Turbulence Dynamics and Mixing due to Gravity Waves in the Lower and Middle Atmosphere

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Abstract

Internal gravity waves contribute to turbulence generation and mixing throughout the atmosphere due to instability accompanying amplitude growth with altitude and/or reduction of intrinsic phase speeds in shear flows. The instability processes accounting for turbulence generation depend on wave and mean flow structures, but in general exhibit an altitude dependence due to increasing wave scales with increasing altitude. Near the tropopause and away from major sources, characteristic vertical scales are a few km, vertical group velocities are small, and the dominant instability appears to be a Kelvin-Helmholtz (KH) shear instability due to inertia-gravity wave motions. At greater altitudes, characteristic vertical scales increase, causing larger vertical group velocities, larger wave energy fluxes, and a preference for faster convective instabilities of the motion field. This paper will review both the reasons for a change in instability character with altitude and the dynamics accompanying the transition to turbulence via convective wave breaking and shear instability of the motion field. We will also describe the different implications of these wave field instabilities for turbulent mixing and transport of momentum, heat, and constituents.

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1. Introduction

Internal gravity waves are ubiquitous in the atmosphere, arise from a variety of sources, and have a number of important effects. Dominant sources at lower altitudes include convection, orography, and wind shear (Fritts and Nastrom, 1992). Among the more significant effects are forcing of the mean and large-scale circulation and thermal structure, large-scale transport of heat, momentum, and constituents, and turbulent mixing and diffusion of these quantities. The first two topics are the subjects of other papers in this volume by Holton and Alexander (2000) and Garcia (2000). The dynamics underlying the latter is the subject of the present paper.

Gravity wave effects vary with altitude for a number of reasons. Most important is the growth of wave amplitudes due to decreasing density with increasing altitude. This causes increases of wave fluxes (per unit mass) with increasing altitude which force the large-scale circulation, structure, and transport. Increasing wave amplitudes also increase the tendency for wave instability, thus increasing turbulence intensities and mixing with increasing altitude. The sensitivity of gravity waves to the environments through which they propagate leads to a complex interplay between transport and mixing processes. While the role of gravity waves in large-scale transport processes is becoming better understood, our knowledge of competitive turbulent mixing and diffusion processes is not as advanced. Importantly, such an understanding can only be achieved through detailed assessments of the dynamics of turbulence transitions and the turbulence cascade. Our purpose here is to review recent progress in these areas and provide an appreciation for the dynamics accounting for turbulent mixing and diffusion in the atmosphere.

Instability processes accompanying gravity wave propagation in the atmosphere have been studied for many years using a variety of methods. As a result, we now understand in general terms the mechanisms accounting for wave instability and turbulence generation and the factors that govern their relative importance with altitude. The two primary mechanisms for turbulence production are Kelvin-Helmholtz (KH) shear instability and wave breaking via convective instability, with shear instability typically more prevalent for lower intrinsic frequencies and wave breaking (via convective instability) more prevalent at higher intrinsic frequencies (Hodges, 1967; Fritts, 1984, 1989; Dunkerton, 1984, 1989, 1997; Fritts and Rastogi, 1985; Fritts and Yuan, 1989). Other instabilities also occur at small and large wave amplitudes, primarily resonant wave-wave interactions (McComas and Bretherton, 1977; Yeh and Liu, 1981; Sonmor and Klaassen, 1997). However, these appear not to contribute directly to the generation of turbulence and mixing and will not be discussed further here.

Despite our earlier understanding of the mechanisms leading to turbulence generation, the details of the instability and turbulence dynamics and their implications for transport and mixing have remained beyond our grasp until recently. Laboratory, theoretical, and numerical studies have made important contributions to our understanding of the initial transition to turbulence and three-dimensional (3D) motions in stratified and/or sheared flows (Thorpe, 1985, 1987; Klaassen and Peltier, 1985; Metcalfe et al., 1987; Andreassen et al., 1994; Fritts et al., 1994, 1996a, b; Winters and D'Asaro, 1994; Palmer et al., 1994, 1996; Caulfield and Peltier, 1994). Likewise, significant progress has occurred in describing the character of vortex structures and interactions in numerical and observational studies of homogeneous turbulence in sheared and unsheared flows (Hopfinger et al., 1982; Maxworthy et al., 1985; Rogers and Moin, 1987; Vincent and Meneguzzi, 1994; Melander and Hussain, 1994; Cadot et al., 1995; Schoppa et al., 1995; Arendt et al., 1997a, b). Until recently, however, an understanding of the transition to turbulence, and of the dynamics within turbulent flows, has been beyond our grasp. This has changed with the recent availability of very large computational resources. Current simulations, for example, now achieve resolutions as high as $\sim 1000^3$, span more than a decade of scales within the "inertial range" of turbulence (i.e., turbulence dynamics for which both density stratification and viscosity are negligible), and promise to provide, with complete analysis, continuing insights into the nature of turbulence dynamics and mixing.

Our objectives in this paper are both to argue for a different statistical role of the dominant gravity wave instability processes with altitude and to outline the vorticity dynamics accompanying these instabilities and their implications for turbulent mixing. Vorticity is used to characterize the transitional and turbulent flows because it provides the simplest and most insightful view of these complex nonlinear dynamics. The first objective is addressed using linear theory and a simple spectral model that conforms closely to observations in Sections 2 and 3. The second objective comprises two parts. We first review what has been learned about the vortex dynamics of turbulence from detailed simulations and analysis of wave breaking in Section 4; we then describe briefly in Section 5 the vortex dynamics accompanying a high-resolution simulation of KH instability employing an incompressible model and comment on the apparent similarity of these two flows. Where appropriate, we also examine the implications of these turbulence sources for mixing, large-scale modeling, and competition with wave-driven transport processes. Our conclusions are presented in Section 6.

2. Linear Theory

a. Dispersion and polariation relations

While gravity waves are, in general, subject to instabilities at large and small amplitudes (see, e.g., Müller et al., 1986; Sonmor and Klaassen, 1997), it appears to be only local, as opposed to global, instabilities that lead directly to turbulence. Fortunately, linear theory provides an adequate description of the tendency for local instability of a gravity wave field. It is sufficient to assume for our purposes that variations in the mean wind and temperature fields are sufficiently slow on wave scales that wave propagation is described adequately by the WKB approximation (Gill, 1982). Also assuming an inviscid fluid of scale height H and steady motions of the form $e^{ik(x-ct)}$ (taking wave propagation to be in the x direction without loss of generality), where $\vec{k} = (k, m)$ is the wavenumber vector, we define $\omega = k(c - \bar{u})$ as the intrinsic frequency, and c and $\bar{u}$ are the horizontal phase speed and mean wind along the direction of wave propagation. Then the dispersion and polarization relations may be written (Fritts, 1984)

$$m^2 = \frac{k^2(N^2 - \omega^2)}{(\omega^2 - f^2)} - \frac{1}{4H^2}, \tag{0.1}$$

$$w' = \frac{-ku'}{m}, \tag{0.2}$$

$$v' = \frac{-ifu'}{\omega}, \tag{0.3}$$

and

$$\frac{\theta'}{\bar{\theta}_z} = \frac{-iw'}{\omega}, \tag{0.4}$$

or alternatively, in terms of u' and the intrinsic phase speed,

$$\frac{\theta'_z}{\bar{\theta}_z} = \frac{-u'}{(c - \bar{u})} = -a, \tag{0.5}$$

where (u, v, w) are component velocities, θ is potential temperature, N is the buoyancy frequency, $f = 2\Omega \sin\phi$ is the inertial frequency, Ω is the earth's rotation rate, ϕ is latitude, primes and subscripts denote perturbation quantities and derivatives, and u' varies approximately as (Bender and Orszag, 1978)

$$u' \sim m^{1/2} e^{i\int m dz} \tag{0.6}$$

in response to gradual variations of N and $\bar{u}$ with altitude. Linear gravity wave dynamics and propagation are described in greater detail by Holton and Alexander (2000) and will not be repeated here.

