have significant momentum flux amplitudes (not shown).

We now ray trace the GWs into the atmosphere according to the procedure detailed in section 4.1. We insert each GW spectrum at  $\mathcal{N} = 4000$  random locations and times. (Here  $\mathcal{N}$  is large enough to ensure that the reconstructed GW field in the thermosphere converges and is smooth. It is much larger than that needed in



**Figure 11.** T' at x = 0 and z = 250 km for the GWs excited by an ocean wave packet with  $\tau_F = 14$  min,  $\lambda_H = 190$  km, and  $\chi = 30$  min every 2.5 min from t = 70.75 to 190.75 min. Profiles are offset by 10 K. The upper x axis shows the time t. The dotted line shows the location of the ocean wave packet.

the troposphere because of wave dispersion.) We specify x'', y'', and t from equations (75) to (76), weight each GW's amplitude by equation (77), and set the launch altitude to be z = 0.1 km. Here we only ray trace the southward propagating GWs (i.e., with  $l \le 0$ ).

In order to understand the basic features of the GW response in the thermosphere, we choose a simple, hyperbolic background temperature profile given by equation (10) of *Vadas and Fritts* [2006] for ray tracing:

$$\overline{T}(z) = \overline{T}_{\max} + (\overline{T}_0 - \overline{T}_{\max}) \left[ \frac{1}{2} \left( 1 - \tanh\left(\frac{z - z_\Delta}{\Delta}\right) \right) \right]^B,$$
(88)

with  $\overline{T}_0 = 238$  K,  $\overline{T}_{max} = 1000$  K,  $\mathcal{B} = 0.2$ ,  $z_{\Delta} = 112$  km, and  $\Delta = 16$  km. We determine the pressure and density using the ideal gas law and equations (2)–(4) in *Vadas* [2007] for  $\mu$ ,  $X_{MW}$ , and  $\gamma$ . The background wind is set to zero here for simplicity and to more easily understand the results. The background atmosphere is shown in Figure 8.

We ray trace the GWs from the initial height up to z = 300 km. The 4-D grid that we employ for reconstructing the GW field spans x = [-2500, 2500] km, y = [-2500, 2500] km, z = [200, 300] km, and t = [0.2, 5.2] h. The number of cells in x, y, z, and t in this grid are  $N_x = N_y = 125$ ,  $N_z = 25$ , and  $N_t = 120$ , yielding cell sizes of  $\Delta_x = \Delta_y = 40$  km,  $\Delta_z = 4$  km, and  $\Delta_t = 2.5$  min, respectively. We ray trace the upward and southward propagating GWs only.

Figure 9 shows the reconstructed T' at z = 250 km. Southward propagating GWs with horizontal wavelengths of  $\lambda_H \sim 200$  km are seen, with maximum  $T' \sim 10$  K at t = 90 min. Note that the GWs which arrive at  $t \sim 70$  min have larger  $|\lambda_z|$  (but much smaller amplitudes) than those that arrive at t = 90 min. Figure 10 shows the maximum values of T' at z = 250 km and the average GW periods, wavelengths, and phase speeds at these maxima. There are approximately five distinct GW packets which reach this altitude over this 5 h period. They arrive at t = 80-120, 135-150, 165-185, 200-225, and 240-270 min with amplitudes of  $T' \sim 10$ , 2, 0.9, 0.2, and 0.03 K, respectively. The corresponding GW periods for these packets are  $\tau_r = 14-18$ , 22-23, 26, 29-30, and 33 min, respectively. The earliest GWs have the largest vertical wavelengths:  $|\lambda_z| \sim 250$  km. The GWs in the first to fifth packets have the fastest to slowest intrinsic phase speeds of  $|c_y| = c_{|H} = \omega_{|r}/k_H \sim 220$ , 140, 120, 110, and 95 m/s, respectively. Thus, the GWs in the first wave packet are the fundamental GWs having the same period and phase speed as the ocean wave. The GWs with  $\tau_{|r} = 12.3$  min are not present even though the buoyancy period is smaller than 12.3 min at z = 250 km, because they violated the restriction that  $\nu$  change slowly enough (i.e.,  $|\lambda_z| < 4\pi \nu/(d\nu/dz)$ ) [*Vadas*, 2007], which resulted in their automatic removal by the

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**Figure 12.** T' at x = 0 and z = 250 km created by a subset of the GWs excited by an ocean wave packet with  $\tau_F = 14 \text{ min}$ ,  $\lambda_H = 190 \text{ km}$ , and  $\chi = 30 \text{ min}$ . (a) T' for those GWs with  $\tau_{Ir} = 12 - 16 \text{ min only}$ . (b) 4T' for those GWs with  $\tau_{Ir} = 16 - 21 \text{ min only}$ . (c) 10T' for those GWs with  $\tau_{Ir} = 21 - 24 \text{ min only}$ . (d) 30T' for those GWs with  $\tau_{Ir} = 24 - 28 \text{ min only}$ . The T' profiles are shown every 2.5 min from t = 70.75 to 190.75 min and are offset by 20 K. The upper x axes show the time t.

