Turbulent rotating Rayleigh-Bénard convection (with comments on $\frac{2}{7}$)

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1 Introduction

Consider a uniformly rotating Boussinesq fluid confined between horizontal plates. If the lower plate is hotter than that above, the fluid in between will experience a buoyant force that may drive it to convective motion. For a sufficiently large temperature drop between the plates, intense plumes (which themselves generate shear and its associated turbulence) spontaneously develop. The ensuing dynamics of these plumes, their mutual interaction, and global transport properties are all affected directly, or indirectly, by the rate of rotation of the fluid layer. My discussion will concentrate on these effects, and will report numerical solutions which illuminate similarities to and differences from non-rotating convection.

2 Non-rotating turbulent convection

2.1 Experimental findings

Non-rotating Rayleigh-Bénard convection is a classic problem experiencing a resurgence recently due to experimental observation of some remarkable and unexpected collective behavior of its plumes. A fundamental question involves the heat transport through the layer for a specified temperature drop. Heslot, Castaing & Libchaber (1987) reported the stabilization (*i.e.*, consistent circulation in a particular direction) and reduced heat transport of a single convection cell once thermal plumes appear, while Krishnamurti (1995) discusses the generation of mean shearing motions by plumes which cluster together and march in "soldier-row" formation across a large $(40 \times 40 \times 1)$ experimental cell. Krishnamurti's large-scale turbulent patterns exhibit an even more drastic reduction in the heat transport than that seen by Heslot *et al.*

With these new observations comes the theoretical challenge of explaining the reduction in the heat transport due to the presence of plumes (Castaing *et al.* 1989; She 1989; Shraiman & Siggia 1990; Yakhot 1992). I will review two of four existing theories which attempt to explain the fundamental physical behavior of laboratory plumey convection, which in recent years has acquired the name "hard turbulence:" Castaing *et al.* 1989 and Shraiman & Siggia 1990. In particular, these theories strive to derive the observed heat-transport law, $Nu \propto Ra^{2/7}$.[1] Here $Nu = H/(\kappa\Delta/L)$ is the Nusselt number or normalized heat flux H, and $Ra = g\alpha\Delta L^3/(\nu\kappa)$ is the Rayleigh number or normalized temperature drop Δ. g and α are the local acceleration due to gravity and the fluid's thermal expansion coefficient. L is the layer depth.

[1] While Libchaber's group repeatedly observes the 2/7-law for turbulent convection in both small $(A = 1)$ and large $(A = 6.7)$ containers — A is the width-to-height or aspect ratio — Krishnamurti's recent results favor a 1/4-law. Aside from geometry (K's apparatus is rectilinear while L's is cylindrical), the major difference between the experiments is the fluid: liquid water $(\sigma \approx 7)$ for K and Helium gas $(\sigma \approx 0.7)$ for L, where the Prandtl number $\sigma = \nu/\kappa$ is the ratio of fluid viscosity ν to thermal diffusivity κ. The much shorter diffusion time, and hence the effectively longer experimental run-time for water (and/or the value of σ) may play a role in establishing K's clustered rows of plumes. As yet no turbulent heat-transport theory attempts to explain K's observed 1/4-law.

2.2 Heat-transport theories

The first of these theories extends earlier notions of Priestley (1959) (also Malkus 1963; Howard 1966) which characterize turbulent convection as efficient and advective in the bulk, but solely diffusive near the boundaries where the velocity is zero. The thin "boundary layers" each span half of the temperature drop Δ, giving $H = \kappa\Delta/(2\lambda) \rightarrow Nu = L/(2\lambda)$ where λ is the boundary-layer thickness. Priestley's remaining assumption is a depth L (or equivalently Ra) sufficiently large that $\lambda \neq \lambda(L)$, resulting in $Nu \propto Ra^{1/3}$ which has actually been observed for Ra below that at which the 2/7-law emerges (Wu *et al.* 1988). Nevertheless, the large range over which the 2/7-law has been observed opens an ever-widening gap from the 1/3-law, the ratio of the two laws exceeding 2.4 at $Ra = 10^{15}$. The contribution by Castaing *et al.* was to conjecture a "mixing zone" where plumes grow out of the boundary layers, then mix through turbulent agitation before merging with the bulk fluid. They propose 1) that plume dynamics is dominated by viscous drag $\nu u_c/\lambda^2 \approx g\alpha\Delta$, and 2) that turbulent convective motions are as Prandtl (1932) suggests, advective $H = u_c\Delta_c$ with kinetic energy of the dominant eddies gained at the expense of buoyancy work as they traverse the layer $u_c^2/2 \approx g\alpha\Delta_c L$. Here u_c and Δ_c are the characteristic velocity and temperature contrast of the dominant turbulent eddies; $\Delta/2$, u_c, and λ are estimates of a plume's temperature, final velocity, and thickness. By solving this system of four equations for the four unknowns (H, λ, u_c, and Δ_c), $Nu \propto Ra^{2/7}$, as well as other scaling laws, is obtained.