b. Gravity wave amplitudes and instability

Eq. (0.5) provides the most intuitive measure of wave amplitude relative to that required for nominal convective instability within the wave field (Hodges, 1967); a gravity wave is convectively stable when $a < 1$ and convectively unstable when $a > 1$, assuming that instability depends only on vertical motions and gradients. This theory was generalized by Hines (1971, 1988), who considered the potential for "slantwise static instability", or SSI, as opposed to strictly vertical instability. Hines concluded, with his assumptions, that SSI always dominates over vertical convective instability, and that slower instability growth rates can occur for $a < 1$. Unless growth rates are competitive with wave intrinsic frequencies, however, it is difficult to see how such instabilities would achieve finite amplitude. Indeed, our own recent simulations of gravity wave instability dynamics suggest that wave amplitudes must exceed $a = 1$ in the absence of rotation in order for instability to occur, due to the propagation of the unstable phase of the wave, in line with the Hines predictions for growth rates exceeding ω. Such studies should also permit assessments of the character of the instability and the validity of the Hines SSI theory.

Alternatively, we may ask under what circumstances the wave field is dynamically, as opposed to convectively, unstable. The possibility of a dynamical, i.e., Richardson number based, instability at a lower wave amplitude than a corresponding convective instability relies on rotation (nonzero f) or additional mean shear. Assuming no mean shear, the nominal amplitude required for a Richardson number, $Ri = 1/4$, is (Dunkerton, 1984; Fritts and Rastogi, 1985)

$$a_d = \frac{2(1 - f^2/\omega^2)^{1/2}}{1 + (1 - f^2/\omega^2)^{1/2}}. \tag{0.7}$$

For $\omega >> f$, $a_d \simeq 1$ and the conditions for convective and dynamical instability are essentially identical. However, a_d decreases to zero as $\omega \to f$, reflecting the tendency for potential temperature perturbations to be suppressed relative to perturbation wind shears for lower-frequency wave motions. The tendency for dynamical instability of inertia-gravity waves was assessed without and with mean shear by Fritts and Yuan (1989) and Yuan and Fritts (1989) using linear stability analysis; more recently, LeLong and Dunkerton (1998a) evaluated the threshold for dynamical instability numerically. As above, amplitudes somewhat larger than a_d were found to be necessary to support instability due to propagation of the wave motion. LeLong and Dunkerton (1998a) also noted only a weak bias in favor of KH instability at the most unstable phase of the wave motion based on linear theory. These studies were extended to more general linear instabilities by Dunkerton (1997), while a theoretical link between the instabilities at large wave amplitudes and the resonant triad interactions identified at smaller amplitudes in oceanic and atmospheric contexts was established by Sonmor and Klaassen (1997).

c. Gravity wave energy density and flux

To assess the importance of the dominant wave instabilities with altitude, we need to estimate the rate of wave energy dissipation relative to wave energy density with altitude. The ratio of these quantities provides an estimate of the time scale for wave dissipation which can be compared with the wave period. Presumably, if the wave energy dissipation rate is large, rapid instabilities are favored; if, on the other hand, the wave energy dissipation rate is small, then instabilities occurring at smaller wave amplitudes have a greater potential to impose amplitude constraints.

Defining the total wave energy (per unit mass)to be E, the wave energy flux (per unit mass) for a monochromatic gravity wave is given by

$$F_E = c_{gz} E, \tag{0.8}$$

where c_{gz} is the vertical group velocity, given by

$$c_{gz} = -\frac{\omega}{m}(1 - \frac{\omega^2}{N^2} - \frac{f^2}{\omega^2}), \tag{0.9}$$

where we have neglected terms varying as $1/4H^2$ and f^2/N^2.

Following Fritts and VanZandt (1993), we then assume that all of the energy removed from the wave field (whether monochromatic or a broad spectrum) is dissipated and equate the energy dissipation rate, ϵ, to the divergence of wave energy flux (per unit mass)

$$\epsilon = -\frac{1}{\rho}\frac{\partial}{\partial z}[\rho F_E]. \tag{0.10}$$

Importantly, the altitude dependencies of E and ϵ vary depending on whether we assume inviscid monochromatic propagation or adhere to the observed statistical description of the wave spectrum. Given our statistical approach to turbulence generation and effects, we will employ the latter in our estimates of wave scales and growth with altitude. However, we note that individual wave packets often account for wave instability and thus consider single-wave instability dynamics in our discussion of turbulence transitions.

d. The inertial range of turbulence

Turbulence arising due to wave instability processes has often been assumed to have an "inertial range" character at sufficiently small scales. These scales are necessarily smaller than the scales of the waves within which instabilities and turbulence arise. They are also assumed smaller than the scale above which buoyancy plays a strong role, termed the "buoyancy" or "outer" scale, and given by $L_B \sim 10(\epsilon/N^3)^{1/2}$ (Weinstock, 1978, 1981). At smaller scales, the inertial range of turbulence is constrained by viscosity, with a corresponding "inner" scale given by $l_0 \sim C(\nu^3/\epsilon)^{1/4}$, where ν is kinematic viscosity and $C \sim 7$ to 10 depending on the model employed for the viscous range at scales dominated by dissipation (Heisenberg, 1948; Tatarskii, 1971). Between these limits, theory and some experiments suggest a 1D turbulence energy spectrum of the form, $E(k) \sim k^{-5/3}$. Thus, the spatial extent of the inertial range depends on N, ν, and ϵ, all of which vary with altitude. In the lower stratosphere, where ϵ and ν are small (see below), $l_0 \sim 10^{-1}$ m and $L_B \sim 50$ m. In the upper mesosphere, where ϵ and ν are much larger, $l_0 \sim 20$ m and $L_B \sim 300$ m (Lübken et al., 1993; Lübken, 1997). As a result, the anticipated inertial range is quite broad in the lower stratosphere, ~ 2 to 3 decades of scales, while it is much more restricted at higher altitudes, extending ~ 1 decade near the mesopause. In all cases, however, the turbulent mixing and diffusion occurs at scales that are far smaller than the scales of the gravity wave motions themselves. As such, simulations which address turbulent mixing must achieve far higher resolution than required to assess other effects of gravity wave energy and momentum fluxes and wave dissipation. While not addressed here, such simulations also enable assessments of the departures of geophysical turbulence from inertial range turbulence theory in the presence of stratification and shear.

3. Variation of Wave Parameters with Altitude

Given extensive observations over the last few decades, we now have a reasonable understanding of the characteristic scales and amplitudes of gravity waves throughout the lower and middle atmosphere. We also have a fair understanding of the distribution of mean wave energy with observed frequency, but more limited knowledge of intrinsic properties of the wave field. This understanding and a canonical model of the gravity wave spectral variations with altitude are reviewed below. This approach does not imply that there is always a broad spectrum of gravity waves present. Indeed, the wave spectrum is more likely composed of a large number of individual wave packets which may or may not be superposed (see, i.e., Riggin et al., 1997). The spectral description nevertheless provides a more accurate view of the variation of wave amplitudes and scales with increasing altitude than linear inviscid theory for monochromatic waves.

a. Observations

Observations or inferences of gravity wave parameters over the last few decades are much too numerous to review here. Thus, we will summarize only the general spectral characteristics of the wave field, its evolution with altitude, and recent evidence for specific wave instability processes at various altitudes. A reader interested in a survey of the earlier observational literature is referred to the reviews by Fritts (1984, 1989).