ray trace model. The difference GWs constitute the third wave packet. The largest-amplitude continuum GWs arrive at t = 100-120 min at the end of the first wave packet and have  $\tau_{lr} \sim 16-18$  min,  $c_H \sim 170-210$  m/s, and  $T' \sim 4-8$  K. The rest of the continuum GWs constitute the second, fourth, and fifth wave packets with correspondingly smaller amplitudes. Note that  $2\pi H \sim 235$  km at z = 250 km. Therefore, all of the largest-amplitude GWs (contained in the first wave packet) violate the Boussinesq approximation.

Figure 11 shows T' slices at x = 0 and z = 250 km. Although the phase of T' mostly moves southward in time, it moves northward at t = 113 - 123 min during the time when the first continuum GWs arrive. We better understand this result by ray tracing the GWs having  $\tau_{lr} = 12 - 16$ , 16 - 21, 21 - 24, and 24 - 28 min separately. The ranges  $\tau_{lr} = 12 - 16$  and 24 - 28 min contain the fundamental and difference GWs, respectively, while the ranges  $\tau_r = 16 - 21$  and 21 - 25 min contain continuum GWs. Figure 12 shows the results. For each range, the phase of T' descends southward in time, as expected, with  $\lambda_H \sim 200$  km. Therefore, the combination of the fundamental and continuum GWs causes the northward phase progression at  $t \sim 120$  min in Figure 11. We also see explicitly from Figure 12 that GWs with larger periods reach z = 250 km further from the excitation location (see equation (67)). Note also from Figure 11 that the GWs arrive simultaneously with the ocean



**Figure 13.** Maximum T' at z = 250 km for the GWs excited by an ocean wave packet with  $\tau_F = 14$  min,  $\lambda_H = 190$  km,  $\chi = 30$  min, and  $\sigma_z = 1$  m. Here the vertical grid spacing  $\Delta z$  used to calculate the GW spectrum is varied. The three overlapping blue lines show the results for  $\Delta z = 0.5$ , 1, and 2 m, the green line shows  $\Delta z = 4$  m, and the two overlapping red lines show  $\Delta z = 8$  and 16 m.

wave below them at  $t \sim 80-90$  min. Finally, we note from Figures 10 and 11 that there are essentially no GWs at z = 250 km prior to t = 70 min.

#### 5.2. Dependence of the GW Solution on $\Delta z$

We calculate the same GW spectrum as in Figure 7 except that we vary the vertical grid size from  $\Delta z = 0.5$  to 16 m. The background atmosphere we use is the same as in Figure 7. We then ray trace the southward propagating GWs from z = 0.1 km into the thermosphere using the same ray trace grid and background atmosphere as in Figure 9. The maximum T' at z = 250 km is shown in Figure 13. The results are identical for  $\Delta z = 0.5$ , 1, and 2 m. However, T' is somewhat larger for  $\Delta z = 4$  m and is much smaller for  $\Delta z = 8$  and 16 m. Thus, the solution converges when  $\Delta z \leq 2\sigma_z$ . In order to accurately resolve an ocean surface wave packet



**Figure 14.** Maximum T' at z = 250 km for the GWs excited by an ocean wave packet with  $\tau_F = 14$  min,  $\lambda_H = 190$  km, and  $\chi = 30$  min. The solid, dashed, and dashed-dotted lines show  $\sigma_z = 1, 2, \text{ and } 4$  m, respectively. In each case,  $\Delta z = 2\sigma_z$ .



**Figure 15.** Same as Figure 7 but for an ocean wave packet with  $\lambda_H = 190$  km,  $\tau_F = 20$  min, and  $\chi = 50$  min.

then, it is necessary to employ a vertical grid spacing of  $\Delta z = 2\sigma_z$  (or smaller) when calculating the excited GW spectrum.

#### 5.3. Dependence of the Thermospheric Response on $\sigma_r$

We calculate the same GW spectrum as in Figure 7 except that we vary the Gaussian depth of the vertical body force from  $\sigma_z = 1$  to 4 m. As before, the ocean wave height is  $h_0 = 1.3$  m. We also set  $\Delta z = 2\sigma_z$ . The background atmosphere we use is the same as in Figure 7. We then ray trace the southward propagating GWs from z = 0.1 km into the thermosphere using the same background atmosphere as in Figure 9. Figure 14 shows the maximum T' at z = 250 km. We see that T' increases approximately linearly with  $\sigma_z$ , with a similar overall shape. The maximum values are  $T' \sim 9$ , 22, and 40 K for  $\sigma_z = 1, 2, and 4$  m, respectively. Although we believe that  $\sigma_z = 0.5$  to 10 m is a reasonable estimate for ocean waves with heights of a fraction of a meter to several meters, we do not know which  $\sigma_z$  best describes the coupled ocean-atmosphere interface for medium-scale ocean waves. Future comparison with 630 nm airglow [e.g., *Makela et al.*, 2011], GPS-derived total electron content, and other types of observations may result in the determination of this parameter.