Shraiman & Siggia also derive the 2/7-law, but now using only the one feature of hard turbulence absent from the picture proposed by Castaing *et al.*: the stable large-scale circulation. In fact, Shraiman & Siggia propose the 2/7-law emerges precisely when the dominant circulation is sufficiently fast to produce shear-flow turbulence near the boundaries. They estimate the dissipation in the layer as $\epsilon \approx u_*^3/L$, where $u_* = \sqrt{\nu(\partial U/\partial z)_{wall}}$ is the "friction velocity," U being the mean velocity parallel to the boundary (wall) and z the normal direction. By suggesting a thermal boundary layer λ thinner than the viscous sub-layer adjacent to the wall, where the mean velocity profile is linear $U = z\,(\partial U/\partial z)_{wall}$, the heat equation can be simplified and the result $Nu \propto (\partial U/\partial z)_{wall}^{1/3}$ obtained. Employing the exact relation $\epsilon L^4/\kappa^3 = \sigma\, Nu\,(Ra - 1)$, the scaling laws $Nu \propto Ra^{2/7}$ and $(\partial U/\partial z)_{wall} \propto Ra^{6/7}$ result. Though the role of plumes appears secondary within Shraiman & Siggia's boundary-layer analysis, it is actually the collective action of the plumes as they establish the large-scale circulation which makes the turbulent shear possible. In addition, in the bulk fluid the plumes carry practically all of the heat flux.

The stark contrast between these two approaches points to the caution with which one must digest scaling theories for turbulent fluids: derivation of a known result does not guarantee the underlying assumptions sufficient to realize the behavior in nature. Even greater care is required when an *unverified* scaling law is "derived" through simple scaling arguments — but this is not the case here.

3 Rotating turbulent convection

Rotating the fluid layer alters the dynamics of plumes dramatically. For experiments or simulations with rotation rates Ω comparable to (or greater than) the convective velocity divided by L, individual plumes approximately conserve angular momentum as they move from the boundaries, amplifying cyclonic vorticity while contracting horizontally. In this way, buoyancy and vortex stretching share a mutual feedback, producing a flow-field dominated by strongly cyclonic plumes. Experiments by Rossby (1969) and Zhong, Ecke & Steinberg (1993) indicate that increased rotation enhances the heat transport slightly; they propose Ekman pumping as a possible cause.[2] Another important difference between rotating and non-rotating convection is the decrease in inter-plume spacing resulting from external rotation. Figure 1 shows the development of the flow field (temperature T and vertical vorticity ω) as Ra is increased while fixing the layer's convective Rossby number $Ro = \sqrt{Ra/(\sigma\, Ta)}$; $Ta = (2\Omega L^2/\nu)^2$ is the Taylor number.[3] This figure appears in Julien, Legg, McWilliams & Werne (1995); it depicts numerical solutions of rotating Rayleigh-Bénard convection with no-slip top and bottom and periodic side boundaries. The domain aspect ratio is $2 \times 2 \times 1$.[4] The line marked at the bottom of each column indicates the horizontal scale of convection predicted by linear stability (Chandrasekhar 1961). As Ra and Ta are increased, chaotic roll-convection gives way to isolated vortex/plumes with decreasing horizontal scale. At high Ra, plume-plume (or equivalently, vortex-vortex) interactions become increasingly likely, given higher plume-vorticity and shortened distance between neighbors. Because the plumes are primarily aligned vertically, the vortical interactions which ensue involve the co-rotation of cyclonic vorticity, with plumes circulating about each other in horizontal planes. The result is lateral stirring tending to mix the thermal content of a plume with its surrounding fluid, thus weakening the plume and intensifying the temperature of the background. The result is a mean temperature gradient not present for non-rotating convection. Figure 2 depicts this gradient which persists for increasing Ra at fixed Ro. (Note the thinning boundary layers near $z = \pm 1$.)