Estimates of gravity wave scales, amplitudes, and spectra have been made with many instrument types. The most extensive measurements to date have been made with various atmospheric radars, primarily stratosphere-troposphere (ST) or mesosphere-stratosphere-troposphere (MST) VHF or UHF radars, though other systems have also made important contributions. These systems typically provide data only below ~ 25 km and between ~ 65 and 85 km at lower latitudes or in winter, or between ~ 80 and 90 km at higher latitudes in summer. Other data sets have been collected with lidars, balloons, rockets, aircraft, and passive optical instrumentation. Together, these techniques have defined in broad terms the main features of the mean gravity wave spectrum and its evolution with altitude. All of these measurement techniques also have their own measurement biases or limitations, however, so we must often be careful in interpreting and, in particular, in intercomparing these data. For example, limited altitude coverage, especially at greater altitudes, limited temporal extent, and windowing and mean or linear trend removal in the data analysis all act to underestimate spectral variance at lower wavenumbers and frequencies. Likewise, geophysical variations of the wave spectrum or the mean wind and stability profiles with altitude may introduce biases in spectral amplitudes and/or slopes (VanZandt and Fritts, 1989; Eckermann, 1990, 1995).

Frequency spectra of horizontal (or radial) velocity and temperature are defined primarily with radar and lidar data, typically vary as $E(\omega) \sim \omega^{-p}$, and exhibit slopes varying from $p \sim 1$ to 2 for frequencies between f and N, with most slopes falling closer to 5/3 (see, e.g., Balsley and Carter, 1982). These slopes appear in most cases to vary little with altitude, though there are cases in which such variations are observed. Importantly, this implies that there is little or no tendency for filtering or instability processes to alter the distribution of wave frequencies with altitude. At greater altitudes, various tidal structures contribute spectral peaks often extending well above the mean gravity wave spectral form. Because horizontal wind and temperature variance tend to be concentrated near inertial frequencies, Doppler effects are small and the intrinsic and Doppler-shifted frequency spectra will have nearly the same structures (Fritts and VanZandt, 1987).

Frequency spectra of vertical velocity, on the other hand, exhibit considerable variability with altitude, primarily because these spectra are highly sensitive to Doppler-shifting effects. Assuming that the majority of spectral variance is due to gravity waves, we expect from the dispersion relation that intrinsic frequency spectra of vertial velocities will exhibit a slope of approximately $2-p$, thus having a peak at frequencies just below N for a characteristic value of $p \sim 5/3$. Thus, vertical velocity variance is concentrated near N and is easily Doppler shifted to both higher and lower apparent frequencies, making these spectra perhaps the most sensitive measure of Doppler-shifting effects. Indeed, limited studies using vertical velocity frequency spectra have shown a tendency for waves to be propagating preferentially against the local mean flow (Fritts and Wang, 1991; VanZandt et al., 1991), in agreement with inferences from momentum flux measurements (see, e.g., Vincent and Reid, 1983; Fritts and Vincent, 1987; Tsuda et al., 1990).

Vertical wavenumber spectra have been measured with many instrument types. As noted above, however, limited altitude range results in a lack of sensitivity to motions with larger vertical scales. This causes those waves having larger vertical group velocities, thus potentially large energy and momentum fluxes, to be systematically underrepresented in such data sets. At the other extreme, resolution and measurement accuracy limits impose constraints on the range of scales that can be reliably measured. This suggests caution in assessments of characteristic scales, spectral slopes, and spectral amplitudes at large m. Nevertheless, multiple data sets suggest characteristic scales varying from $\sim$ 2 to 5 km in the lower stratosphere to $\sim$ 10 to 30 km near the mesopause (see Fritts, 1984, 1989). Spectral slope estimates at higher m also vary, but typical values range from -2.5 to -3 (Dewan et al., 1984). Importantly, while there are departures from universal amplitudes and slopes, particularly under strongly sheared conditions (Eckermann, 1995), there is remarkable conformity of spectral amplitudes and slopes at high m with the expectations of the various saturation theories that have been offered to date (Dewan and Good, 1986; Allen and Vincent, 1995).

Horizontal wavenumber spectra are more challenging to measure unambiguously, as they mix spatial and temporal sampling. But assuming that the speed of the measurement platform is large compared to wave horizontal phase speeds (this is not true for the larger vertical scales), horizontal wavenumber spectra are obtained which are again in reasonable agreement with theory and other spectral measurements (VanZandt, 1982; Gardner et al., 1993).

b. Simple Models

To allow for estimates of energy fluxes and dissipation rates, we describe here a canonical spectrum that appears to have many of the attributes of atmospheric spectra and their evolution with altitude. As noted above, this does not imply that the gravity wave spectrum is broad. Rather, it insures that our estimates are as consistent as possible with observed scales and variations with altitude in the atmosphere. We also note that because of the gravity wave dispersion relation, the various spectra are related. Thus, it is necessary only to have accurate characterizations of two spectra, intrinsic frequency and vertical wavenumber, for example, in order to infer the associated gravity wave fluxes of momentum and energy. For this, however, we need to employ the argument above that intrinsic and observed frequency spectra are likely very similar, due to the occurrence of the majority of the variance at low frequencies (for horizontal velocities and temperature).

Spectral models have evolved with time, as we have identified spectral features and constraints more precisely. For example, we now believe the vertical wavenumber spectrum is more highly peaked near the characteristic vertical wavenumber, or scale, than was described initially (i.e., the spectrum is more nearly monochromatic). Likewise, early models of this spectrum assumed a slope at small m which implied large or infinite vertical wave energy flux. These limitations have now been largely removed, and a spectral description that appears to capture most of the character of the gravity wave field is assumed (for convenience, but with some observational basis) to be separable in frequency and vertical wavenumber and is given by (Fritts and VanZandt, 1993)

$$E(\mu, \omega, \phi) = E_0 A(\mu) B(\omega) \Phi(\phi), \tag{0.11}$$

with

$$A(\mu) = A_0 \frac{\mu^s}{1 + \mu^{s+t}} \tag{0.12}$$

and

$$B(\omega) = B_0 \omega^{-p}, \tag{0.13}$$

where A_0, B_0, and $\Phi(\phi)$ are chosen such that $\int_0^\infty A(\mu) d\mu = 1$, $\int_f^N B(\omega) d\omega = 1$, and $\int_0^{2\pi} \Phi(\phi) d\phi = 1$.

For mid-latitudes and $(s, t, p, f/N) = (1, 3, 5/3, 1/200)$ this yields

$$F_E \simeq \frac{NE_0}{18m_*}. \tag{0.14}$$

The factor $N/18m_* \simeq \overline{(1/m)}\overline{\omega}$ may be interpreted as an effective vertical group velocity $\overline{c}_{gz}$. With $m_* \sim 3$ rad km^{-1} near the tropopause and $\sim 1/3$ rad km^{-1} near the mesopause, $\overline{c}_{gz} \sim 0.5$ and 5 m s^{-1}, respectively. With $E_0 \sim 10$ and 10^3 m^2 s^{-2}, this implies $F_E \sim 5$ and 5×10^3 m^3 s^{-3}, respectively, near the tropopause and mesopause.