#### 5.4. Thermospheric Response to a Longer Duration Ocean Wave Packet

In order to better understand the effect the sum GWs have on the thermosphere, we consider a longer duration ocean wave packet with a sum period > 13 min. Such GWs could be excited (i.e., are allowed by the GW dispersion relation) and could propagate to z = 250 km. We model an idealized ocean wave packet with  $\lambda_H = 190$  km,  $\tau_F = 20$  min, and  $\chi = 50$  min. We also choose  $w_{ocean} = 0.01$  m/s,  $\theta = 90^\circ$ ,  $\sigma_z = 1.0$  m,  $x_L = \lambda_{H}$ ,  $y_L = 3\lambda_H$ ,  $x_0 = y_0 = 0$ , and  $z_0 = 4.0$  m. This model ocean wave has a somewhat larger height of  $h_0 = 1.9$  m from equation (66) and has a phase speed of 158 m/s. Note that the result for this same ocean surface wave but with a smaller amplitude of  $h_0 = (1.9/4)$  m = 0.5 m, for example, can easily be obtained from the results shown here by dividing all GW amplitudes (e.g., T') by a factor of 4. The dependence of the GW spectrum on  $\lambda_H$  and  $\lambda_z$ , however, will be unchanged. Additionally, all propagation times (into the thermosphere), phase speeds, distances traveled, etc, will be unchanged.



**Figure 16.** Same as Figure 10 but for the GWs excited by an ocean wave packet with  $\tau_F = 20 \text{ min}$ ,  $\lambda_H = 190 \text{ km}$ , and  $\chi = 50 \text{ min}$ .

We generate the GW spectrum from the force given by equations (70) and (82) using a grid with  $n_x = 4$ ,  $n_y = 2$ ,  $n_z = 131072$ , x = [-95, 95] km, y = [-47.5, 47.5] km, z = [-121, 141] km,  $\Delta x = \Delta y = 47.5$  km, and  $\Delta z = 2$  m. As before, the grid extends to z = -121 km to prevent truncating the GW spectrum in  $|\lambda_z|$ . The background atmosphere is the same as in Figure 7.

Figure 15 shows the excited GW spectrum. The spectrum only consists of GWs having  $\lambda_{H} = 190$  km. The sum, fundamental, and difference GWs have periods of  $\tau_{lr} = 14$ , 20, and 33 min and horizontal phase speeds of  $c_{lH} = 226$ , 158, and 96 m/s, respectively. The sum GWs are seen in Figure 15 with  $|\lambda_{z}| \sim 135$  km. This is followed by continuum GWs with  $\tau_{lr} = 16.5$  min,  $|\lambda_{z}| \sim 85$  km, and  $c_{lH} \simeq 190$  m/s. The fundamental GWs follow with  $|\lambda_{z}| \sim 60$  km. The rest of the continuum GWs follow with  $\tau_{lr} \sim 21-60$  min,  $|\lambda_{z}| \sim 15-55$  km, and  $c_{lH} \simeq 60-150$  m/s. The continuum GWs have  $\omega_{lr} < |\alpha|$  and have momentum flux amplitudes which decrease rapidly as  $\tau_{lr}$  increases for  $\tau_{lr} > \tau_{F}$ . The difference GWs only appear as a small bump in the spectrum. The momentum fluxes are largest for the fundamental GWs, and decrease rapidly with increasing  $\tau_{lr}$ . Note that if we increase  $n_x$  but keep  $\Delta x = 47.5$  km, we find that only those GWs with  $\lambda_{H} = 190$  km have nonnegligible momentum fluxes (not shown).

We ray trace the GWs from the initial height up to z = 300 km. The 4-D grid that we employ for reconstructing the GW field spans x = [-2500, 2500] km, y = [-2500, 2500] km, z = [200, 300] km, and t = [0.2, 5.2] h.



**Figure 17.** Same as Figure 11 but for the GWs excited by an ocean wave packet with  $\tau_F = 20 \text{ min}$ ,  $\lambda_H = 190 \text{ km}$ , and  $\chi = 50 \text{ min}$ .