Despite these marked differences between rotating and non-rotating convection (vortical interactions, mean thermal gradient, Ekman pumping, *etc.*), the 2/7-law remains for simulations with no-slip vertical boundaries (figure 3). Stress-free boundaries, on the other hand, exhibit Priestley's 1/3-law. Conclusions then are 1) the detailed dynamics of plumes are relatively unimportant to establishing the 2/7-law, and 2) boundary details are crucial. Both of these conclusions are consistent with Shraiman & Siggia (1990), and the second casts doubt on the fundamental nature of

[2]Ekman pumping occurs when a vortex tube terminates at a solid boundary. For a tube in geostrophic balance (*i.e.*, with the radial pressure gradient balancing centrifugal acceleration), pumping of fluid away from the boundary occurs at the vortex core due to the fact that the fluid velocity (and hence the centrifugal acceleration) vanishes at the wall, creating a pressure imbalance.

[3]Ro is the ratio of the rotation timescale $(2\Omega)^{-1}$ to the convective timescale $\sqrt{L/(g\alpha\Delta)}$. For a given value of Ro, the influence of rotation on convection should be roughly constant.

[4]Simulations are conducted using a pseudo-spectral, Fourier-Chebyshev "tau"-method. The influence-matrix with tau-corrections is employed (Kleiser & Schumann 1980; Werne 1995). All simulations resolve the Kolmogorov length-scale for the turbulence.

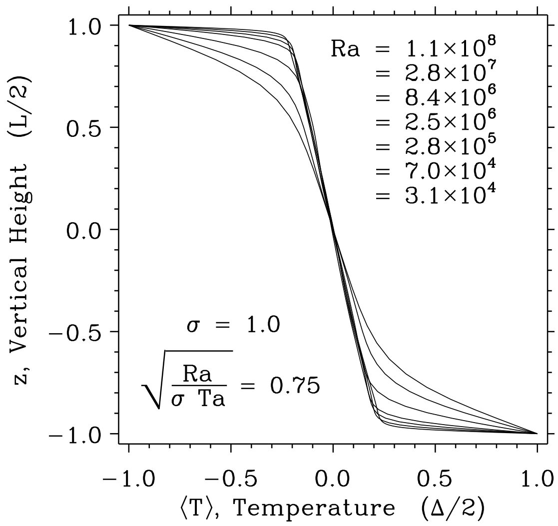

Figure 2: Temperature profiles.

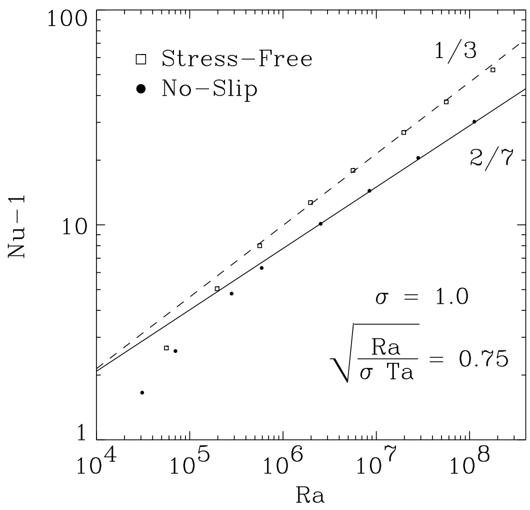

Figure 3: Heat transfer versus *Ra*.

Castaing *et al.* (1989), since that theory cannot distinguish stress-free from no-slip boundaries. The key point is that the majority of the heat transport through the boundaries takes place in strong shear regions between plumes, not at plume-sites. This is consistent with earlier findings (Werne 1993).

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