We see from the form of Eq. (0.14) that wave energy density and energy flux vary with both altitude and N. Such variations are especially important in regions of rapidly increasing N, where the result is increased wave dissipation and flux divergences (VanZandt and Fritts, 1989). For our purposes, however, it is sufficient to assume that N is constant or slowly varying, in which case wave dissipation is dominated by the systematic variations of the wave spectrum with height, yielding

$$\epsilon \simeq \frac{NE_0}{18m_*}(\frac{1}{H} - \frac{3}{2H_E}), \tag{0.15}$$

with a dissipation time scale of

$$T_d \sim E_0/\epsilon \sim \frac{18m_*}{N}(\frac{1}{H} - \frac{3}{2H_E})^{-1}. \tag{0.16}$$

Though there are uncertainties in E_0, m_*, and the fraction of wave energy that is directly dissipated as a result of wave interaction and instability processes with height, this expression provides a useful estimate of the variation of the dissipation time scale with altitude. More specifically, since m_* varies as $E_0^{-1/2}$, the time scale likewise varies as $E_0^{-1/2}$, causing wave energy to be extracted from the wave field ~ 10 times more rapidly near the mesopause than near the tropopause, largely because of the increase of the vertical group velocity (or vertical wave scale) with altitude.

For characteristic wave scales and energies (see above), Eq. (0.16) yields $T_d \sim 100$ hours for the lower stratosphere and ~ 10 hours near the mesopause. Importantly, the time scale for wave dissipation in the lower stratosphere is *longer* than all intrinsic wave periods at middle to high latitudes, whereas the time scale near the mesopause is *shorter* than the energy-containing wave periods. The implications are relatively slow and systematic wave dissipation in the lower stratosphere, allowing slower instability processes to be operative, and relatively faster wave dissipation at higher altitudes, often requiring significant energy dissipation within a wave period. Hence, we expect that KH shear instabilities, which are favored at wave amplitudes below the nominal amplitude for convective instability, will be preferred at lower altitudes and away from significant wave sources. On the other hand, convective instabilities appear to be more likely at greater altitudes, where intermittent wave dissipation must shed excess energy more rapidly, and in the vicinity of strong sources of high-frequency waves at lower altitudes (Lilly and Kennedy, 1973; Alexander et al., 1995; Alexander, 1996).

Alternatively, for measured wave energy densities of $\sim$ 300 to 1000 m^2 s^{-2} and corresponding energy dissipation rates of $\sim 10^{-2}$ to 10^{-1} W/kg near the mesopause (Hocking, 1996; Lübken, 1997), we obtain a time scale of $T_d \sim 3$ to 10 hours. In the lower stratosphere, with $E \sim 3$ to 10 m^2 s^{-2} and $\epsilon \sim 10^{-5}$ to 10^{-4} W/kg (Lilly and Lester, 1974; Hocking, 1985, 1996), we obtain time scale estimates of $T_d \sim 30$ to 100 hours, generally consistent with our spectral estimates above.

4. Gravity Wave Breaking Dynamics

Gravity wave breaking typically occurs accompanying the most energetic (and largest amplitude) components of the wave spectrum. Near the mesopause, typical vertical wavelengths are $\sim$ 10 to 30 km and corresponding unstable layer depths are $\sim$ 1 to a few km, though larger scales are occasionally observed. As noted above, unstable layer depths typically constrain the depth and the outer scale, L_B, of the resulting turbulence.

Numerical studies of wave breaking over the last few years have defined the dynamics of both the transition to turbulence and the initial turbulence cascade (Andreassen et al., 1994, 1998; Fritts et al., 1994, 1996a, 1998; Isler et al., 1994; Winters and D'Asaro, 1994). These studies were constrained by numerical resolution, however, and thus did not address either inertial range dynamics or the implications of wave breaking for mixing. The implications of these results for the dominant instability processes have nevertheless been supported by parallel observational studies (Fritts et al., 1993; Swenson and Mende, 1994; Hecht et al., 1997). Our purposes in this section are to summarize the dynamics of wave breaking reported elsewhere and to provide a preview of mixing implications based on more recent and higher-resolution studies.

a. Review of Wave Breaking Dynamics

We will begin by reviewing the dynamics of wave breaking for waves of high intrinsic frequencies, for which convective instability dominates, from a vorticity perspective. This approach was found by Andreassen et al. (1998) and Fritts et al. (1998) to provide considerable insights into both the initial instability processes and the subsequent turbulence dynamics. We will also employ a quantity corresponding approximately to the minimum pressures in the flow in our discussion of the vorticity dynamics, given its ability to characterize vortex structures at many stages of their evolution (see Andreassen et al., 1998, for further details).

The simulation discussed here was performed using a compressible model solving the Navier-Stokes equations in a horizontally-periodic domain, having inflow/outflow boundary conditions in the vertical, subject to a spectral formulation of viscosity and thermal diffusivity, and employing a mean streamwise (along the direction of wave propagation) shear flow to confine wave instability within a high-resolution domain. Additional details of the numerical code and the simulation are provided by Andreassen et al. (1998).

The character of convective wave breaking is illustrated with isosurfaces of potential temperature through the breaking region shortly after initial convective instability and approximately one buoyancy period later in Plate 1. The direction of wave propagation (the streamwise direction) is out of the page and to the right. At early stages, wave breaking is quasi-2D and resembles a wave breaking on a beach. As instabilities evolve, however, the flow becomes increasingly complex. The vorticity field accompanying this evolution is displayed volumetrically in Plate 2 for an interval extending for ~ 1.4 buoyancy periods beyond initial convective instability and spanning the transition from largely two-dimensional (2D) to fully 3D flow. Views of the vorticity field are from below, with the streamwise direction to the right and the spanwise (across the flow) direction down.

In the absence of rotation, spanwise mean velocity, and 3D noise, the initial vorticity distribution is entirely spanwise and comprises vortex sheets due to both wave and mean velocity shears. As the wave field achieves convective instability, however, the 3D noise seeds counter-rotating, streamwise-aligned convective rolls (see the top two panels in Plate 2). These rolls are confined initially to the convectively unstable phase of the wave field, derive their vorticity (and energy) from baroclinic tendencies, and occupy the full depth of the unstable region of the flow (Fritts et al., 1996a; Andreassen et al., 1998). For reference, the depth of the unstable region is $\sim 1/5$ of the vertical wavelength of the overturning gravity wave and would typically be ~ 1 to a few km near the mesopause (Fritts et al., 1993; Hecht et al., 1997). The convective rolls immediately decouple from the unstable phase of the gravity wave (the wave experiences phase propagation, while the streamwise vortices undergo mutual advection relative to the 2D flow) and advecting vortex pairs begin to impinge on adjacent spanwise vortex sheets. Divergent spanwise flow ahead of advecting vortex pairs leads to local intensification (via vortex stretching) and thinning and wrapping (via vertical and spanwise advection) of the vortex sheets around the streamwise vortex pairs. As described more completely by Andreassen et al. (1998), the consequence is a succession of intensified spanwise vortex sheets which individually become susceptible to spanwise-localized KH instabilities having largely spanwise vorticity. These secondary KH instabilities lead to spanwise vortex tubes which link to the streamwise rolls accounting for their formation. The result is a succession of intertwined vortex loops at each site of initial spanwise vortex sheet intensification (see Plate 2c). These vortex loops bear a close resemblance to the "horseshoe" and "hairpin" vortices that arise in a variety of turbulent flows and thus may have implications for the evolution of a broad class of flows.