This grid has  $N_x = N_y = 125$ ,  $N_z = 25$ , and  $N_t = 120$ , yielding cell sizes of  $\Delta_x = \Delta_y = 40$  km,  $\Delta_z = 4$  km, and  $\Delta_t = 2.5$  min. We sprinkle the GW spectrum shown in Figure 15 at  $\mathcal{N} = 4000$  random locations and times and then ray trace all upward and southward propagating GWs from z = 0.1 km into the thermosphere using the same background atmosphere as in Figure 9. Figure 16 shows the maximum values of T' at z = 250 km, as well as the average periods, wavelengths, and phase speeds at those maxima. We see that approximately five distinct GW packets reach this altitude over this 5 h period having  $\tau_{lr} = 14$  to 35 min. The sum, fundamental, and difference GWs have maximum T' at t = 80-110 min, t = 140 min, and t = 260-290 min, respectively, with T' ~10, 5, and 0.1 K, respectively. The largest amplitudes (i.e., T' ~8-10 K) occur at t = 90-130 min from the sum and continuum GWs and have  $\tau_{lr} = 14-18$  min,  $|\lambda_z| = 150-250$  km,  $c_H = |c_v| \sim 170-230$  m/s, and  $|c_{z}| \sim 140-280$  m/s. For this ocean wave packet then, the largest thermospheric response is created by the sum and continuum GWs with periods of  $\tau_{lr} \sim 14-18$  min, not the fundamental GWs, even though the fundamental GWs dominate the excited GW spectrum (see Figure 15). This is because the GWs with  $\tau_{lr}$  ~14–18 min have much larger  $|\lambda_z|$  and  $c_H$  and therefore propagate more deeply into the thermosphere prior to dissipating [Vadas, 2007; Miyoshi and Fujiwara, 2008]. Note that the fundamental GWs are beginning to dissipate at  $z \simeq 250$  km because  $c_{H} = 158$  m/s [Vadas, 2007]. Figure 17 shows T' at x = 0 and z = 250 km. The arrival of the different wave packets is evident. The phase of T' mostly moves southward but moves northward at t ~130-145 min when continuum and fundamental GWs arrive simultaneously. Importantly, the sum and earliest continuum GWs arrive ~50-60 min before the ocean wave and have much larger horizontal phase speeds.

#### 6. Conclusions and Discussion

In this paper, we derived the compressible f plane linear solutions for the excitation of GWs and AWs from ocean surface wave packets in an isothermal, windless, and nondissipative atmosphere. Here we modeled the ocean wave packet as a vertical body force at the ocean surface, with horizontal and temporal scales characteristic of the wave packet. (Note that diffusive effects from the Ekman layer or planetary boundary layer were neglected here.) The temporal portion of the forcing included the fundamental ocean wave frequency  $\omega_F$  multiplied by a single  $\sin^2(\pi t/\chi)$  envelope for  $t = [0, \chi]$ . The GW amplitudes were proportional to the ocean wave packet height. The frequencies of the excited GWs included the fundamental ocean wave frequency, the sum and difference frequencies with the packet duration, and a continuum of frequencies. Note that if the solution for a more complicated temporal envelope function is desired instead, the envelope function could be written as a sum of  $\sin^2(\pi t/\chi)$  functions; then the solutions could be added

together linearly. This would likely yield a GW spectrum that is more "blurred" than the cases analyzed here. The total solutions would then be analyzed in a way similar to that done in this paper.

We also derived the linear compressible f plane solutions excited by steady state ocean waves (Appendices B and C). We again modeled the ocean waves using a vertical body force oscillating at the frequency  $\omega_F$ . These solutions included the fundamental and a continuum of frequencies. Because the ocean wave packet solutions better describe localized ocean waves such as the leading edge of tsunamis, we focused on the ocean wave packet solutions in this paper.

In order to include a realistic atmosphere which incorporates spatially and temporally varying background temperature, wind, and thermospheric dissipation, we input the GW solutions into a 3-D ray trace model. We developed a new scheme to randomly sprinkle hundreds or thousands of GW spectra into the region occupied by the ocean wave packet, assign each GW a unique phase that accounts for its location, time, period, and wave vector, and reduce each GW's amplitude according to its location and time in relation to the packet's peak location and time. We then ray traced all GWs to higher altitudes. During ray tracing, we saved the average momentum fluxes, wave vectors, frequencies, and GW phases in *xyzt* cells within a 4-D grid in the atmosphere. We then reconstructed the GW field on this grid using the GW dissipative dispersion relations. This is allowed for the determination of the temperature, wind, and neutral density perturbations created by the GWs. Because the solutions are linear, the reconstructed GW amplitudes are proportional to the ocean wave packet height  $h_0$ . We then determined the ray trace normalization factor needed to convert the spectral to real-space amplitudes via comparison with the exact solutions in the troposphere.