The series of intertwined vortex loops arising from initial convective and secondary dynamical instabilities condition the flow for a rapid subsequent evolution because of the close proximity of adjacent vortices. This further evolution is displayed in the lower four panels of Plate 2. Though the time required for this further evolution spans only ~ 0.7 buoyancy periods and the vortex dynamics are somewhat viscously constrained, the increase in flow complexity is dramatic. The vortex loops present in Plate 2c and 2d are fragmented into pieces, and new smaller vortex tubes continue to be formed. Indeed, the small-scale flow at later times appears nearly isotropic locally and displays little evidence of its initial state. Additionally, the maximum vorticity and enstrophy at smaller scales of motion increase throughout the simulation, both because of the initial cascade from larger to smaller scales and because enstrophy is larger at the smaller scales within an inertial range of turbulence (an inertial range of turbulence having a one-dimensional

(1D) energy spectrum of the form $E(k) \sim k^{-5/3}$ also has shear or enstrophy spectra that vary as $S(k) \sim k^{1/3}$). The dynamics accounting for this further evolution toward a more chaotic and isotropic vortex field are reviewed in the remainder of this section.

b. Primary and Secondary Vortex Dynamics

The processes that appear to govern the evolution of the vortex field and the cascade of energy and enstrophy to smaller scales of motion within a breaking gravity wave have been discussed in detail by Fritts et al. (1998). Here, we will simply summarize these findings and illustrate the key processes of these dynamics. A number of these processes can be seen to occur in the latter stages of the full vorticity evolution (Plate 2) or more clearly in the closeup images of the vortex field spanning the evolution from Plate 2d to 2f in Plate 3 (and compensated for mean advection). Based on our assessments of importance, the dominant processes include 1) orthogonal vortex stretching, 2) vortex sheet and tube formation, and 3) vortex-core oscillations, termed Kelvin waves or twist waves, exhibiting azimuthal wavenumber 0 and 2 structures. Processes that also occur frequently, but appear to be of secondary importance to the turbulence casade, include 1) vortex reconnection, 2) twist waves exhibiting azimuthal wavenumber 1 structure, and 3) vortex pairing.

1. Orthogonal vortex stretching

The key element in vortex intensification on short time scales is vortex stretching, either of the ambient shear field by localized vortices or of localized vortices by other localized and closely spaced vortex structures. To be effective, the local component of vorticity, ω_i, and the diagonal component of the strain tensor, S_{ii}, must contribute a source, $\omega_i S_{ii}$, which exceeds other baroclinic, compressional, and viscous contributions. This stretching source is especially strong when the vorticity lines of two neighboring vortex structures are nearly orthogonal. Physically, the stretching is due to a flow divergence along the vorticity. Noting that the vorticity lines of the vortex sheets are predominantly spanwise (vertical in Plate 2) and that the vorticity lines of a vortex tube are roughly along the tube, we see that there are many sites where vortex structures in Plate 2 have approximately orthogonal alignments and undergo local intensification. When the stretching involves a vortex sheet, roll-up and tube formation can result, as discussed above.

2. Vortex sheet and tube formation

Formation and intensification of vortex sheets and their subsequent roll-up into vortex tubes were identified as initial steps in the transition to and within turbulent flow by Betchov (1957) and Lundgren (1982). Our wave breaking results and the previous discussion of Plate 2 show that these processes are also of central importance in a stratified fluid. Sheet roll-up occurs where they have intensified due to vortex stretching by the flow of a neighboring orthogonal vortex. The roll-up proceeds as described above, whether as part of the initial turbulence transition or the subsequent flow evolution.

3. Mode-zero twist waves

As observed in the latter evolution displayed in Plate 2, vortex loops arising from secondary KH instability tend to fragment into pieces. One of the causes of this fragmentation is the presence of mode-zero (azimuthal wavenumber $m = 0$) twist waves of large amplitude. Examples of mode-zero twist waves can be seen on many of the spanwise-aligned vortices in Plates 2 and 3. In each case, the mode-zero twist waves are initiated through axial stretching and compression of a vortex by a near-orthogonal neighbor. In the case of many of the spanwise vortices, mode-zero twist waves appear to be excited at both ends of the vortex simultaneously (see the top two images of Plate 3), to propagate inward from the sites of excitation, and to jointly cause an axial convergence which weakens the vortex to such an extent that its center is obliterated. The prevalence and consequences of mode-zero twist waves in our simulation suggests this to be a key mechanism for vortex breakup and the evolution toward smaller scales of motion.

The propagation and characteristics of twist waves were first described by Kelvin (1880), while Arendt et al. (1997b) recognized the role of twist waves in the interpretation of our wave breaking vortex dynamics. In general, twist waves are dispersive traveling waves on vortex tubes. The axisymmetric (mode-zero) modes propagate by twisting the vortex lines of the tube, thereby creating an axial flow. The axial flow changes the enstrophy of the tube by axial stretching and compression. This, in turn, changes the rotation rate of the tube which then changes the twist.

4. Mode-two twist waves

A second type of twist wave that plays a major role in the fragmentation of vortex loops is a mode-two (azimuthal wavenumber $m = 2$) wave. These waves are typically excited where a vortex tube is perturbed near its end by another vortex structure (either a tube or sheet). The clearest and most long-lived examples of mode-two twist waves in our numerical solution arise due to upstream perturbations of the more axially-uniform streamwise vortex tubes (see the upper left portions of Plate 2e to 2h and Plate 3). In each case, a mode-two twist wave with a small amplitude is excited on a vortex tube and propagates via self advection, though the propagation of the wave is difficult to quantify because of streamwise flow advection. The waves also grow until they are sufficiently large to unravel the vortex tube on which they propagate into a pair of intertwined helices.

The propagation of a mode-two twist wave excited by a Gaussian pinch is shown from two perspectives at several times (in circulation time units) in the lower panel of Plate 4. Note that the direction of propagation is determined by the direction of the vorticity in the tube and the sense of helicity of the wave packet. In these images, vorticity is to the right and the velocity field due to the vortex tube and its perturbation imposes rightward phase motion for left-handed helicity and leftward phase motion for right-handed helicity. The most conspicuous mode-two twist waves seen at the upper left of the images in Plate 2 have left-handed helicity and positive-x vorticity (Plate 2f) and right-handed helicity and negative-x vorticity (Plate 2h) therefore both propagate in the positive-x direction. The propagation of mode-two twist waves in the wave breaking simulation is discussed in greater detail by Arendt et al. (1997a, b).

5. Secondary vortex dynamics

Vortex dynamics identified as present, but less significant, to the turbulence evolution includes vortex reconnection, mode-one twist waves, and vortex pairing. Vortex reconnections play a clear role in our wave breaking simulation, but they typically change the topology, rather than the scale or the intensity of the vortex structures. Apart from the interesting dynamics of reconnection itself, however, these events appear to play a minor role in the turbulence evolution.

Mode-one (azimuthal wavenumber $m = 1$) twist waves perturb a vortex tube into a single helix and propagate through self-advection, as in the mode-zero and mode-two cases. The propagation of a mode-one twist wave excited by a Gaussian displacement to a vortex tube is illustrated from two perspectives at several times in the upper panel of Plate 4. A clear example of a mode-one twist wave in our simulation is on the vortex slanted at $\sim 45°$ at the lower right of the expanded images in Plate 3. The mode-one twist wave is apparently excited by the close orthogonal neighboring vortex underlying it at its lower end. The flow of this orthogonal neighbor bends the tube initially into a single wiggle, and the self-dynamics of the tube transforms the single wiggle into a full helix thereafter. Apart from increasing flow complexity, however, mode-one twist waves appear to contribute little to the turbulence cascade.

Vortex pairing is the mutual (merging) interaction of two parallel vortex tubes and is understood reasonably well on the basis of 2D laboratory and numerical studies. We observe several cases of vortex pairing in our 3D results. Generally they resemble their 2D analogs, with the vortices approaching closely, mutually advecting, and wrapping around each other. The pairing proceeds between adjacent loops, with one loop being advected over and within the other loop (because of the direction of vorticity in the initial vortex sheet). Pairing thus resembles leap-frogging vortex rings. In our simulation, other processes often occur on faster timescales than pairing, thus relegating it to minor importance. It is possible, however, that the pairing events facilitate other vortex interactions (i.e., twist waves) by driving more intense vortex structures closer together.

c. Turbulent Mixing

The results discussed here provide some qualitative insights for mixing due to wave breaking. However, a more complete assessment must await our current higher-resolution studies which are addressing much less viscous flows. Though turbulence evolves quickly, with most of the turbulence transition and cascade occupying less than a buoyancy period, the region of active mixing and transport is not confined to the location of initial instability. Instead, the turbulence is advected out of the unstable phase of the wave motion and evolves thereafter in stable stratification. Thus, except at the earliest stages, vertical transport tends to be down gradient and no region of the flow appears to remain well mixed at later stages. This aspect of turbulence due to wave breaking contrasts sharply with the mixing and transport accompanying KH instability discussed below. In this simulation, turbulent diffusion comprises molecular diffusion across large-amplitude, small-scale structures (and shears). The role of vortices in turbulent diffusion is simply to efficiently bring fluids of different character or composition into close proximity, thus greatly enhancing the molecular diffusion operative on otherwise much smaller gradients. While it is too early to advertise the results of higher-resolution simulations, current results appear to support the implications of earlier theoretical efforts suggesting inefficient mixing and a turbulent Prandtl number greater than 1 (Fritts and Dunkerton, 1985; Coy and Fritts, 1988; McIntyre, 1989). We expect, however, because turbulence does persist after being advected out of the unstable phase of the wave motion, that non-negligible mixing will occur under stably-stratified conditions with largely vertical gradients of heat, momentum, and constituents.

5. KH Shear Instability Dynamics and Mixing

While KH shear instability due to inertia-gravity wave motions in the stratosphere has been inferred from observations and simulated in numerical models (Fritts and Rastogi, 1985; Coulman et al., 1995; LeLong and Dunkerton, 1998a, b), these methods provide few insights into the implications for turbulence and mixing. Instead, such an understanding relies on numerical simulations focussing specifically on KH instability dynamics, due to their enormous demand for computational resources. Thus, we must make some assumptions about the environment in which KH instability arises and the generic effects of such events in other environments. Further studies will be required to assess the validity of this approach.

A great deal has been learned about the dynamics of KH instability in recent years. Theoretical and laboratory studies have contributed significantly to our understanding of the underlying 2D dynamics of unstable shear flows and of the transition to 3D flow in stratified and unstratified fluids (see Thorpe, 1987, for a review). However, it has required high-resolution numerical studies to describe the transition to turbulence and the resulting mixing more completely. Though a number of authors have addressed the transition to 3D accompanying KH instability (Metcalfe et al., 1987; Palmer et al., 1994, 1996; Caulfield and Peltier, 1994; Fritts et al., 1996b), only the study by Werne and

Fritts (1999a, b) has captured the emergence of an inertial range of turbulence following instability to date.

The KH simulation described by Werne and Fritts was performed for an environment having uniform stratification and a velocity profile of the form $\bar{u}(z) = u_0 \tanh(z/h)$, with $Ri = N^2h^2/u_0^2 = 0.05$, $Re = u_0h/\nu = 2000$, and $Pr = \nu/\kappa = 1$, where ν and κ are kinematic vicosity and thermal diffusivity. The simulation employed a spectral formulation of the incompressible Navier-Stokes equations solved in a horizontally-periodic domain of wavelength $L = 4\pi h$, corresponding to the most unstable linear eigenfunction of the viscous equations, with domain dimensions (1, 0.3, 2) L and spectral resolution of (720, 240, 1440) for the most vigorous portion of the turbulence evolution. Additional details about the forcing are provided by Werne and Fritts (1999a).

a. Initial Vorticity Dynamics

The evolution of the flow is shown in Plate 5 with streamwise-vertical cross sections of enstrophy and potential temperature at times of $t = 60$, 90, 160, and 300. Times are in units of h/u_0, where $t = 28$ corresponds to a buoyancy period. The left panels show the initial instability in the outer billow, the expansion of turbulence throughout the billow core, and the horizontal homogenization and decay of the turbulence layer. The right panels show the corresponding evolution of the thermal field. Note, in particular, the strong thermal gradients within the billow prior to vigorous turbulence and mixing. As turbulence expands, however, thermal gradients within the billow are quickly annihilated, leaving a nearly homogeneous billow core with sharp thermal gradients at the edges of the mixed region. As the turbulence expands horizontally, the mean profile takes on similar structure, again confining large thermal gradients to the edges of the mixed layer.

Vortical structures arising within the KH billow are displayed in Plate 6 in a volumetric fashion at times of 60, 70, 90, 105, 120, and 160 spanning the transition to turbulence and approach to isotropy. Enstrophy is used to display the vorticity field in these plates because it depicts vortex structures having both tube-like and sheet-like character. At early times, the flow is seen to trigger streamwise-aligned (vortex cores along the 2D flow) convective rolls downstream of the entrainment regions of the outer billow (see the initial vortex structures at $t = 60$ in Plate 6), consistent with earlier theoretical predictions and laboratory and numerical studies (Thorpe, 1987). Unlike earlier numerical KH studies that were viscously constrained, the subsequent evolution reveals an increasing isotropy and an expanding range of scales of turbulence enabled by the higher initial Reynolds number of the flow. The effective turbulence Reynolds number defined in terms of the turbulence layer width, $\sim$ 6h, the velocity difference, $\sim 2u_0$, and the kinematic viscosity is $Re_{turb} \sim 24{,}000$.

Following secondary convective instability, the vortex structures become increasingly 3D, largely through the interactions of adjacent vortices. These interactions parallel in important respects those observed in our studies of vortex dynamics accompanying wave breaking discussed above. As in wave breaking, KH billow roll-up results in adjacent regions of convectively unstable and sheared stable fluid, arising in large part from baroclinic tendencies acting on fluid being entrained into the billow core (Fritts et al., 1996b). The dynamics of adjacent convectively unstable and sheared stable layers manifest themselves in two ways. Where counter-rotating streamwise convective rolls are closely spaced, they undergo mutual advection and continued intensification via streamwise stretching. The convective rolls also lead to spanwise divergence at the locations of adjacent spanwise vortex sheets, leading to spanwise-localized thinning and intensification of these sheets. As in wave breaking, the spanwise vortex sheets are wrapped around the streamwise convective rolls accounting for sheet intensification. Intensification and thinning occurs on both the upper spanwise vortex sheet and on the (countersign) vortex sheet on which the streamwise-aligned convective rolls arise.