We then calculated the results for several medium-scale ocean wave packets with  $\lambda_H = 190$  km,  $\chi = 30-50$  min, and  $\tau_F \sim 14-20$  min. Such wave packets excite GWs similar to those observed by *Makela et al.* [2011]. The excited GW spectra contained a mix of discrete and continuum frequencies. The discrete frequencies occurred at  $\omega_F \pm 2\pi/\chi$  and  $\omega_F$ , which corresponded to the sum, difference, and fundamental frequencies. The continuum GWs had frequencies which ranged from very small values ( $\tau_{lr} > 100$  min) to almost  $\omega_F + 2\pi/\chi$ . The GW momentum flux spectrum peaked between  $\omega_F$  and  $\omega_F + 2\pi/\chi$  and decreased rapidly for decreasing  $\omega_{lr}$  (for  $\omega_{lr} < \omega_F$ ). The excited GWs had phase speeds of  $c_{lH} \sim 50-260$  m/s. The sum GWs had phase speeds that were much larger than that of the fundamental (frequency) GWs by  $\lambda_H/\chi$ . If the sum GWs were not allowable by the GW dispersion relation, then GWs with the smallest allowable wave periods were excited instead.

We then ray traced the excited GWs into a thermosphere with an exospheric temperature of  $\overline{T} = 1000$  K and reconstructed the GW field at z = 250 km. We chose to examine this altitude because it is the approximate center of the 630 nm airglow layer, and GWs with  $|\lambda_z| > 100$  km can be detected in this layer. We found that GWs with  $c_{lH} \ge 100$  m/s propagated to z = 250 km. We also found that these GWs tended to arrive as wave packets. However, except for those GWs with frequencies of  $\omega_{F_1} |\alpha|$ , and  $|\beta|$ , this may have been due to the fact that  $\lambda_z$  was discretized in our Fortran 90 model. If the sum GWs were allowed and the sum frequency was less than the buoyancy frequency at z = 250 km, we found that the observation site 50–60 min before the fundamental GWs.

We modeled an ocean wave packet with  $\tau_F = 20 \text{ min}$ ,  $\lambda_H = 190 \text{ km}$ ,  $\chi = 50 \text{ min}$ ,  $\sigma_z = 1 \text{ m}$ , and height  $h_0 = 1.9 \text{ m}$ . The fundamental GWs had  $c_{IH} = 158 \text{ m/s}$ , while the sum GWs (with  $\tau_{Ir} = 14 \text{ min}$ ) had much larger phase speeds of  $c_{IH} = 226 \text{ m/s}$ . The highest-frequency continuum GWs had  $c_{IH} = 160-220 \text{ m/s}$ . We found that the highest-frequency continuum GWs arrived at z = 250 km after the sum GWs but before the fundamental GWs. At this altitude,  $T' \sim 10$ , 8, and 5 K for the sum, highest-frequency continuum, and fundamental GWs, respectively. Therefore, although the fundamental GWs dominated the excited GW spectra, the sum and highest-frequency continuum GWs dominated the thermospheric response with  $c_{IH} \sim 170-230 \text{ m/s}$ ,  $\tau_{Ir} \sim 14-18 \text{ min}$ , and  $|\lambda_z| \sim 150-250 \text{ km}$ . (Note that such large- $\lambda_z$  GWs can be detected, in principle, in the 630 nm layer.) The sum and highest-frequency continuum GWs. In contrast, the difference GWs had  $c_{IH} \sim 96 \text{ m/s}$  and  $\tau_{Ir} = 33 \text{ min}$  and arrived  $\sim 2 \text{ h}$  after the fundamental GWs with very small amplitudes. The continuum GWs had  $\tau_{Ir} = 16 \text{ min}$  to at least 35 min, although T' was quite small for GWs with  $\tau_{Ir} \geq 30 \text{ min}$ . Note that T' is proportional to the height  $h_0$  (or the vertical velocity  $w_{ocean}$ ) of the ocean wave packet.

We also found that T' was approximately linearly proportional to the Gaussian depth of the vertical body force. Because typical tsunami amplitudes are a fraction of a meter to several meters on the open ocean

[Salmon, 2014], we chose  $\sigma_z = 1$ , 2, and 4 m. For these values, the maximum T' at z = 250 km was  $T' \sim 9$ , 22, and 40 K, respectively, for an ocean wave packet with  $\lambda_H = 190$  km,  $\tau_F = 14$  min,  $\chi = 30$  min, and  $h_0 = 1.3$  m. (Because the GW response is linearly proportional to  $h_0$ , T' would decrease by a factor of 2 for a smaller wave height of  $h_0/2$ , for example.) We do not know which  $\sigma_z$  best describes the ocean-atmosphere interface for these medium-scale ocean waves; however, we believe that  $\sigma_z = (0.5 - 10)h_0$  provides a reasonable estimate. Comparison with 630 nm airglow observations may result in the determination of  $\sigma_z$  in the future.

We emphasize that the results for the same ocean surface waves but with smaller heights  $h_{\text{new}}$  can be trivially obtained from the results shown in this paper by multiplying all GW amplitudes (e.g., T') by  $h_{\text{new}}/h_0$ . The dependence of the GW spectrum on  $\lambda_H$  and  $\lambda_z$  would be unchanged for this new wave height  $h_{\text{new}}$ . Therefore, all propagation times (into the thermosphere), phase speeds, distances traveled, etc., would be unchanged. We emphasize that only the GW amplitudes are affected by decreasing the ocean surface wave height.