Spanwise-localized intensification of the vortex sheets has two effects in our KH simulations. It causes the vortex sheets to become dynamically unstable to secondary (spanwise-localized) KH instability. The vortex sheets in turn perturb the streamwise vortices, resulting in the excitation of twist waves at larger scales. The evolution of a vortex sheet in the lower portion of the KH billow which exhibits several of these responses in shown in the left images of Plate 7. The relative importance of these processes is observed to be very sensitive to initial conditions for the 3D flow. In the entrainment regions, the deformation and roll-up of the spanwise vortex sheets leads to attachment of the emerging spanwise vortices to the streamwise vortices accounting for sheet intensification. Where the vortex structures are more uniform along the streamwise flow, the result is a series of spanwise vortices and the attachment of each to the associated streamwise vortices (see the second and third images on the left in Plate 7), much as described above for the transition to turbulence in wave breaking. Where streamwise variations are greater, twist waves on the streamwise vortices are more readily excited (see below). In either case, the tendency for vortex sheets to intensify and evolve toward vortex tubes is widespread and operates at many scales. Though there are vortex sheets that either survive to later times or continue to be created via stretching, the majority of the vorticity at later stages of the turbulent flow is associated with vortex tubes.

b. Dynamics of the Turbulence Cascade

As discussed above, it was proposed previously that twist waves provide the primary means by which energy and enstrophy are cascaded toward smaller scales within a turbulent flow (Arendt et al., 1997b; Fritts et al., 1998). The present results provide the first opportunity to test this hypothesis and establish the extent to which the same cascade dynamics prevail when dissipation does not play a primary role on scales at which vortices arise (see the discussion in Sec. 2d above).

In the KH simulation, near-orthogonal vortex alignments arise due to the secondary and tertiary flow instabilities discussed above. Close proximity of adjacent vortices then leads to vortex perturbations which excite various twist wave modes. As shown in Plate 4, twist waves manifest themselves as vortices exhibiting axial variations in vorticity and axial velocity (mode zero) and single or double helices (mode one and mode two). Of these, the mode-one and mode-two helical structures are easiest to identify, though mode-zero twist waves are surely present and may play a greater role in vortex breakdown. In each case, the mechanism of propagation is self advection, as discussed above; helicity implies a component of induced velocity along the vortex tube which accounts for the phase propagation of the twist wave.

The first clear evidence of twist-wave vortex dynamics in the KH simulation can be seen in Plate 7 accompanying vortex-sheet intensification and secondary KH formation, which shows the evolution of the vortex field at the bottom of the billow for times of $t = 78$ and thereafter. A mode-one twist wave having right-handed helicity is seen to emerge on a vortex along the left top of the images from $t = 79.2$ to 80.3. Note that the twist wave amplifies with time, while undergoing an interaction with a neighboring vortex at later stages. A second mode-one twist wave with the same helicity is observed on a much larger scale at the right end of the vortex sheet in the first two images on the left in Plate 7.

An example of mode-two twist wave dynamics is displayed in the right panels of Plate 7. This sequence shows the evolution from $t = 95.2$, where mode-two twist wave excitation is due to the perturbation of the initial vortex by its neighbor. Following excitation, the direction of propagation is to the left because of the sign of the helicity and vorticity.

Like the vortex dynamics within a breaking wave then, the generation and evolution of turbulence due to KH instability involves a succession of vortex dynamics. Following secondary convective instability and tertiary KH instability of spanwise-localized vortex sheets, the vorticity field is complex, 3D, and composed primarily of vortex tubes. The subsequent vortex interactions drive the flow toward ever smaller scales and increasingly complex vortex structures. Sheets are stretched and roll up to form new tubes; tubes interact and excite twist waves which unravel and fragment the parent tubes until they are arrested by viscous and diffusive effects.

c. Turbulent Mixing

The effects of momentum and heat transport and turbulent mixing are shown in Figure 1 with spanwise-averaged vertical profiles of $T(z)$, $u(z)$, and Ri at various times. Profiles of $T(z)$ and $u(z)$ are shown at the streamwise midpoint of the billow (solid lines) and through the braid between adjacent billows (dashed lines) in Figure 1a and 1b at $t =$ 50, 60, 70, 90, 105, 120, 160, and 300. Corresponding profiles of Ri determined from the (streamwise and spanwise) mean profiles, $\bar{T}(z)$ and $\bar{u}(z)$, are shown in Figure 1c. Profiles of mechanical and thermal energy dissipation rates, ϵ and χ, are displayed at these same times in Figure 2a and 2b.

Mixing accompanies vigorous vortex dynamics, obliterates thermal gradients within the billow core, and pushes large-scale thermal and velocity gradients to the edges of the turbulent layer. Mixing at the center of the shear layer is thus fairly complete, but remains confined to the initial shear layer. This is in contrast to mixing due to gravity wave breaking, which largely follows the unstable phase of the wave motion, is thus less complete, but extends over a larger vertical depth.

The edge regions of KH billows and the resulting turbulence layer remain sites of active entrainment dynamics at smaller scales following mixing of the billow cores. They are also the regions of KH billow turbulence most easily observed using radar and acoustic sounding methods in atmospheric and oceanic measurements of such dynamics because of their high refractive-index gradients (Eaton et al., 1995; Seim and Gregg, 1995; Hill et al., 1999; Gibson-Wilde et al., 2000). However, the mixing accompanying these tertiary instabilities is less complete because of the much smaller energy dissipation rates at these sites. This causes a weakening and spreading of the edge gradients, and of the associated constituent profiles, until the small-scale structures have decayed and the flow has restratified (see the last images in Plate 5 and Figures 1 and 2.

The implications for the inertia-gravity wave motions within which KH billows arise are presumably similar, though the simulations to define this have yet to be performed. When KH instability dynamics are vigorous, we expect this to result in a well mixed layer and accompanying momentum, heat, and constituent transport. Because such events are spatially localized, they will alter the inertia-gravity wave structure locally, likely impacting wave amplitude only weakly and causing a readjustment of the flow via radiation of other waves at scales comparable to the event scale. Successive KH mixing events would seem likely accompanying readjustment, though this is speculative at this time.

6. Summary and Conclusions

Our purposes in this paper were twofold. We first provided evidence that the character of gravity wave instability undergoes a systematic variation with altitude. We then described in some detail the dynamics of the transition to turbulence accompanying wave breaking and KH shear instability and, to the extent presently possible, the implications for turbulent mixing.

The character of the gravity wave spectrum and its variations with altitude suggest that wave energy dissipation increases more rapidly than wave energy density (per unit mass) as altitude increases. This is because wave energy flux varies as the product of wave energy density, E, and vertical group velocity, c_{gz}, with the vertical group velocity itself increasing as $c_{gz} \sim m_*^{-1/2} \sim E^{1/2}$. Thus, as E increases by ~ 100 from the tropopause to the mesopause, the energy flux increases by ~ 1000, causing energy to be removed from the wave field ~ 10 times more quickly at the higher altitudes. The corresponding time scales for wave energy dissipation (assuming all removed energy cascades to turbulence) are ~ 30 to 100 hours near the tropopause and ~ 3 to 10 hours near the mesopause.

The implications of these estimates are a tendency for slower, systematic KH shear instabilities due to lower-frequency motions in the lower stratosphere and a tendency for more rapid convective instabilities of higher-frequency wave motions at greater altitudes. Radar and balloon observations in the stratosphere over many years and more recent optical measurements at mesopause altitudes provide supporting data for these inferences.