Finally, we found that it is necessary to include compressibility for calculating the GW spectrum excited by high-frequency, medium-scale ocean wave packets. For packets with  $\lambda_H = 190$  km,  $\tau_F = 14-20$  min, and  $\chi = 30-50$  min, the sum, highest-frequency continuum, and fundamental GWs had  $|\lambda_z| \sim 50-260$  km at the ocean surface, which did not satisfy  $|\lambda_z| << 2\pi H$ . Thus, the Boussinesq approximation is not satisfied for the GWs that are most important in the thermosphere. This is true in general for any lower atmospheric GWs that can propagate to z = 250 km [*Vadas*, 2007].

An important conclusion of this paper is that if an ocean wave packet has a sum period  $\geq 10-12$  min, then the excited sum and/or highest-frequency continuum GWs have much faster horizontal phase speeds than that of the ocean wave, and thus reach a fixed latitude/longitude well before the ocean wave packet. This effect may have already been observed; indeed, some "early" GWs occurred in the 630 nm airglow layer an hour before the Tohoku tsunami reached Hawaii [Makela et al., 2011]. Although a tsunami does slow down as it approaches land, this effect cannot account for this 1 h difference. Makela et al. [2011] considered several possible explanations for these early GWs. The first was that they were caused by infrasonic waves generated by the earthquake. This possibility was dismissed because the wave direction was incorrect. Additionally, infrasound waves have  $\tau_r < 5$  min. The second was that the tsunami itself created these early waves. This possibility was dismissed because the available coupled GW/ocean wave models could not reproduce these early waves [Occhipinti et al., 2006, 2008; Kherani et al., 2009]. (However, those models were steady state and assumed that only GWs with frequency  $\omega_F$  were excited by an ocean surface wave.) The third was that the early GWs were created by ocean waves not included in the ocean models. From ocean buoy data, they discovered the presence of high-frequency ocean waves which preceded the tsunami but which were not included in the ocean models. Although they could not determine  $\lambda_H$  for these waves, Makela et al. [2011] tentatively concluded that the early GWs might have been created by these early ocean waves. Given the fact that a tsunami is a highly localized ocean wave packet, and that wave packets excite GWs with phase speeds much greater than the ocean wave, we suggest the possibility that these early GWs might have been the sum and/or higher-frequency continuum GWs excited by the Tohoku tsunami. Such "ultrafast" GWs may prove valuable for early tsunami detection using this observational technique. Work is currently being done to determine the thermospheric response to GWs excited by realistic tsunamis using the wave packet model presented in this paper.

#### **Appendix A: Special Cases of the Ocean Wave Packet Solutions**

We now derive special cases of the ocean wave packet solutions derived in section 3.2. When k = l = 0, the horizontal velocity perturbations decouple from the vertical velocity, pressure, and density perturbations. Then the horizontal velocity perturbations are zero:

$$\widetilde{\xi}_F(t) = 0, \tag{A1}$$

$$\widetilde{\sigma}_{F}(t) = 0.$$
 (A2)

When k = l = 0 and f = 0, then  $s_1 = 0$ . The forced solutions in this case are the same as equations (45)–(49) but with the following substitutions:  $A \rightarrow A'$ ,  $B \rightarrow B'$ ,  $C \rightarrow C'$ ,  $D \rightarrow 0$ ,  $E \rightarrow 0$ ,  $F \rightarrow F'$ , and  $G \rightarrow G'$  where

$$A' = \frac{(\mathcal{Z} - 1)}{\left(-\omega_2^2 + \omega_F^2\right)},\tag{A3}$$

**AGU** Journal of Geophysical Research: Space Physics

10.1002/2015JA021430

$$B' = \frac{(Z-1)}{2(-\omega_2^2 + \alpha^2)},$$
 (A4)

$$C' = \frac{(Z-1)}{2(-\omega_2^2 + \beta^2)},$$
(A5)

$$F' = \frac{1}{\omega_2} \left( \frac{(\omega_F - \mathcal{Z}p_2(\omega_F))}{(\omega_F^2 - \omega_2^2)} + \frac{(\alpha - \mathcal{Z}p_2(\alpha))}{2(\alpha^2 - \omega_2^2)} + \frac{(\beta - \mathcal{Z}p_2(\beta))}{2(\beta^2 - \omega_2^2)} \right),$$
(A6)

$$G' = -\frac{\mathcal{Z}}{\omega_2} \left( \frac{q_2(\omega_F)}{(\omega_F^2 - \omega_2^2)} + \frac{q_2(\alpha)}{2(\alpha^2 - \omega_2^2)} + \frac{q_2(\beta)}{2(\beta^2 - \omega_2^2)} \right).$$
(A7)

For example, the scaled pressure perturbation is

$$\widetilde{\psi}_{F}(t) = -\frac{\delta \widetilde{F}_{zs}}{\chi} \left\{ A' \sin \omega_{F} t + B' \sin \alpha t + C' \sin \beta t + F' \sin \omega_{2} t + G' \cos \omega_{2} t \right\}.$$
(A8)

#### **Appendix B: Steady State Ocean Wave Solutions**

For completeness sake, we now derive the f plane compressible solutions to a steady state ocean surface wave that is modeled as a vertical body force. Special case solutions for this force are given in Appendix C. We choose  $\mathcal{F}$  to be a real function which oscillates sinusoidally in time for  $t = [-\infty, \infty]$ :

$$\mathcal{F}(t) = \sin(\omega_F t). \tag{B1}$$

Here  $\omega_F$  is the frequency of the plane ocean surface wave. The Laplace transform of equation (B1) is

$$\mathcal{L}_F = \frac{\omega_F}{s^2 + \omega_F^2}.$$
(B2)

We plug equation (B2) into equation (39) and solve for  $\tilde{\psi}$ ,  $\tilde{\xi}$ ,  $\tilde{\sigma}$ ,  $\tilde{\eta}$ , and  $\tilde{\phi}$  in a manner similar to that done in section 3.2. The final steady state solutions for all time  $t = [-\infty, \infty]$  are

$$\widetilde{\xi}_{SS}(t) = i\omega_F[g(\gamma - 1) - ic_s^2 m_s] \widetilde{F}_{zs} \left\{ k \left[ \frac{M}{(\omega_F^2 - f^2)} \cos \omega_F t + \frac{O}{(\omega_1^2 - f^2)} \cos \omega_1 t + \frac{Q}{(\omega_2^2 - f^2)} \cos \omega_2 t \right] + lf \left[ \frac{M}{\omega_F (\omega_F^2 - f^2)} \sin \omega_F t + \frac{O}{\omega_1 (\omega_1^2 - f^2)} \sin \omega_1 t + \frac{Q}{\omega_2 (\omega_2^2 - f^2)} \sin \omega_2 t \right] \right\},$$
(B3)

$$\widetilde{\sigma}_{SS}(t) = i\omega_F[g(\gamma - 1) - ic_s^2 m_s] \widetilde{F}_{zs} \left\{ I \left[ \frac{M}{(\omega_F^2 - f^2)} \cos \omega_F t + \frac{O}{(\omega_1^2 - f^2)} \cos \omega_1 t + \frac{Q}{(\omega_2^2 - f^2)} \cos \omega_2 t \right] -kf \left[ \frac{M}{\omega_F (\omega_F^2 - f^2)} \sin \omega_F t + \frac{O}{\omega_1 (\omega_1^2 - f^2)} \sin \omega_1 t + \frac{Q}{\omega_2 (\omega_2^2 - f^2)} \sin \omega_2 t \right] \right\},$$
(B4)

$$\widetilde{\eta}_{SS}(t) = \frac{\omega_F \widetilde{F_{zS}}}{\left(N_B^2 - \omega_F^2\right)} \cos \omega_F t - \omega_F \left[c_s^2 \left(m^2 + \frac{1}{4H^2}\right) - N_B^2\right] \widetilde{F_{zS}} \\ \times \left\{\frac{M}{\left(N_B^2 - \omega_F^2\right)} \cos \omega_F t + \frac{O}{\left(N_B^2 - \omega_1^2\right)} \cos \omega_1 t + \frac{Q}{\left(N_B^2 - \omega_2^2\right)} \cos \omega_2 t\right\},$$
(B5)

$$\widetilde{\psi}_{SS}(t) = -\omega_F \left[ g(\gamma - 1) - ic_s^2 m_s \right] \widetilde{F_{zs}} \left\{ \frac{M}{\omega_F} \sin \omega_F t + \frac{O}{\omega_1} \sin \omega_1 t + \frac{Q}{\omega_2} \sin \omega_2 t \right\},$$
(B6)

10.1002/2015JA021430

$$\widetilde{\phi}_{SS}(t) = \frac{g(\gamma - 1)\widetilde{F_{zs}}}{c_s^2 \left(N_B^2 - \omega_F^2\right)} \sin \omega_F t + \frac{\omega_F}{c_s^2} \left[g(\gamma - 1) - ic_s^2 m_s\right] \widetilde{F_{zs}} \times \left\{ \frac{\left(\omega_F^2 - im_s(\gamma - 1)g\right)M}{\omega_F \left(N_B^2 - \omega_F^2\right)} \sin \omega_F t + \frac{\left(\omega_1^2 - im_s(\gamma - 1)g\right)O}{\omega_1 \left(N_B^2 - \omega_1^2\right)} \sin \omega_1 t + \frac{\left(\omega_2^2 - im_s(\gamma - 1)g\right)Q}{\omega_2 \left(N_B^2 - \omega_2^2\right)} \sin \omega_2 t \right\},$$
(B7)