Our second purpose was to describe the dynamics of the turbulence transition and the turbulence cascade accompanying both wave breaking and KH shear instability. This included a review of previous, relatively low-resolution simulations of wave breaking and more recent, higher-resolution studies of both wave breaking and KH instability. Remarkably, the turbulence evolutions in the two flows appear to be parallel in many respects. The clearest examples are the similar vortex sheet dynamics displayed in the upper images in Plate 2 and the left images in Plate 7 and the excitation of twist waves throughout both simulations.

In both wave breaking and KH instability, the transition to 3D structure is triggered by an instability comprising counter-rotating streamwise (or shear-aligned) rolls where the 2D flow is convectively unstable. In both flows, these streamwise vortices occur adjacent to spanwise vortex sheets due to mean and perturbation shears. The interactions of adjacent streamwise vortex tubes and spanwise vortex sheets causes local (spanwise) intensification of the vortex sheets, a wrapping of these sheets around pairs of streamwise vortex tubes, and a secondary shear (KH) instability of the intensified sheets. This results in additional spanwise vortices which link with the streamwise vortices responsible for their formation, yielding groups of intertwined vortex loops at each site of initial spanwise shear intensification.

The evolution subsequent to intertwined loop formation is rapid and is driven by interactions among vortex loops (and sheets to a lesser extent) in close proximity. These interactions excite perturbations on adjacent vortices that are most naturally described as twist waves which propagate along the vortex tubes and may attain large amplitudes. Mode-zero and mode-two twist waves (those with azimuthal wavenumbers 0 and 2) appear to play a large role in the breakup and fragmentation of vortex tubes and the cascade of energy and enstrophy to smaller scales of motion. Mode-zero twist waves introduce large axial flow variations which lead to vortex bursting; mode-two twist waves effectively unravel larger vortices into two intertwined vortices of smaller size. Other vortex interactions, primarily mode-one twist waves, vortex pairing, and reconnection also occur, but are judged to be less significant in the evolution of the vorticity field.

While their turbulence dynamics are similar, the implications of turbulence for mixing and transport due to wave breaking and KH instability are quite different. Wave breaking leads to turbulence which is not confined to the initial layer of instability, thus contributing to down-gradient transport in other portions of the wave field due to advection of the turbulence (or more correctly the propagation of the initially unstable wave phase) away from the site of turbulence generation. KH turbulence, on the other hand, leads to efficient mixing of the layer of initial billow development, with little or no influence elsewhere.

Though a complete evaluation of the relative roles of these processes in mixing and transport must await higher-resolution studies quantifying the statistical effects of a series of such events, we can at least speculate on their likely roles based on simulations to date. To the extent that mixing by KH instability is fairly complete within the shear layer, and that successive shears tends to occur where stratification is largest, this suggests that such mixing should be fairly efficient relative to the small energy dissipation rates characteristic of the lower stratosphere. Mixing due to wave breaking is likely most vigorous within the unstable phase of the wave motion where vertical gradients are weak, but will also extend with smaller ϵ (hence smaller intensity and vertical depth) into the stable phase of the wave motion. This suggests less efficient mixing than for KH instability of the same intensity (comparable ϵ) and scale. However, wave breaking dominates turbulence generation at greater altitudes and for larger vertical energy fluxes, group velocities, and vertical scales. Hence, such events are likely capable of mixing and turbulent transport over extended vertical (and horizontal) scales at the larger values of ϵ anticipated at higher altitudes. Both ϵ and mixing efficiency are likely to be highly dependent on the amplitude of the unstable wave, with sporadic, large-amplitude events accounting for more mixing and transport than continuous weaker events yielding the same mean wave energy densities. The tentative implication for parameterization in large-scale models is more efficient mixing (for given ϵ) at lower altitudes where KH instability is predominant, with decreasing efficiency at higher altitudes where wave breaking is predominant.

More quantitative descriptions of turbulent mixing will require further simulations and analysis, but are now feasible with current computational capabilities. Numerical tracer advection and dispersion studies are anticipated and will offer a potential to compare with similar studies in the laboratory and the atmospheric boundary layer. Together, these capabilities promise significant improvements in our understanding of, and parameterization of, turbulent mixing and diffusion in the near future.

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Plate and Figure Captions

Plate 1. Isosurfaces of potential temperature for the wave breaking simulation shortly after initial convective instability (left) and approximately one buoyancy period later (right). The streamwise direction (wave propagation direction) is out of the page and to the right.

Plate 2. Volume renderings of vortex cores from below with positive x to the right spanning ~ 1.4 buoyancy periods following initial convective instability left to right and top to bottom. Color and opacity scales are shown at the bottom. The opacity scale extends from transparent (bottom) to opaque (top); both scales show minimum pressure on the left.

Plate 3. As in Plate 2 for a subdomain at the upper left of the full domain including $\sim 1/3$ of the domain in x and $1/2$ of the domain in y and extending from the image in 1d to 1f. The subdomain is advected with the mean 2D motion to track individual vortex structures. Color and opacity scales are shown at the bottom.

Plate 4. Propagation and dispersion of mode-one (top) and mode-two (bottom) twist waves excited by a Gaussian pinch and displacement, respectively. Times are in units of circulation times. Note that modes having opposite helicity propagate in opposite directions.

Plate 5. Streamwise-vertical cross sections of vorticity magnitude (left) and potential temperature (right) showing the evolution of a KH billow from the onset of 3D instability through the expansion of turbulence, the cascade toward smaller scales of motion, and the restratification of the flow as turbulent motions subside. Times are 60, 90, 160, and 300, top to bottom.

Plate 6. Perspective volumetric views of vorticity magnitude throughout the evolution and turbulent breakdown of a KH billow. Times are 60, 70, 90, 105, 120, and 160 in units of h/U_0, small values of vorticity are transparent or blue, large values of vorticity are yellow or red, and only the central 1/3 of the vertical domain is displayed.

Plate 7. As in Plate 6, but showing closeup views of mode-one and mode-two twist waves at two stages of the evolution. The left panels show the mode-one twist wave seen

occurring on a vortex sheet along the bottom of the billow at times of $t = 78.0$, 79.2, 80.3, and 82.1. The right panels show the initiation and amplification of a mode-two twist wave occurring from $t = 95.2$ to 97.5. Colors and opacities are as in Plate 6.

Figure 1. Spanwise-averaged vertical profiles of $u(z)$ (top), $T(z)$ (middle), and Ri (bottom) at times of $t = 50$, 60, 70, 90, 105, 120, 160, and 300. Profiles of $u(z)$ and $T(z)$ are shown at the streamwise midpoint of the billow (solid lines) and through the braid between adjacent billows (dashed lines). Ri profiles are derived using streamwise and spanwise mean profiles, $\bar{u}(z)$ and $\bar{T}(z)$.

Figure 2. Streamwise- and spanwise-averaged profiles of energy and thermal dissipation rates, ϵ and χ (top and bottom, respectively), at times of $t = 50$, 60, 70, 90, 105, 120, 160, and 300. Note that ϵ remains large at the center of the turbulence layer, while χ quickly achieves its maxima at the edges because of rapid obliteration of thermal gradients by vigorous mixing within the layer. Successive profiles are offset by a uniform increment so as to display the varying structure and magnitude of the kinetic and thermal dissipation with time.