where the subscript "SS" denotes the steady state solution, and where the time-independent coefficients are

$$M = -\frac{\left(\omega_F^2 - f^2\right)}{\left(-\omega_1^2 + \omega_F^2\right)\left(-\omega_2^2 + \omega_F^2\right)},$$
(B8)

$$O = -\frac{\left(-\omega_1^2 + f^2\right)}{\left(-\omega_2^2 + \omega_1^2\right)\left(-\omega_1^2 + \omega_F^2\right)},$$
(B9)

$$Q = \frac{\left(-\omega_2^2 + f^2\right)}{\left(-\omega_2^2 + \omega_1^2\right)\left(-\omega_2^2 + \omega_F^2\right)}.$$
(B10)

Equations (B3)–(B7) show that the frequencies of the GWs excited by a plane ocean surface wave include the fundamental frequency  $\omega_F$  and a continuum of frequencies.

#### Appendix C: Special Cases of the Steady State Ocean Wave Solutions

We now derive special cases of the steady state ocean wave solutions presented in Appendix B. When k = l = 0, the horizontal velocity perturbations decouple from the vertical velocity, pressure, and density perturbations. Then

$$\widetilde{\xi}_{SS}(t) = 0,$$
 (C1)

$$\widetilde{\sigma}_{SS}(t) = 0.$$
 (C2)

When k = l = 0 and f = 0, then  $s_1 = 0$ . The steady state solutions in this case are

$$\widetilde{\eta}_{SS}(t) = \frac{\omega_F \widetilde{F_{zS}}}{\left(N_B^2 - \omega_F^2\right)} \cos \omega_F t - \omega_F \left[c_s^2 \left(m^2 + \frac{1}{4H^2}\right) - N_B^2\right] \widetilde{F_{zS}} \\ \times \left\{\frac{M'}{\left(N_B^2 - \omega_F^2\right)} \cos \omega_F t + \frac{Q'}{\left(N_B^2 - \omega_2^2\right)} \cos \omega_2 t\right\},\tag{C3}$$

$$\widetilde{\psi}_{SS}(t) = -\omega_F[g(\gamma - 1) - ic_s^2 m_s] \widetilde{F}_{zs} \left\{ \frac{M'}{\omega_F} \sin \omega_F t + \frac{Q'}{\omega_2} \sin \omega_2 t \right\},$$
(C4)

$$\widetilde{\phi}_{SS}(t) = \frac{g(\gamma - 1)\widetilde{F_{zS}}}{c_s^2 \left(N_B^2 - \omega_F^2\right)} \sin \omega_F t + \frac{\omega_F}{c_s^2} \left[g(\gamma - 1) - ic_s^2 m_s\right] \widetilde{F_{zS}} \\ \times \left\{ \frac{\left(\omega_F^2 - im_s(\gamma - 1)g\right)M'}{\omega_F \left(N_B^2 - \omega_F^2\right)} \sin \omega_F t + \frac{\left(\omega_2^2 - im_s(\gamma - 1)g\right)Q'}{\omega_2 \left(N_B^2 - \omega_2^2\right)} \sin \omega_2 t \right\},$$
(C5)

where

$$M' = -\frac{1}{-\omega_2^2 + \omega_F^2},$$
 (C6)

$$Q' = \frac{1}{-\omega_2^2 + \omega_F^2}.$$
 (C7)

#### **Appendix D: Modeling a Propagating Plane Ocean Wave**

In section 3.2, we assumed a temporal dependence for the body force of  $\mathcal{F} \propto \sin \omega_F t$ . For  $\theta = 0$ , this can be thought of as being created by two ocean surface waves traveling simultaneously in the +x and -x directions. In order to create a single ocean surface wave using the solutions from section 3.2, we must linearly add the solutions from two distinct phase-shifted forces. If a plane ocean surface wave has zonal and meridional wave numbers k and l, respectively, where

$$k = k_H \cos \theta, \quad l = k_H \sin \theta,$$
 (D1)

then its variation in time and space is proportional to

$$\cos(\omega_F t - kx - ly) = \sin(\omega_F t)\sin(kx + ly) + \sin(\omega_F t_{shift})\cos(kx + ly),$$
 (D2)

where  $t_{shift} = t + \pi/(2\omega_F)$ . Thus, the first force has spatial dependence  $F_z \propto \sin(kx + ly)$  and begins at t = 0, while the second force has spatial dependence  $F_z \propto \cos(kx + ly)$  and begins at  $t = \pi/(2\omega_F)$ . Note that the second force is phase shifted in time by one fourth of a period and is phase shifted in space by one fourth of a wavelength.

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